Widely-linear frequency-shift compensation of CFO and I/Q imbalance in OFDMA/SC-FDMA systems

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***Abstract*—This paper considers the problem of jointly compensating carrier frequency offsets (CFOs) and in-phase/quadrature-phase (I/Q) imbalance effects in orthogonal frequency-division multiple-access (OFDMA) or single-carrier frequency-division multiple-access (SC-FDMA) systems. Specifically, we propose a widely-linear time-varying minimum mean-output energy compensation scheme, by deriving its data-dependent frequency-shift implementation. The performance of the proposed receiver is assessed by Monte Carlo computer simulations.**

***Index Terms*—Carrier frequency offset (CFO), frequency-shift (FRESH) filtering, in-phase/quadrature-phase (I/Q) imbalance, orthogonal frequency-division multiple-access (OFDMA), single-carrier frequency-division multiple-access (SC-FDMA), widely-linear processing.**

I. Introduction

In recent years, multiple-access (MA) techniques based on orthogonal frequency-division modulation (OFDM), so called OFDMA schemes, have been thoroughly investigated. In an OFDMA system, according to a given carrier allocation scheme, each user is assigned a subset of orthogonal subcarriers. Major advantages of OFDMA include ease to cope with multipath effects through simplified equalization and resource allocation flexibility. For these reasons, OFDMA schemes have been incorporated in several wireless high-speed standards, such as IEEE 802.16 [1], 3GPP LTE [2], and the forthcoming cognitive-based IEEE 802.22 [3] standard. Moreover, a modification of OFDMA, known as SC-FDMA or SC-IFDMA [4], [5], has been adopted in the LTE uplink, because of its low peak-to-average transmit power ratio (PAPR) features.

Performances of OFDMA/SC-FDMA schemes are very sensitive to *carrier frequency offsets* (CFOs) between transmitters and receivers, which disrupt subcarrier orthogonality, giving rise to intercarrier interference (ICI) and multiple-access interference (MAI). Moreover, as in any bandpass modulation scheme, analog front-end *in-phase/quadrature-phase* (I/Q) imbalances [6] are further sources of degradation. Many CFO compensation schemes for OFDMA/SC-FDMA systems have been recently proposed, mostly with reference to the challenging uplink case (see [7] for a comprehensive survey). Moreover, I/Q compensation schemes for OFDMA/SC-FDMA systems have been proposed in [8], under the simplifying assumption of perfect time and frequency synchronization.

In the literature, less attention has been devoted to the problem of *joint* compensation of CFO and I/Q imbalance, which has been tackled (see e.g. [9], [10]) only in the less challenging single-user OFDM scenario. In [11], [12], we considered the uplink of an SC-IFDMA system operating over a highly-dispersive channel and affected by time and frequency errors, as well as transmitter I/Q imbalances. This work was later extended [13] in order to take into account and compensate for receiver I/Q imbalance as well. The widely-linear (WL) minimum mean-output energy (MMOE) based equalizers proposed in [13] were implemented under ideal conditions, i.e., by assuming perfect knowledge of all the second-order statistics involved in the receiver synthesis.

In this paper, we reformulate the synthesis of [13] in a more general OFDMA/SC-FDMA uplink scenario, by proposing new data-dependent implementations of the MMOE-based receivers. Particularly, since the equalization structures turn out to be time-varying, we resort to the frequency-shift (FRESH) [14], [15], [16] approach to reduce the computationally-intensive time-varying estimation problem into simpler time-invariant ones. The performance of the WL-MMOE-FRESH receiver is assessed through Monte Carlo simulations and compared with their ideal versions.

II. System model

Let us[1] consider the uplink of an OFDMA/SC-FDMA systems with M subcarriers, where $K \leq K_{\text{m}}$ active users are served by a common base-station (BS). Each user employs a disjoint set of $M_{\text{u}} \triangleq M/K_{\text{m}}$ subcarriers, and both user devices and the BS are equipped with single-antenna transceivers.

The data-block $\mathbf{s}_k(n) \in \mathbb{C}^{M_{\text{u}}}$ of the kth user in the nth symbol period, with $k \in \{1, 2, \ldots, K\}$ and $n \in \mathbb{Z}$, is precoded by an invertible matrix $\mathbf{F}_{\text{pre}} \in \mathbb{C}^{M_{\text{u}} \times M_{\text{u}}}$. The OFDMA scheme

[1]Besides standard notations, we adopt the following ones: for any pair of random processes $\mathbf{x}(n) \in \mathbb{C}^N$ and $\mathbf{y}(n) \in \mathbb{C}^M$, $\mathbf{R}_{\mathbf{xy}}(n) \triangleq \text{E}[\mathbf{x}(n)\,\mathbf{y}^H(n)] \in \mathbb{C}^{N \times M}$ and $\mathbf{R}_{\mathbf{xy}*}(n) \triangleq \text{E}[\mathbf{x}(n)\,\mathbf{y}^T(n)] \in \mathbb{C}^{M \times M}$ denote the (possibly time-varying) cross-correlation and conjugate cross-correlation matrices, respectively; $\text{symm}(\mathbf{A}) \triangleq \frac{1}{2}(\mathbf{A} + \mathbf{A}^T)$; $\text{hcat}(\cdot)$ and $\text{vcat}(\cdot)$ denote horizontal/vertical concatenation of matrices/vectors; $\otimes$ denotes Kronecker product; $\langle \cdot \rangle$ denotes temporal averaging.

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is obtained by setting $\mathbf{F}_{\text{pre}} = \mathbf{I}_{M_u}$, whereas SC-FDMA is obtained by choosing $\mathbf{F}_{\text{pre}} = \mathbf{W}_{\text{dft}}$, where $\mathbf{W}_{\text{dft}} \in \mathbb{C}^{M_u \times M_u}$ is the unitary M_u-point discrete Fourier transform (DFT) matrix. We assume that: **(a1)** $\mathbf{s}_k(n)$ is a zero-mean circularly-symmetric complex (ZMCSC) random vector, with correlation matrix $\mathbf{R}_{\mathbf{s}_k \mathbf{s}_k} = \sigma_s^2 \mathbf{I}_{M_u}$, and $\mathbf{s}_{k_1}(n_1)$ is statistically independent of $\mathbf{s}_{k_2}(n_2)$ for $k_1 \neq k_2$ and/or $n_1 \neq n_2 \in \mathbb{Z}$. The precoded block $\widetilde{\mathbf{s}}_k(n) \triangleq \mathbf{F}_{\text{pre}} \, \mathbf{s}_k(n) \in \mathbb{C}^M$ is subject to subcarrier mapping, modeled by matrix $\mathbf{P}_k \in \mathbb{R}^{M \times M_u}$. After mapping, the block $\mathbf{P}_k \, \widetilde{\mathbf{s}}_k(n) \in \mathbb{C}^M$ undergoes conventional OFDM precoding, consisting of an M-point inverse DFT (IDFT) followed by CP insertion. The resulting time-domain block is given by

$$\mathbf{u}_k(n) = \mathbf{T}_{\text{cp}} \, \mathbf{W}_{\text{IDFT}} \, \mathbf{P}_k \, \mathbf{F}_{\text{pre}} \, \mathbf{s}_k(n) \tag{1}$$

where $\mathbf{T}_{\text{cp}} \triangleq [\mathbf{I}_{\text{cp}}^T, \mathbf{I}_M]^T \in \mathbb{R}^{P \times M}$ describes the insertion of the CP of length L_{cp}, with $P \triangleq M + L_{\text{cp}}$ and $\mathbf{I}_{\text{cp}} \in \mathbb{R}^{L_{\text{cp}} \times M}$ obtained by collecting the last L_{cp} rows of $\mathbf{I}_M$, whereas $\mathbf{W}_{\text{IDFT}} \in \mathbb{C}^{M \times M}$ denotes the unitary M-point IDFT matrix. After digital-to-analog conversion operating at rate $1/T_c = P/T$, where T denotes the symbol period, the analog baseband signal obtained from $\mathbf{u}_k(n)$ is up-converted to radio-frequency using a local oscillator, which is prone to I/Q imbalance. The effects of frequency-flat I/Q imbalance on the kth transmitted signal is described [8] in the baseband model by assuming perfect up-conversion of a distorted version $\widetilde{\mathbf{u}}_k(n)$ of (1), i.e.,

$$\widetilde{\mathbf{u}}_k(n) \triangleq \alpha_k \, \mathbf{u}_k(n) + \beta_k \, \mathbf{u}_k^*(n) \tag{2}$$

where $\alpha_k \triangleq \cos(\delta\phi_k) + j\delta a_k \sin(\delta\phi_k)$ and $\beta_k \triangleq \delta a_k \cos(\delta\phi_k) - j \sin(\delta\phi_k)$, with δa_k the amplitude and $\delta\phi_k$ the phase mismatch of the kth transmitter.

After propagation over the wireless channels, the received signal at the BS is down-converted to baseband, sampled at rate $1/T_c$, and subject to CP removal to suppress IBI. In the absence of I/Q receiver imbalance, the vector $\mathbf{r}(n) \in \mathbb{C}^M$ of the baseband received samples can be written as

$$\mathbf{r}(n) = \sum_{k=1}^{K} e^{j \frac{2\pi}{M} \epsilon_k n P} \, \mathbf{\Omega}_k \, \mathbf{H}_k \, \widetilde{\mathbf{u}}_k(n) + \mathbf{w}(n) \tag{3}$$

where $\epsilon_k = \Delta f_k M T_c$ denotes the normalized CFO of the kth user ($|\epsilon_k| \leq 0.5$), $\mathbf{\Omega}_k \triangleq \text{diag}[e^{j \frac{2\pi}{M} \epsilon_k L_{\text{cp}}}, e^{j \frac{2\pi}{M} \epsilon_k (L_{\text{cp}}+1)}, \ldots, e^{j \frac{2\pi}{M} \epsilon_k (L_{\text{cp}}+M-1)}] \in \mathbb{C}^{M \times M}$, $\mathbf{H}_k \in \mathbb{C}^{M \times P}$ is the lower-triangular Toeplitz channel matrix, whose first column and row are, respectively, $[h_k(L_{\text{cp}}), 0, \ldots, 0]^T$ e $[h_k(L_{\text{cp}}), h_k(L_{\text{cp}} - 1), \ldots, h_k(0), 0, \ldots, 0]$, where $h_k(\cdot)$ is the impulse response of the discrete-time channel, modeled as finite-impulse response (FIR) filter, whose maximum order $L_{\max}$ is assumed to not exceed the CP length, i.e., $L_{\max} \leq L_{\text{cp}}$, and $\mathbf{w}(n) \in \mathbb{C}^M$ accounts for thermal noise. It is assumed that: **(a2)** $\mathbf{w}(n)$ a Gaussian ZMCSC random vector, statistically independent of $\mathbf{s}_k(n)$, with time-invariant correlation matrix $\mathbf{R}_{\mathbf{ww}} = \sigma_w^2 \, \mathbf{I}_M$, and $\mathbf{w}(n_1)$ statistically independent of $\mathbf{w}(n_2)$ for $n_1 \neq n_2$.

Similarly to (2), frequency-flat I/Q imbalance effects at the receiver can be modeled as

$$\widetilde{\mathbf{r}}(n) = \alpha_R \, \mathbf{r}(n) + \beta_R \, \mathbf{r}^*(n) \tag{4}$$

where α_R and β_R are defined similarly to α_k and β_k. It is worth noting that (2) and (4) can be interpreted as *widely-linear* [18] transformations of $\mathbf{u}_k(n)$ and $\mathbf{r}(n)$, respectively.

By substituting (2) and (3) into (4), one has

$$\begin{aligned}
\widetilde{\mathbf{r}}(n) = \sum_{k=1}^{K} \big[& e^{j \frac{2\pi}{M} \epsilon_k n P} \, \alpha_R \, \alpha_k \, \mathbf{\Omega}_k \, \mathbf{H}_k \\
& + e^{-j \frac{2\pi}{M} \epsilon_k n P} \, \beta_R \, \beta_k^* \, \mathbf{\Omega}_k^* \, \mathbf{H}_k^* \big] \mathbf{u}_k(n) \\
& + \sum_{k=1}^{K} \big[e^{j \frac{2\pi}{M} \epsilon_k n P} \, \alpha_R \, \beta_k \, \mathbf{\Omega}_k \, \mathbf{H}_k \\
& + e^{-j \frac{2\pi}{M} \epsilon_k n P} \beta_R \, \alpha_k^* \, \mathbf{\Omega}_k^* \, \mathbf{H}_k^* \big] \mathbf{u}_k^*(n) + \mathbf{d}(n)
\end{aligned} \tag{5}$$

where $\mathbf{d}(n) \triangleq \alpha_R \, \mathbf{w}(n) + \beta_R \, \mathbf{w}^*(n)$.

It is convenient to further elaborate on quantities $\mathbf{\Omega}_k \, \mathbf{H}_k \, \mathbf{u}_k(n)$ and $\mathbf{\Omega}_k^* \, \mathbf{H}_k^* \, \mathbf{u}_k(n)$, appearing in (5) with their conjugate versions. Indeed, taking into account (1), the fact that $\mathbf{H}_k \mathbf{T}_{\text{cp}}$ and $\mathbf{H}_k^* \mathbf{T}_{\text{cp}}$ are circulant matrices, and straightforward properties of $\mathbf{P}_k$, we get

$$\mathbf{\Omega}_k \, \mathbf{H}_k \mathbf{u}_k(n) = \mathbf{\Omega}_k \, \mathbf{W}_{\text{IDFT}} \mathbf{P}_k \mathbf{\Lambda}_k \mathbf{F}_{\text{pre}} \, \mathbf{s}_k(n) \tag{6}$$

$$\mathbf{\Omega}_k^* \, \mathbf{H}_k^* \mathbf{u}_k(n) = \mathbf{\Omega}_k^* \, \mathbf{W}_{\text{IDFT}} \, \mathbf{P}_k \widetilde{\mathbf{\Theta}}_k \mathbf{F}_{\text{pre}} \, \mathbf{s}_k(n) \tag{7}$$

where $\widetilde{\mathbf{\Lambda}}_k \in \mathbb{C}^{M_u \times M_u}$ and $\widetilde{\mathbf{\Theta}}_k \in \mathbb{C}^{M_u \times M_u}$ are diagonal matrices containing samples of the DFT of $h_k(\cdot)$ and $h_k^*(\cdot)$, respectively, evaluated in correspondence of the subcarriers assigned to the kth user. By defining vectors $\mathbf{a}_k(n) \triangleq \alpha_k e^{j \frac{2\pi}{M} \epsilon_k n P} \widetilde{\mathbf{\Lambda}}_k \mathbf{F}_{\text{pre}} \, \mathbf{s}_k(n)$ and $\mathbf{a}_{\text{mir},k}(n) \triangleq \beta_k e^{j \frac{2\pi}{M} \epsilon_k n P} \widetilde{\mathbf{\Theta}}_k^* \mathbf{F}_{\text{pre}}^* \, \mathbf{s}_k^*(n)$ we can rewrite (5) as

$$\begin{aligned}
\widetilde{\mathbf{r}}(n) = {} & \alpha_R \sum_{k=1}^{K} \mathbf{\Omega}_k \mathbf{W}_{\text{IDFT}} \mathbf{P}_k \, \mathbf{a}_k(n) \\
& + \alpha_R \sum_{k=1}^{K} \mathbf{\Omega}_k \mathbf{W}_{\text{IDFT}}^* \mathbf{P}_k \, \mathbf{a}_{\text{mir},k}(n) + \beta_R \sum_{k=1}^{K} \mathbf{\Omega}_k^* \mathbf{W}_{\text{IDFT}}^* \mathbf{P}_k \, \mathbf{a}_k^*(n) \\
& + \beta_R \sum_{k=1}^{K} \mathbf{\Omega}_k^* \mathbf{W}_{\text{IDFT}} \mathbf{P}_k \, \mathbf{a}_{\text{mir},k}^*(n) + \mathbf{d}(n) \, .
\end{aligned} \tag{8}$$

Finally, let $\mathbf{\Psi} \triangleq \text{hcat}[\{\mathbf{\Omega}_k \mathbf{W}_{\text{IDFT}} \mathbf{P}_k\}_{k=1}^K] \in \mathbb{C}^{M \times K M_u}$ and $\mathbf{\Psi}_{\text{mir}} \triangleq \text{hcat}[\{\mathbf{\Omega}_k \mathbf{W}_{\text{IDFT}}^* \mathbf{P}_k\}_{k=1}^K] \in \mathbb{C}^{M \times K M_u}$, and $\mathbf{a}(n) \triangleq \text{vcat}[\{\mathbf{a}_k(n)\}_{k=1}^K] \in \mathbb{C}^{K M_u}$ e $\mathbf{a}_{\text{mir}}(n) \triangleq \text{vcat}[\{\mathbf{a}_{\text{mir},k}(n)\}_{k=1}^K] \in \mathbb{C}^{K M_u}$, one has

$$\begin{aligned}
\widetilde{\mathbf{r}}(n) = {} & \alpha_R \, \mathbf{\Psi} \, \mathbf{a}(n) + \alpha_R \, \mathbf{\Psi}_{\text{mir}} \, \mathbf{a}_{\text{mir}}(n) \\
& + \beta_R \, \mathbf{\Psi}^* \, \mathbf{a}^*(n) + \beta_R \, \mathbf{\Psi}_{\text{mir}}^* \, \mathbf{a}_{\text{mir}}^*(n) + \mathbf{d}(n) \, .
\end{aligned} \tag{9}$$

III. Receiver synthesis

To counteract the degrading effects of I/Q imbalances and CFOs on system performance, we employ a two-stage receiver, composed by a multiuser CFO compensation scheme with I/Q mitigation capabilities, synthesized under the MMOE criterion, followed by a bank of per-user MMSE equalizers. Since the synthesis involves second-order statistics of the complex received signal $\widetilde{\mathbf{r}}(n)$, we preliminarily derive analytical expressions of the correlation $\mathbf{R}_{\widetilde{\mathbf{r}}\widetilde{\mathbf{r}}}(n) \in \mathbb{C}^{2M \times 2M}$ and conjugate correlation $\mathbf{R}_{\widetilde{\mathbf{r}}\widetilde{\mathbf{r}}^*}(n) \in \mathbb{C}^{2M \times 2M}$ matrices involved. Taking into account (9) as well as **(a1)** and **(a2)**, it can be easily shown that

$$\begin{aligned}\mathbf{R}_{\widetilde{\mathbf{r}}\widetilde{\mathbf{r}}}(n) &= |\alpha_{\text{R}}|^2\, \mathbf{\Psi}\, \mathbf{R}_{\mathbf{aa}}\, \mathbf{\Psi}^H + |\alpha_{\text{R}}|^2\, \mathbf{\Psi}_{\text{mir}}\, \mathbf{R}_{\mathbf{a}_{\text{mir}}\mathbf{a}_{\text{mir}}}\, \mathbf{\Psi}_{\text{mir}}^H \\ &+ |\beta_{\text{R}}|^2\, \mathbf{\Psi}^*\, \mathbf{R}_{\mathbf{aa}}^*\, \mathbf{\Psi}^T + |\beta_{\text{R}}|^2\, \mathbf{\Psi}_{\text{mir}}^*\, \mathbf{R}_{\mathbf{a}_{\text{mir}}\mathbf{a}_{\text{mir}}^*}\, \mathbf{\Psi}_{\text{mir}}^T \\ &+ 4\, \text{symm}\{\text{Re}[\alpha_{\text{R}}\beta_{\text{R}}^*\, \mathbf{\Psi}\, \mathbf{R}_{\mathbf{aa}_{\text{mir}}^*}(n)\, \mathbf{\Psi}_{\text{mir}}^T]\} + \mathbf{R}_{\mathbf{dd}} \quad (10)\end{aligned}$$

$$\begin{aligned}\mathbf{R}_{\widetilde{\mathbf{r}}\widetilde{\mathbf{r}}^*}(n) &= 2\alpha_{\text{R}}\beta_{\text{R}}\, \text{Re}(\mathbf{\Psi}\, \mathbf{R}_{\mathbf{aa}}\, \mathbf{\Psi}^H) \\ &+ 2\alpha_{\text{R}}\beta_{\text{R}}\, \text{Re}(\mathbf{\Psi}_{\text{mir}}\, \mathbf{R}_{\mathbf{a}_{\text{mir}}\mathbf{a}_{\text{mir}}}\, \mathbf{\Psi}_{\text{mir}}^H) \\ &+ 2\alpha_{\text{R}}^2\, \text{symm}\{\mathbf{\Psi}\, \mathbf{R}_{\mathbf{aa}_{\text{mir}}^*}(n)\, \mathbf{\Psi}_{\text{mir}}^T\} \\ &+ 2\beta_{\text{R}}^2\, \text{symm}[\mathbf{\Psi}^*\, \mathbf{R}_{\mathbf{aa}_{\text{mir}}^*}^*(n)\, \mathbf{\Psi}_{\text{mir}}^H] \quad (11)\end{aligned}$$

where

$$\mathbf{R}_{\mathbf{aa}} = \sigma_s^2\, \text{diag}(\{|\alpha_k|^2\, \widetilde{\mathbf{\Lambda}}_k\, \widetilde{\mathbf{\Lambda}}_k^H\}_{k=1}^K) \quad (12)$$

$$\mathbf{R}_{\mathbf{a}_{\text{mir}}\mathbf{a}_{\text{mir}}} = \sigma_s^2 \text{diag}(\{|\beta_k|^2 \widetilde{\mathbf{\Theta}}_k\, \widetilde{\mathbf{\Theta}}_k^H\}_{k=1}^K) \quad (13)$$

$$\mathbf{R}_{\mathbf{aa}_{\text{mir}}^*}(n) = \sigma_s^2\, \text{diag}(\{e^{j\frac{2\pi}{M}2\epsilon_k nP}\alpha_k\, \beta_k \widetilde{\mathbf{\Lambda}}_k\, \widetilde{\mathbf{\Theta}}_k^H\}_{k=1}^K)\,. \quad (14)$$

Two remarks are now in order. First, as a consequence of I/Q imbalances, the conjugate autocorrelation matrix $\mathbf{R}_{\widetilde{\mathbf{r}}\widetilde{\mathbf{r}}^*}(n)$ of $\widetilde{\mathbf{r}}(n)$ is nonzero and, thus, the received vector $\widetilde{\mathbf{r}}(n)$ turns out to be improper [17]; in this case, WL processing schemes [18], which jointly elaborate $\widetilde{\mathbf{r}}(n)$ and its complex conjugate counterpart $\widetilde{\mathbf{r}}^*(n)$, are expected to significantly improve the performance upon linear ones. In addition, eq. (14) shows that data exhibit almost periodic statistics, whose *cycle frequencies* [14], [19] are related to the user CFOs. This in turn implies that both $\mathbf{R}_{\widetilde{\mathbf{r}}\widetilde{\mathbf{r}}}(n)$ and $\mathbf{R}_{\widetilde{\mathbf{r}}\widetilde{\mathbf{r}}^*}(n)$ are time-variant matrices, suggesting that a time-varying processing should be employed.

Let $\mathbf{F}(n) \in \mathbb{C}^{2KM_{\text{u}} \times 2M}$, the WL first stage is modeled as

$$\mathbf{z}(n) = \mathbf{F}(n)\, \widetilde{\mathbf{z}}(n) \quad (15)$$

where $\widetilde{\mathbf{z}}(n) \triangleq \text{vcat}[\widetilde{\mathbf{r}}(n), \widetilde{\mathbf{r}}^*(n)] \in \mathbb{C}^{2M}$ is

$$\begin{aligned}\widetilde{\mathbf{z}}(n) &= \underbrace{\begin{bmatrix}\alpha_{\text{R}}\, \mathbf{\Psi} & \beta_{\text{R}}\, \mathbf{\Psi}_{\text{mir}}^* \\ \beta_{\text{R}}^*\, \mathbf{\Psi} & \alpha_{\text{R}}^*\, \mathbf{\Psi}_{\text{mir}}^*\end{bmatrix}}_{\mathbf{\Phi}_0 \in \mathbb{C}^{(2M)\times(2KM_{\text{u}})}} \underbrace{\begin{bmatrix}\mathbf{a}(n) \\ \mathbf{a}_{\text{mir}}^*(n)\end{bmatrix}}_{\boldsymbol{\xi}_0(n) \in \mathbb{C}^{2KM_{\text{u}}}} \\ &+ \underbrace{\begin{bmatrix}\beta_{\text{R}}\, \mathbf{\Psi}^* & \alpha_{\text{R}}\, \mathbf{\Psi}_{\text{mir}} \\ \alpha_{\text{R}}^*\, \mathbf{\Psi}^* & \beta_{\text{R}}^*\, \mathbf{\Psi}_{\text{mir}}\end{bmatrix}}_{\mathbf{\Phi}_1 \in \mathbb{C}^{(2M)\times(2KM_{\text{u}})}} \underbrace{\begin{bmatrix}\mathbf{a}^*(n) \\ \mathbf{a}_{\text{mir}}(n)\end{bmatrix}}_{\boldsymbol{\xi}_1(n) \in \mathbb{C}^{2KM_{\text{u}}}} + \underbrace{\begin{bmatrix}\mathbf{d}(n) \\ \mathbf{d}^*(n)\end{bmatrix}}_{\boldsymbol{\eta}(n) \in \mathbb{C}^{2M}} \\ &= \mathbf{\Phi}_0\, \boldsymbol{\xi}_0(n) + \mathbf{\Phi}_1\, \boldsymbol{\xi}_1(n) + \boldsymbol{\eta}(n)\,.\end{aligned}$$

To compensate for the CFOs of all the users, it would be sufficient to impose the *zero-forcing* (ZF) condition $\mathbf{F}(n)\, \mathbf{\Phi}_0 = \mathbf{I}_{2KM_{\text{u}}}$, which admits solution if $\mathbf{\Phi}_0$ is full-column rank, i.e., **(c1)**: $2M \geq 2KM_{\text{u}}$ (which is always verified since $K_{\text{m}} \geq K$) and **(c2)**: $\text{rank}(\mathbf{\Phi}_0) = 2KM_{\text{u}}$. However, when $M > KM_{\text{u}}$ (which happens when the system is not fully loaded, i.e., when some subcarriers are not utilized) the ZF solution is not unique, thus the remaining degrees of freedom can be exploited to mitigate I/Q imbalance effects. In [13] we proposed to synthesize $\mathbf{F}(n)$ by minimizing the mean-output-energy at the filter output, subject to the ZF constraint:

$$\mathbf{F}_{\text{mmoe}}(n) = \arg\min_{\mathbf{F}(n)} \text{E}[\|\mathbf{z}(n)\|^2] \quad \text{s.t.} \quad \mathbf{F}(n)\, \mathbf{\Phi}_0 = \mathbf{I}_{2KM_{\text{u}}} \quad (16)$$

whose solution can be expressed as

$$\mathbf{F}_{\text{mmoe}}(n) = [\mathbf{\Phi}_0^H\, \mathbf{R}_{\widetilde{\mathbf{z}}\widetilde{\mathbf{z}}}^{-1}(n)\, \mathbf{\Phi}_0]^{-1}\, \mathbf{\Phi}_0^H\, \mathbf{R}_{\widetilde{\mathbf{z}}\widetilde{\mathbf{z}}}^{-1}(n) \quad (17)$$

where $\mathbf{R}_{\widetilde{\mathbf{z}}\widetilde{\mathbf{z}}}(n)$ is given by

$$\mathbf{R}_{\widetilde{\mathbf{z}}\widetilde{\mathbf{z}}}(n) = \begin{bmatrix}\mathbf{R}_{\widetilde{\mathbf{r}}\widetilde{\mathbf{r}}}(n) & \mathbf{R}_{\widetilde{\mathbf{r}}\widetilde{\mathbf{r}}^*}(n) \\ \mathbf{R}_{\widetilde{\mathbf{r}}\widetilde{\mathbf{r}}^*}^*(n) & \mathbf{R}_{\widetilde{\mathbf{r}}\widetilde{\mathbf{r}}}^*(n)\end{bmatrix}. \quad (18)$$

After the first stage, having restored subcarrier orthogonality, the user can be separated by simply picking out the $(2M_{\text{u}})$-dimensional sub-vectors of $\mathbf{z}(n)$. Thus, assuming that the resulting MAI is small, the kth-user data vector is given by

$$\mathbf{q}_k(n) \triangleq \mathbf{R}_k\, \mathbf{z}(n) \approx \widetilde{\mathbf{C}}_k(n)\, \mathbf{s}_k(n) + \mathbf{R}_k\, \mathbf{F}_{\text{mmoe}}(n)\, \boldsymbol{\eta}(n) \quad (19)$$

where $\mathbf{R}_k \triangleq \mathbf{I}_2 \otimes [\mathbf{O}_{M_{\text{u}}\times(k-1)M_{\text{u}}}, \mathbf{I}_{M_{\text{u}}}, \mathbf{O}_{M_{\text{u}}\times(K-k)M_{\text{u}}}] \in \mathbb{R}^{2M_{\text{u}}\times 2KM_{\text{u}}}$ and

$$\widetilde{\mathbf{C}}_k(n) \triangleq \begin{bmatrix} e^{j\frac{2\pi}{M}\epsilon_k(nP+L_{\text{cp}})}\alpha_k \widetilde{\boldsymbol{\mathcal{H}}}_k \\ e^{-j\frac{2\pi}{M}\epsilon_k(nP+L_{\text{cp}})}\beta_k^* \widetilde{\boldsymbol{\mathcal{H}}}_{\text{mir},k}^* \end{bmatrix}. \quad (20)$$

In the second stage, an estimate $\hat{\mathbf{s}}_k(n)$ of the symbol block of the kth user can be obtained as $\hat{\mathbf{s}}_k(n) = \mathbf{G}_k(n)\, \mathbf{q}_k(n)$, where the per-user time-varying filtering matrix $\mathbf{G}_k(n) \in \mathbb{C}^{M_{\text{u}}\times 2M_{\text{u}}}$ is synthesized according to the MMSE criterion as

$$\mathbf{G}_{k,\text{mmse}}(n) = \sigma_s^2 \widetilde{\mathbf{C}}_k^H(n)\, \mathbf{R}_{\mathbf{q}_k\mathbf{q}_k}^{-1}(n)\,. \quad (21)$$

IV. WL-MMOE-FRESH IMPLEMENTATION

Both the MMOE (17) and the MMSE (21) equalizers derived in the previous section are time-varying filters, hence they can be conveniently implemented by using the FRESH [14], [15], [16] approach. As for the MMOE stage, let us express the time-varying $\mathbf{F}(n)$ by its *polyperiodic* expansion

$$\mathbf{F}(n) = \sum_{q=1}^{Q} \mathbf{F}^{(\gamma_q)}\, e^{j2\pi\gamma_q n} \quad (22)$$

where $\mathbf{F}^{(\gamma_q)} \in \mathbb{C}^{2KM_{\text{u}}\times 2M}$ are the coefficients of the representation, and $\boldsymbol{\gamma} = [\gamma_1, \gamma_2, \ldots, \gamma_Q]^T \in \mathbb{R}^Q$ is a vector of properly chosen cycle frequencies; we will assume without restrictions (see later) that $\gamma_1 = 0$. By substituting (22), the output of the filter (15) can be conveniently rewritten as

$$\mathbf{z}(n) = \sum_{q=1}^{Q} \mathbf{F}^{(\gamma_q)}\, [\widetilde{\mathbf{z}}(n)\, e^{j2\pi\gamma_q n}] \quad (23)$$

which can be interpreted as a bank of time-invariant filters operating on frequency-shifted version of $\widetilde{\mathbf{z}}(n)$. Eq. (23) can be expressed more compactly, by defining $\mathbf{F}^{(\boldsymbol{\gamma})} \triangleq \text{hcat}[\{\mathbf{F}^{(\gamma_q)}\}_{q=1}^Q] \in \mathbb{C}^{2KM_{\text{u}}\times 2MQ}$ and $\widetilde{\mathbf{z}}_{\boldsymbol{\gamma}}(n) \triangleq \text{vcat}[\{\widetilde{\mathbf{z}}(n)\, e^{j2\pi\gamma_q n}\}_{q=1}^Q] \in \mathbb{C}^{2MQ}$, as

$$\mathbf{z}(n) = \mathbf{F}^{(\boldsymbol{\gamma})}\, \widetilde{\mathbf{z}}_{\boldsymbol{\gamma}}(n) \quad (24)$$

The advantage of the FRESH formulation is that the filter to be singled out, i.e., the matrix $\mathbf{F}^{(\gamma)}$, is time-invariant.

A key point is how to reformulate the ZF constraint in (16). By taking into account (22), one has

$$\sum_{q=1}^{Q} [\mathbf{F}^{(\gamma_q)} \, \mathbf{\Phi}_0] \, e^{j2\pi\gamma_q n} = \mathbf{I}_{2KM_u} \, . \tag{25}$$

The left-hand sum is a linear combination of complex exponentials, which are linearly independent functions. Therefore, remembering that $\gamma_1 = 0$, eq. (25) is satisfied if and only if

$$\mathbf{F}^{(\gamma_q)} \, \mathbf{\Phi}_0 = \begin{cases} \mathbf{I}_{2KM_u} \, , & \text{for } q = 1 \, ; \\ \mathbf{O}_{2KM_u \times 2KM_u} \, , & \text{for } q = 2, \ldots, Q \, . \end{cases} \tag{26}$$

Eq. (26) can be rewritten in compact form as $\mathbf{F}^{(\gamma)} \, \widetilde{\mathbf{\Phi}}_0 = \widetilde{\mathbf{I}}$, where $\widetilde{\mathbf{\Phi}}_0 = \mathbf{I}_Q \otimes \mathbf{\Phi}_0 \in \mathbb{C}^{2MQ \times 2KM_u Q}$ and $\widetilde{\mathbf{I}} \triangleq [\mathbf{I}, \mathbf{O}, \ldots, \mathbf{O}] \in \mathbb{R}^{2KM_u \times 2KM_u Q}$. Thus, by straightforward matrix algebra, it can be shown that the FRESH formulation of the MMOE criterion (16) is

$$\mathbf{F}^{(\gamma)}_{\text{mmoe}} = \arg\min_{\mathbf{F}^{(\gamma)}} \langle \mathrm{E}[\|\mathbf{z}(n)\|^2] \rangle \quad \text{s.t.} \quad \mathbf{F}^{(\gamma)} \, \widetilde{\mathbf{\Phi}}_0 = \widetilde{\mathbf{I}} \tag{27}$$

which, by substituting (24), can be rewritten as

$$\mathbf{F}^{(\gamma)}_{\text{mmoe}} = \arg\min_{\mathbf{F}^{(\gamma)}} \text{trace}[\mathbf{F}^{(\gamma)} \, \mathbf{R}_{\mathbf{z}_\gamma \mathbf{z}_\gamma} \, \mathbf{F}^{(\gamma)H}] \quad \text{s.t.} \quad \mathbf{F}^{(\gamma)} \, \widetilde{\mathbf{\Phi}}_0 = \widetilde{\mathbf{I}} \tag{28}$$

where $\mathbf{R}_{\mathbf{z}_\gamma \mathbf{z}_\gamma} \triangleq \langle \mathrm{E}[\mathbf{z}_\gamma(n) \, \mathbf{z}_\gamma^H(n)] \rangle \in \mathbb{C}^{2MQ \times 2MQ}$ is the time-averaged correlation matrix of $\mathbf{z}_\gamma(n)$. Problem (28) can be solved by standard Lagrange multipliers calculus yielding

$$\mathbf{F}^{(\gamma)}_{\text{mmoe}} = \widetilde{\mathbf{I}} \, (\widetilde{\mathbf{\Phi}}_0^H \, \mathbf{R}^{-1}_{\mathbf{z}_\gamma \mathbf{z}_\gamma} \, \widetilde{\mathbf{\Phi}}_0)^{-1} \widetilde{\mathbf{\Phi}}_0^H \, \mathbf{R}^{-1}_{\mathbf{z}_\gamma \mathbf{z}_\gamma} \, . \tag{29}$$

Note that (29) can be implemented from a finite sample-size K_{mmoe} of data by replacing $\mathbf{R}_{\mathbf{z}_\gamma \mathbf{z}_\gamma}$ with its estimate:

$$\widehat{\mathbf{R}}_{\mathbf{z}_\gamma \mathbf{z}_\gamma} \triangleq \frac{1}{K_{\text{mmoe}}} \sum_{n=0}^{K_{\text{mmoe}}-1} \mathbf{z}_\gamma(n) \, \mathbf{z}_\gamma^H(n) \, . \tag{30}$$

To obtain the FRESH versions of the MMSE filters, we express $\mathbf{G}_k(n)$ by its polyperiodic expansion:

$$\mathbf{G}_k(n) = \sum_{t=1}^{T_k} \mathbf{G}_k^{(\delta_t)} \, e^{j2\pi\delta_t n} \tag{31}$$

where $\mathbf{G}_k^{(\delta_t)} \in \mathbb{C}^{M_u \times 2M_u}$ are the coefficients and $\boldsymbol{\delta}_k = [\delta_1, \delta_2, \ldots, \delta_{T_k}]^T \in \mathbb{R}^T$ are the cycle frequencies of the kth user. The output of the MMSE filter can thus be compactly expressed as $\hat{\mathbf{s}}_k(n) = \mathbf{G}_k^{(\boldsymbol{\delta}_k)} \, \widetilde{\mathbf{q}}_k(n)$, where $\mathbf{G}_k^{(\boldsymbol{\delta}_k)} \triangleq \text{hcat}[\{\mathbf{G}_k^{(\delta_t)}\}_{t=1}^{T_k}] \in \mathbb{C}^{M_u \times 2M_u T_k}$ and $\widetilde{\mathbf{q}}_k(n) \triangleq \text{vcat}[\{\mathbf{q}_k(n) \, e^{j2\pi\delta_t n}\}_{t=1}^{T_k}] \in \mathbb{C}^{2M_u T_k}$. With straightforward but tedious calculations, it can be proven that the matrix $\mathbf{G}_k^{(\boldsymbol{\delta}_k)}$ solving the FRESH version of the MMSE criterion is

$$\mathbf{G}_{k,\text{mmse}}^{(\boldsymbol{\delta}_k)} = \sigma_s^2 \, \widetilde{\mathbf{C}}_k^{(\boldsymbol{\delta}_k)H} \, \mathbf{R}^{-1}_{\widetilde{\mathbf{q}}_k \widetilde{\mathbf{q}}_k} \tag{32}$$

where $\widetilde{\mathbf{C}}_k^{(\boldsymbol{\delta}_k)} \triangleq \mathbf{t}_{1,k}^{(\boldsymbol{\delta}_k)} \otimes \widetilde{\mathbf{C}}_{1,k} + \mathbf{t}_{2,k}^{(\boldsymbol{\delta}_k)} \otimes \widetilde{\mathbf{C}}_{2,k}$, with $\widetilde{\mathbf{C}}_{1,k} \triangleq \text{vcat}[\alpha_k \widetilde{\mathbf{\Lambda}}_k \mathbf{F}_{\text{pre}}, \mathbf{O}_{M_u \times M_u}] \in \mathbb{C}^{2M_u \times M_u}$, $\widetilde{\mathbf{C}}_{2,k} \triangleq \text{vcat}[\mathbf{O}_{M_u \times M_u}, \beta_k^* \widetilde{\mathbf{\Theta}}_k \mathbf{F}_{\text{pre}}] \in \mathbb{C}^{2M_u \times M_u}$, $\mathbf{t}_{1,k}^{(\boldsymbol{\delta}_k)} \triangleq \langle \boldsymbol{\delta}_k(n) \, e^{j\frac{2\pi}{M}\epsilon_k n P} \rangle \in \mathbb{C}^{T_k}$, $\mathbf{t}_{2,k}^{(\boldsymbol{\delta}_k)} \triangleq \langle \boldsymbol{\delta}_k(n) \, e^{-j\frac{2\pi}{M}\epsilon_k n P} \rangle \in \mathbb{C}^{T_k}$, $\boldsymbol{\delta}_k(n) \triangleq \text{vcat}[\{e^{j2\pi\delta_t n}\}_{t=1}^{T_k}] \in \mathbb{C}^{T_k}$ and, finally, matrix $\mathbf{R}_{\widetilde{\mathbf{q}}_k \widetilde{\mathbf{q}}_k} \triangleq \langle \mathrm{E}[\widetilde{\mathbf{q}}_k(n) \widetilde{\mathbf{q}}_k^H(n)] \rangle \in \mathbb{C}^{T_k 2M_u \times T_k 2M_u}$ can be estimated from a finite sample-size K_{mmse} of data as in (30).

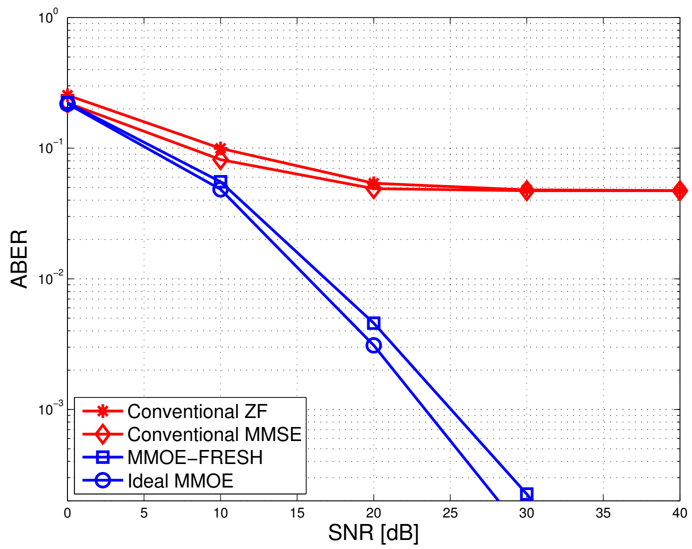

Fig. 1. ABER versus SNR.

V. Numerical Results

We simulated a SC-IFDMA system with a total of $M = 8$ QPSK-modulated subcarrier, employing a CP of length $L_{\text{cp}} = 4$, and accommodating a maximum of $K_{\text{m}} = 4$ users, with only $K = 3$ active users. We considered a frequency-selective Rayleigh fading channel model, where the order of the FIR user channels is $L_{\max} = L_{\text{cp}}$, and the values $h_k(\ell)$ are modeled as independent and identically distributed ZMCSC Gaussian random variables, with variance following an exponentially-decaying power delay profile $\sigma_h(\ell) = \sigma_h^2(0) \exp(-0.8\,\ell)$, where $\sigma_h^2(0)$ ensures energy normalization. All the results are averaged over 100 trials, with each trial employing a different realization of symbols, channels, and noise. The signal-to-noise ratio (SNR) is defined as $\text{SNR} \triangleq \sigma_s^2/\sigma_w^2$. As a figure of merit, we report the average (over all subcarriers) bit-error-rate (ABER) of the proposed WL-MMOE compensation scheme, both in its ideal (i.e., data-independent) version and data-dependent FRESH version. Cycle frequencies vectors $\boldsymbol{\gamma}$ and $\boldsymbol{\delta}_k$ are formed starting from user CFOs, which are assumed to be known, whereas the chosen sample-sizes are $K_{\text{mmoe}} = 500$ and $K_{\text{mmse}} = 250$ symbols. As a comparison, we considered the exact version of the conventional ZF and MMSE receivers. The first user is chosen as a reference, and its CFO is set to $\epsilon_1 = 0$ in all experiments, in order to allow a fair comparison with the conventional ZF and MMSE receivers, which do not compensate for the CFO.

In the first experiment, the CFOs are $\epsilon_1 = 0$, $\epsilon_2 = -\epsilon_3 = 0.20$. Each transmitter exhibits severe I/Q imbalance, with equal $\delta a_k = 0.20$ and $\delta\phi_k = \pi/18$, corresponding to a transmitter IRR of 11.5 dB, whereas the receiver is characterized by $\delta a_R = 0.05$ and $\delta\phi_R = \pi/20$, resulting in a receiver IRR

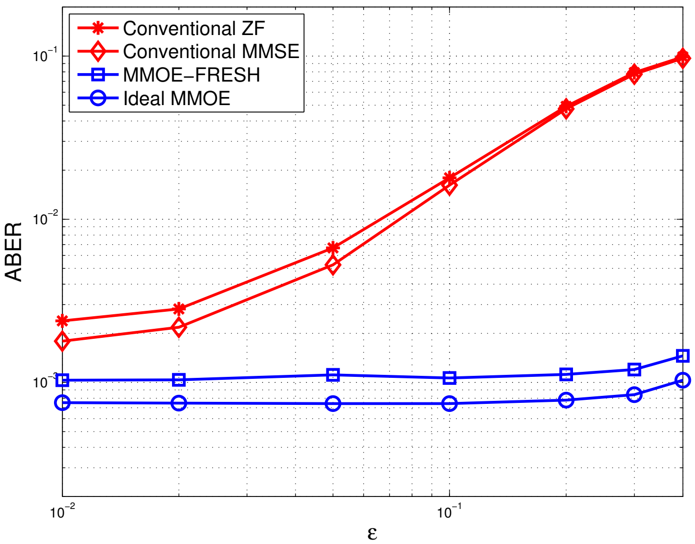

Fig. 2. ABER versus CFO ϵ.

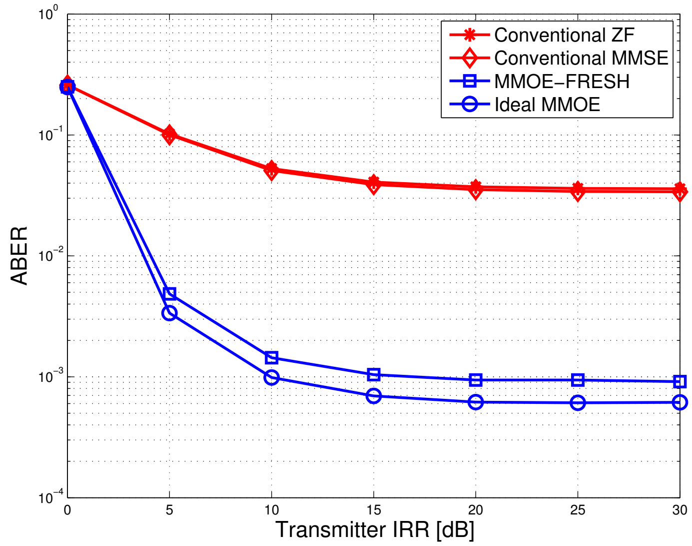

Fig. 3. ABER versus transmitter IRR.

of 15.6 dB. Results of Fig. 1 show that the proposed WL-MMOE-FRESH receiver exhibits good performances, paying only a small gap with respect to the ideal WL-MMOE implementation. Conventional ZF and MMSE receivers perform poorly, exhibiting an unacceptable performance floor.

In the second experiment, we assessed the robustness of the proposed receiver against CFO variations, by varying $\epsilon_2 = -\epsilon_3 = \epsilon$. The values of I/Q imbalances and IRRs are the same as for the first experiment, and the SNR is 25 dB. Results of Fig. 2 show that the ideal and FRESH implementation of the proposed MMOE receiver perform similarly, achieving almost perfect CFO compensation. On the contrary, conventional ZF and MMSE receivers, which do not compensate for the CFOs, work satisfactorily only for values of ϵ not exceeding 0.02.

In the third experiment, we assessed the performance behavior as the transmitter IRR is allowed to vary. In particular, the values of CFOs and of the receive I/Q imbalance are the same as the first experiment, whereas $\delta a_k = 0$ and $\delta \phi_k$ is varied so as to obtain values of IRR in the range $0 \div 40$ dB. Results of Fig. 3 show that both versions of the proposed WL-MMOE receiver perform similarly, outperforming conventional receivers when the IRR exceeds $5 \div 10$ dB and working similarly only for IRR = 0 dB, which is a case of very severe I/Q imbalance.

VI. Conclusions

We tackled the problem of joint CFO and transmitter/receiver I/Q imbalance compensation for an OFDMA/SC-IFDMA wireless network. The data-dependent FRESH implementation of the proposed WL-MMOE compensation scheme perform very close to its ideal version, assuring good performances in a wide range of scenarios.

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