WIDELY-LINEAR BEAMFORMING/COMBINING TECHNIQUES FOR MIMO WIRELESS SYSTEMS

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ABSTRACT

Joint design of beamforming and combining vectors for multiple-input multiple-output (MIMO) wireless systems is considered. The proposed approach employs widely-linear (WL) techniques, aimed at providing substantial performance gains over linear ones when the transmitted signal and/or the disturbance exhibit noncircularity features. Computer simulation results assess the claimed performance gains.

***Index Terms*—** Multiple-input multiple-output (MIMO) systems, diversity combining, transmit beamforming, noncircular random processes.

1. INTRODUCTION

The use of multiple-antenna systems in wireless communications has received significant attention in recent years, due to their ability to provide substantial performance gains with reasonable complexity. When multiple antennas are available only at the receiver, classical *combining* schemes can be employed, such as selection diversity (SDC), equal gain (EGC), and maximal ratio combining (MRC), which assure both array and diversity gain. However, when multiple antennas and channel state information (CSI) are also available at the transmitter, the adoption of *transmit beamforming* techniques, such as selection diversity (SDT), equal gain (EGT), and maximal ratio transmission (MRT), provide additional performance advantages.

Beamforming/combining schemes proposed in the literature [1] adopt a multiple-input multiple-output (MIMO) system model and employ conventional *linear* signal processing, both at the transmitter and the receiver. Recently, *widely-linear* (WL) processing schemes [2], which exploit the possible improper [3] or noncircular nature of many modulation schemes, have been recognized to provide significant advantages over linear ones in many applications, including spatial filtering [4], equalization [5, 6], blind channel identification [7], multiuser detection [8], and many others. In particular, optimal design of WL *block* precoders/decoders for MIMO systems has been considered in [9, 10], showing that the WL approach provides substantial advantages (in terms of multiplexing gain, mean-square error, and bit error rate) when noncircular symbols are transmitted and/or in the presence of noncircular noise.

In this paper, we focus on optimal design of WL *symbol* precoders/decoders (i.e., beamforming/combining vectors), synthesized under the maximum signal-to-noise ratio (SNR) approach. Unlike [9, 10], where a combination of real and complex modulations was considered, we focus our attention on *strongly noncircular* [11] (also referred to as *conjugate symmetric* [12] or *quasi-rectilinear* [13] constellations) which are representative of all memoryless real modulation formats (BPSK, ASK), differential schemes (DBPSK), offset schemes (OQPSK, OQAM), and even (in an approximate sense) modulations with memory (binary CPM, MSK, GMSK). By adopting a complex-valued representation of the input signals, we show that, due to the possible periodically time-varying (PTV) features of the considered modulation formats, the resulting precoding/decoding vectors turn out to be PTV as well, which can be implemented by resorting either to the well-known time-series representation (TSR) or frequency-series representation (FSR) [14].

Notations: The fields of complex and real numbers are denoted with $\mathbb{C}$ and $\mathbb{R}$, respectively; matrices [vectors] are denoted with upper [lower] case boldface letters (e.g., $\mathbf{A}$ or $\mathbf{a}$); the field of $m \times n$ complex [real] matrices is denoted as $\mathbb{C}^{m\times n}$ [$\mathbb{R}^{m\times n}$], with $\mathbb{C}^m$ [$\mathbb{R}^m$] used as a shorthand for $\mathbb{C}^{m\times 1}$ [$\mathbb{R}^{m\times 1}$]; the superscripts $*$, T, H, and -1 denote the conjugate, the transpose, the Hermitian (conjugate transpose), and the inverse of a matrix, respectively; $\mathbf{0}_m \in \mathbb{R}^m$, $\mathbf{O}_{m\times n} \in \mathbb{R}^{m\times n}$, and $\mathbf{I}_m \in \mathbb{R}^{m\times m}$ denote the null vector, the null ma-

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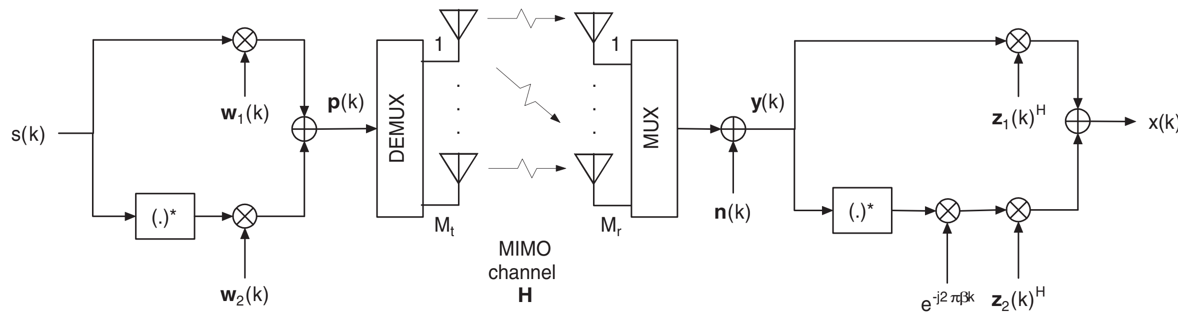

Fig. 1. The MIMO system employing WL transmit/receive beamforming.

trix, and the identity matrix, respectively; $\mathsf{E}[\cdot]$ denotes ensemble averaging; finally, $j \triangleq \sqrt{-1}$ denotes the imaginary unit.

2. MIMO SYSTEM MODEL

Let us consider the $M_t \times M_r$ MIMO communication system model depicted in Fig. 1, which employs M_t transmit and M_r receive antennas. The symbol sequence $s(k) \in \mathbb{C}$, with power $\sigma_s^2 = \mathsf{E}[|s(k)|^2]$, is subject to WL transmit beamforming, which generates the vector $\mathbf{p}(k) \in \mathbb{C}^{M_t}$ to be transmitted from the M_t antennas:

$$\mathbf{p}(k) = \mathbf{w}_1(k)\, s(k) + \mathbf{w}_2(k)\, s^*(k) \tag{1}$$

where $\mathbf{w}_1(k), \mathbf{w}_2(k) \in \mathbb{C}^{M_t}$ are the (possibly time-varying) WL beamforming weight vectors. We assume therein that $s(k)$ is *strongly noncircular* [11], i.e., it satisfies

$$s^*(k) = e^{j2\pi\beta k}\, s(k)\,, \qquad \forall k \in \mathbb{Z} \tag{2}$$

for a suitable $\beta \in [0, 1)$. Signals exhibiting such a property are referred in the literature to as *conjugate symmetric* ones [12] and are widely used in telecommunications. For example, real modulation schemes satisfy (2) with $\beta = 0$, whereas some complex modulation schemes, such as OQPSK, OQAM, and MSK, fulfill (2) with $\beta = 1/2$ [11]. Accounting for (2), eq. (1) can thus be rewritten as

$$\mathbf{p}(k) = \left[\mathbf{w}_1(k) + \mathbf{w}_2(k)\, e^{j2\pi\beta k}\right] s(k) = \mathbf{w}(k)\, s(k) \tag{3}$$

with $\mathbf{w}(k) \triangleq \mathbf{w}_1(k) + \mathbf{w}_2(k)\, e^{j2\pi\beta k} \in \mathbb{C}^{M_t}$. Note that when $s(k)$ is strongly noncircular, WL beamforming boils down to a linear (albeit PTV) one.

Under the assumption of flat-flat fading channel, the signal received by the M_r antennas can be written as

$$\mathbf{y}(k) = \mathbf{H}\, \mathbf{p}(k) + \mathbf{n}(k) = \mathbf{H}\, \mathbf{w}(k) s(k) + \mathbf{n}(k) \tag{4}$$

where $\mathbf{H} \in \mathbb{C}^{M_r \times M_t}$ is the MIMO channel matrix, and $\mathbf{n}(k) = [n_1(k), n_2(k), \ldots, n_{M_r}(k)]^T \in \mathbb{C}^{M_r}$ is the disturbance vector, which will be assumed zero-mean, uncorrelated with $s(k)$, and with power $\sigma_n^2 = \mathsf{E}[|n_i(k)|^2]$. Accounting for (2), it turns out that

$$\mathbf{y}^*(k) = \mathbf{H}^*\, \mathbf{w}^*(k)\, e^{j2\pi\beta k} s(k) + \mathbf{n}^*(k)\,. \tag{5}$$

Thus, a WL estimate $x(k) \in \mathbb{C}$ of $s(k)$ is obtained at the receiver by combining $\mathbf{y}(k)$ and a *derotated* version of $\mathbf{y}^*(k)$:

$$x(k) = \mathbf{z}_1^H(k)\, \mathbf{y}(k) + \mathbf{z}_2^H(k)\, \mathbf{y}^*(k)\, e^{-j2\pi\beta k} \tag{6}$$

where $\mathbf{z}_1(k), \mathbf{z}_2(k) \in \mathbb{C}^{M_t}$ are the WL combining weight vectors. Eq. (6) can be compactly expressed as

$$x(k) = \widetilde{\mathbf{z}}^H(k) \overline{\mathbf{y}}(k) \tag{7}$$

where $\widetilde{\mathbf{z}}(k) = [\mathbf{z}_1^H(k), \mathbf{z}_2^H(k)]^H \in \mathbb{C}^{2M_r}$ and

$$\overline{\mathbf{y}}(k) \triangleq \begin{bmatrix} \mathbf{y}(k) \\ \mathbf{y}^*(k)\, e^{-j2\pi\beta k} \end{bmatrix} = \overline{\mathbf{H}}\, \overline{\mathbf{w}}(k)\, s(k) + \overline{\mathbf{n}}(k) \tag{8}$$

with

$$\overline{\mathbf{w}}(k) \triangleq \begin{bmatrix} \mathbf{w}(k) \\ \mathbf{w}^*(k) \end{bmatrix} \in \mathbb{C}^{2M_t} \tag{9}$$

$$\overline{\mathbf{n}}(k) \triangleq \begin{bmatrix} \mathbf{n}(k) \\ \mathbf{n}^*(k)\, e^{-j2\pi\beta k} \end{bmatrix} \in \mathbb{C}^{2M_r} \tag{10}$$

$$\overline{\mathbf{H}} \triangleq \begin{bmatrix} \mathbf{H} & \mathbf{O}_{M_r \times M_t} \\ \mathbf{O}_{M_r \times M_t} & \mathbf{H}^* \end{bmatrix} \in \mathbb{C}^{2M_r \times 2M_t}\,. \tag{11}$$

Substitution of (8) into (7) yields:

$$x(k) = \widetilde{\mathbf{z}}^H(k) \overline{\mathbf{H}}\, \overline{\mathbf{w}}(k)\, s(k) + \widetilde{\mathbf{z}}^H(k) \overline{\mathbf{n}}(k)\,. \tag{12}$$

The problem at hand is choosing the pair $\{\overline{\mathbf{w}}(k), \widetilde{\mathbf{z}}(k)\}$ in (12) so that $x(k)$ is close to $s(k)$ in some sense. In the following, we will solve it under the maximum SNR criterion.

3. WL MAXIMUM SNR OPTIMIZATION

The (possibly time-varying) SNR in (12) is defined as

$$\mathsf{SNR} \triangleq \sigma_s^2 \frac{|\widetilde{\mathbf{z}}^H(k) \overline{\mathbf{H}} \overline{\mathbf{w}}(k)|^2}{\widetilde{\mathbf{z}}^H(k) \mathbf{R}_{\overline{\mathbf{n}}\overline{\mathbf{n}}}(k)\, \widetilde{\mathbf{z}}(k)} \tag{13}$$

where $\mathbf{R}_{\overline{\mathbf{n}}\overline{\mathbf{n}}}(k) \triangleq \mathsf{E}[\overline{\mathbf{n}}(k)\overline{\mathbf{n}}^H(k)] \in \mathbf{C}^{2M_r \times 2M_r}$ is the (positive definite) disturbance autocorrelation matrix. Maximization of (13) w.r.t. $\widetilde{\mathbf{z}}(k)$ involves standard application of the Cauchy-Schwartz inequality [15], yielding

$$\widetilde{\mathbf{z}}(k) = \alpha\, \mathbf{R}_{\overline{\mathbf{n}}\overline{\mathbf{n}}}^{-1}(k)\, \overline{\mathbf{H}}\, \overline{\mathbf{w}}(k) \tag{14}$$

where $\alpha \in \mathbb{C}$ is an arbitrary value.[1] For $\alpha = 1$, by decomposing $\mathbf{R}_{\overline{\mathbf{n}}\overline{\mathbf{n}}}^{-1}(k) = \mathbf{R}_{\overline{\mathbf{n}}\overline{\mathbf{n}}}^{-1/2}(k)\, \mathbf{R}_{\overline{\mathbf{n}}\overline{\mathbf{n}}}^{-1/2}(k)$, the filter defined by (14) can be interpreted as a noise *pre-whitening* of $\overline{\mathbf{y}}(k)$ [multiplication by $\mathbf{R}_{\overline{\mathbf{n}}\overline{\mathbf{n}}}^{-1/2}(k)$] followed by a *matched filtering* to the pre-whitened version of $\overline{\mathbf{y}}(k)$ [multiplication by $\mathbf{R}_{\overline{\mathbf{n}}\overline{\mathbf{n}}}^{-1/2}(k)\, \overline{\mathbf{H}}\, \overline{\mathbf{w}}(k)$].

By substituting (14) into (13), the maximum SNR turns out to be

$$\mathsf{SNR}_{\max} = \sigma_s^2 \left[\overline{\mathbf{w}}^H(k) \overline{\mathbf{H}}^H \mathbf{R}_{\overline{\mathbf{n}}\overline{\mathbf{n}}}^{-1}(k) \overline{\mathbf{H}} \overline{\mathbf{w}}(k) \right] \tag{15}$$

which should be further maximized w.r.t. $\overline{\mathbf{w}}(k)$.

It is worthwhile to observe at this point that, according to (9), $\overline{\mathbf{w}}(k)$ exhibits conjugate symmetry between its first and second-half components, which can be formally expressed as

$$\overline{\mathbf{w}}(k) = \mathbf{J}_{2M_t} \overline{\mathbf{w}}^*(k) \tag{16}$$

where

$$\mathbf{J}_{2M_t} \triangleq \begin{bmatrix} \mathbf{O}_{M_t \times M_t} & \mathbf{I}_{M_t} \\ \mathbf{I}_{M_t} & \mathbf{O}_{M_t \times M_t} \end{bmatrix} \in \mathbb{C}^{2M_t \times 2M_t} . \tag{17}$$

In the following, any vector satisfying (16) will be referred to as a *conjugate symmetric* one, and the set of all conjugate symmetric $2M_t$-vectors will be denoted as $\mathcal{S}^{2M_t} \subset \mathbb{C}^{2M_t}$.

When maximizing (15), a norm constraint on $\overline{\mathbf{w}}(k)$ must be imposed to avoid the trivial solution $\overline{\mathbf{w}}(k) \to \infty$, i.e., infinite power transmission. Since, by (3), we have $\mathsf{E}[\|\mathbf{p}(k)\|^2] = \|\mathbf{w}(k)\|^2 \sigma_s^2$, we set $\|\mathbf{w}(k)\|^2 = 1$ so that $\mathsf{E}[\|\mathbf{p}(k)\|^2] = \sigma_s^2$. Moreover, according to (9), $\|\overline{\mathbf{w}}(k)\|^2 = 2\|\mathbf{w}(k)\|^2$, hence the equivalent constraint on $\overline{\mathbf{w}}(k)$ is $\|\overline{\mathbf{w}}(k)\|^2 = 2$. Thus, ignoring σ_s^2 in (15), the optimal $\overline{\mathbf{w}}(k)$ can be obtained by solving the following problem:

$$\max_{\overline{\mathbf{w}}(k) \in \mathcal{S}^{2M_t}} \left[\overline{\mathbf{w}}^H(k)\, \boldsymbol{\mathcal{H}}(k)\, \overline{\mathbf{w}}(k) \right] \quad \text{s.t.} \quad \|\overline{\mathbf{w}}(k)\|^2 = 2 \tag{18}$$

where we defined the following Hermitian matrix:

$$\boldsymbol{\mathcal{H}}(k) \triangleq \overline{\mathbf{H}}^H \mathbf{R}_{\overline{\mathbf{n}}\overline{\mathbf{n}}}^{-1}(k) \overline{\mathbf{H}} \in \mathbb{C}^{2M_t \times 2M_t} \tag{19}$$

The standard solution of (18) is [15] the dominant eigenvector $\mathbf{u}(k) \in \mathbb{C}^{2M_t}$ corresponding to the maximum eigenvalue $\lambda_{\max}(k) > 0$ of matrix $\boldsymbol{\mathcal{H}}(k)$, properly scaled so as to satisfy the norm constraint. However, such a solution does not necessarily satisfy the conjugate symmetry constraint (16). To overcome this problem, we exploit the particular structure of $\boldsymbol{\mathcal{H}}(k)$. Indeed, note first that $\mathbf{R}_{\overline{\mathbf{n}}\overline{\mathbf{n}}}(k)$ can be partitioned as

$$\mathbf{R}_{\overline{\mathbf{n}}\overline{\mathbf{n}}}(k) = \begin{bmatrix} \mathbf{R}_{\mathbf{n}\mathbf{n}}(k) & \mathbf{R}_{\mathbf{n}\mathbf{n}^*}(k)\, e^{j2\pi\beta k} \\ \mathbf{R}_{\mathbf{n}\mathbf{n}^*}^*(k)\; e^{-j2\pi\beta k} & \mathbf{R}_{\mathbf{n}\mathbf{n}}^*(k) \end{bmatrix} \tag{20}$$

where $\mathbf{R}_{\mathbf{n}\mathbf{n}}(k) \triangleq \mathsf{E}[\mathbf{n}(k)\mathbf{n}^H(k)] \in \mathbf{C}^{N_r \times N_r}$ and $\mathbf{R}_{\mathbf{n}\mathbf{n}^*}(k) \triangleq \mathsf{E}[\mathbf{n}(k)\mathbf{n}^T(k)] \in \mathbf{C}^{N_r \times N_r}$. The particular symmetry of $\mathbf{R}_{\overline{\mathbf{n}}\overline{\mathbf{n}}}(k)$ is equivalently expressed as

$$\mathbf{R}_{\overline{\mathbf{n}}\overline{\mathbf{n}}}(k)\, \mathbf{J}_{2M_r} = \mathbf{J}_{2M_r}\, \mathbf{R}_{\overline{\mathbf{n}}\overline{\mathbf{n}}}^*(k) \tag{21}$$

from which, by conjugating, taking the inverse, and noting that $\mathbf{J}_{2M_r}^{-1} = \mathbf{J}_{2M_r}$, we obtain

$$\mathbf{R}_{\overline{\mathbf{n}}\overline{\mathbf{n}}}(k)^{-1} \mathbf{J}_{2M_r} = \mathbf{J}_{2M_r} [\mathbf{R}_{\overline{\mathbf{n}}\overline{\mathbf{n}}}^{-1}(k)]^* \tag{22}$$

which shows that also $\mathbf{R}_{\overline{\mathbf{n}}\overline{\mathbf{n}}}^{-1}(k)$ has the symmetry (20). Noting, moreover, that

$$\overline{\mathbf{H}} \mathbf{J}_{2M_t} = \mathbf{J}_{2M_r} \overline{\mathbf{H}}^* , \qquad \overline{\mathbf{H}}^H \mathbf{J}_{2M_r} = \mathbf{J}_{2M_t} \overline{\mathbf{H}}^T \tag{23}$$

it is a simple task to show that

$$\boldsymbol{\mathcal{H}}(k) \mathbf{J}_{2M_t} = \mathbf{J}_{2M_t} \boldsymbol{\mathcal{H}}^*(k) \tag{24}$$

that is, also $\boldsymbol{\mathcal{H}}(k)$ has the symmetry (20). Thus, let us define the symmetric part of the dominant eigenvector $\mathbf{u}(k)$ as

$$\overline{\mathbf{u}}(k) = \tfrac{1}{2}\mathbf{u}(k) + \tfrac{1}{2}\mathbf{J}_{2M_t} \mathbf{u}^*(k) \tag{25}$$

which satisfies the conjugate symmetry constraint (16) by construction. It can be readily verified that $\overline{\mathbf{u}}(k) \in \mathbb{C}^{2M_t}$ is still a dominant eigenvector of $\boldsymbol{\mathcal{H}}(k)$. Indeed, remembering that $\boldsymbol{\mathcal{H}}(k)\, \mathbf{u}(k) = \lambda_{\max}(k)\, \mathbf{u}(k)$ with $\lambda_{\max}(k) > 0$, and taking into account (24), one has:

$$\begin{aligned} \boldsymbol{\mathcal{H}}(k)\, \overline{\mathbf{u}}(k) &= \tfrac{1}{2}\boldsymbol{\mathcal{H}}(k)\, \mathbf{u}(k) + \tfrac{1}{2}\boldsymbol{\mathcal{H}}(k)\, \mathbf{J}_{2M_t} \mathbf{u}^*(k) \\ &= \tfrac{1}{2}\boldsymbol{\mathcal{H}}(k)\, \mathbf{u}(k) + \tfrac{1}{2}\mathbf{J}_{2M_t} \boldsymbol{\mathcal{H}}^*(k) \mathbf{u}^*(k) \\ &= \tfrac{1}{2}\lambda_{\max}(k)\, \mathbf{u}(k) + \tfrac{1}{2}\mathbf{J}_{2M_t} \lambda_{\max}(k) \mathbf{u}^*(k) \\ &= \lambda_{\max}(k)\, \overline{\mathbf{u}}(k) . \end{aligned} \tag{26}$$

It is worth noting that $\overline{\mathbf{w}}(k) = \overline{\mathbf{u}}(k)$ can be easily scaled to satisfy the norm constraint of (18), without affecting its conjugate symmetry, leading hence to the desired solution.

In the following, for the sake of clarity, we summarize the main steps required to build the optimal WL beamforming/combining vectors:

(1) Compute the dominant eigenvector $\mathbf{u}(k)$ of matrix $\boldsymbol{\mathcal{H}}(k) \triangleq \overline{\mathbf{H}}^H \mathbf{R}_{\overline{\mathbf{n}}\overline{\mathbf{n}}}^{-1}(k) \overline{\mathbf{H}}$.

(2) Calculate the symmetric part $\overline{\mathbf{u}}(k)$ of $\mathbf{u}(k)$ and scale it in order to satisfy the norm constraint; the result is the beamforming vector $\overline{\mathbf{w}}(k)$.

[1] The value of α could be varying with k as well, but we neglect this dependency for the sake of notation simplicity.

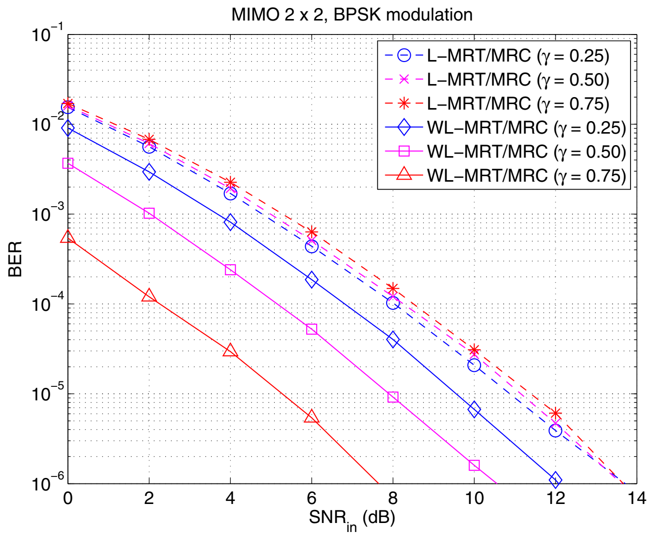

Fig. 2. Average BER versus SNR for BPSK modulation.

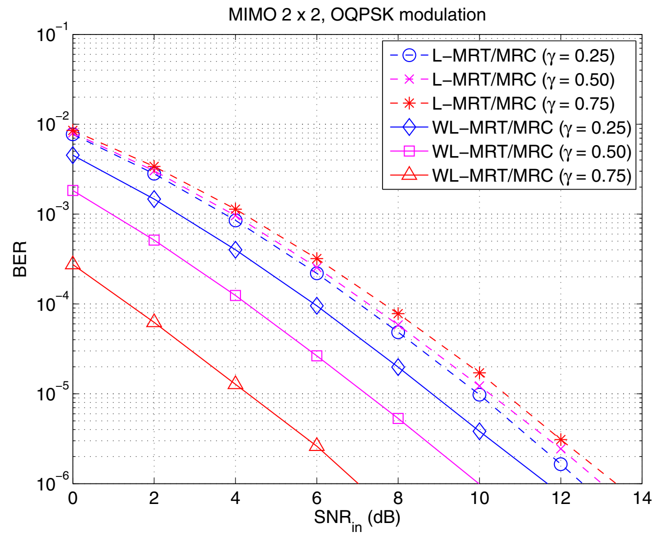

Fig. 3. Average BER versus SNR for OQPSK modulation.

(3) Evaluate the optimal WL combining vector $\widetilde{\mathbf{z}}(k)$ by (14) with an arbitrary $\alpha \in \mathbb{C}$ (e.g., $\alpha = 1$), i.e., $\widetilde{\mathbf{z}}(k) = \mathbf{R}_{\overline{\mathbf{n}}\overline{\mathbf{n}}}^{-1}(k)\,\overline{\mathbf{H}}\,\overline{\mathbf{w}}(k)$.

It should be noted that the proposed approach can be considered as the generalization to the WL case of the linear beamforming/combining MRT/MRC scheme described in [1]. Generalizations to the WL case of other approaches, such as, e.g., the EGT/MRC scheme of [1], are possible and are the subject of current study.

4. NUMERICAL RESULTS

Monte Carlo computer simulations were carried out to assess the performance gain of the proposed WL-MRT/MRC technique over the conventional (i.e., linear) L-MRT/MRC method of [1]. We considered an $M_t = M_r = 2$ MIMO system, transmitting over a flat-flat Rayleigh fading channel, with each element of $\mathbf{H}$ distributed according to $\mathcal{CN}(0, 1)$. The disturbance vector is modeled as a linear combination of spatially-white circular and non-circular disturbance, with $\gamma \in \{0.25, 0.50, 0.75\}$ denoting the proportion (in power) of noncircular disturbance over the total one.

The average (over channel realizations) bit-error rate (BER) versus the input SNR, defined as $\mathsf{SNR}_{\text{in}} = \sigma_s^2/\sigma_n^2$, is plotted in Figs. 2 and 3, for BPSK and OQPSK modulations, respectively. It is worth noting that, whereas for BPSK modulation ($\beta = 0$) the WL-MRT/MRC beamforming/combining vectors are time-invariant, for OQPSK ($\beta = 1/2$) they turn out to be PTV and are implemented by the TSR approach. First, observe that, for both modulations, all the BER curves roughly have the same slope, hence the WL-MRT/MRC transceiver does not exhibit any diversity gain over the L-MRT/MRC one. However, the WL-MRT/MRC transceiver exhibits a SNR performance gain (i.e., an array gain) over the L-MRT/MRC one, which steadily increases with the degree of non-circularity of the disturbance, starting from 2 dB up to 6 dB when γ increases from 0.25 to 0.75. On the contrary, the performance of the L-MRT/MRC transceiver is barely affected by the degree of noncircularity of the disturbance.

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