

where $\mathbf{C} \triangleq \mathbf{G}\mathbf{F}\mathbf{H}\mathbf{F}_0 \in \mathbb{C}^{N_D \times N_B}$ is the *dual-hop* channel matrix and $\mathbf{v} \triangleq \mathbf{G}\mathbf{F}\mathbf{w} + \mathbf{n}$ is the equivalent noise vector at the destination. The composite matrices

$$\mathbf{H} \triangleq [\mathbf{H}_1^T, \mathbf{H}_2^T, \dots, \mathbf{H}_{N_C}^T]^T \in \mathbb{C}^{(N_C N_R) \times N_S} \quad (2)$$

$$\mathbf{G} \triangleq [\mathbf{G}_1, \mathbf{G}_2, \dots, \mathbf{G}_{N_C}] \in \mathbb{C}^{N_D \times (N_C N_R)} \quad (3)$$

collect the *first-* (backward) and *second-hop* (forward) MIMO channel coefficients of all the relays, respectively, whereas the diagonal blocks $\mathbf{F}_i \in \mathbb{C}^{N_R \times N_R}$ of

$$\mathbf{F} \triangleq \text{diag}(\mathbf{F}_1, \mathbf{F}_2, \dots, \mathbf{F}_{N_C}) \quad (4)$$

denote the relaying matrices, and $\mathbf{F}_0 \in \mathbb{C}^{N_S \times N_B}$ represents the source precoding matrix. Finally, $\mathbf{w} \in \mathbb{C}^{N_C N_R}$ and $\mathbf{n} \in \mathbb{C}^{N_D}$ gather the noise samples at all the relays and at the destination, respectively. The vector \mathbf{r} is subject to linear equalization at the destination through the equalizing matrix $\mathbf{D} \in \mathbb{C}^{N_B \times N_D}$, hence yielding an estimate $\hat{\mathbf{b}} \triangleq \mathbf{D}\mathbf{r}$ of the source block \mathbf{b} , whose entries are then subject to minimum-distance (in the Euclidean sense) detection. Increase in spectral efficiency can be obtained by considering *two-way relaying* [6], which is based on establishing bidirectional connections between two or more terminals using one or several half-duplex relays.

To achieve the expected gains, channel state information (CSI) is required at the network nodes, i.e., source, AF relays, and destination. *Full CSI (F-CSI)* is invoked in many papers dealing with optimization of one-way (see, e.g., [7]–[15]) and two-way (see, e.g., [16], [17]) cooperative MIMO networks. Specifically, with reference to the system model (1), F-CSI is tantamount to requiring: (i) instantaneous knowledge of the first-hop channel matrix \mathbf{H} ; (ii) instantaneous knowledge of the second-hop channel matrix \mathbf{G} ; (iii) instantaneous knowledge of the dual-hop channel matrix \mathbf{C} . While the dual-hop channel matrix \mathbf{C} can be directly estimated at the destination by training, separate acquisition of the first- and second-hop matrices \mathbf{H} and \mathbf{G} is more complicated to achieve, both in terms of communication resources and signal overhead, especially in multiple-relay WSNs. Moreover, since channel estimation errors occur in practical situations, robust optimization designs are needed [18], [19], which further complicate system deployment. In resource-constrained WSNs, the use of *partial* CSI (P-CSI) can extend network lifetime and reduce the complexity burden.

Relay selection is a common strategy to reduce signaling overhead and system design complexity in single-input single-output (SISO) cooperative WSNs [20]–[23]. Design of SISO relay selection procedures providing diversity gains – even when F-CSI is not available – has been addressed in [24]–[28]. Such methods rely on P-CSI, since selection of the best relay is based only on instantaneous knowledge of the source-to-relay channels. However, the diversity order of the methods developed in these papers does not scale in the number of relays N_C . For SISO nodes, a P-CSI relay selection scheme has been proposed in [29], yielding full diversity order N_C . However, besides the instantaneous knowledge of

the source-to-relay channels, such a method requires that the selected relay sends instantaneous CSI of the corresponding source-to-relay channel to the destination for optimal decoding. Moreover, the optimization problem in [29] does not admit a closed-form solution and is solved by using a line search algorithm.

It has been shown in [30] that P-CSI relay selection approaches for MIMO nodes, based only on the instantaneous knowledge of \mathbf{H} , do not fully exploit the diversity arising from the presence of multiple relays. Besides instantaneous knowledge of \mathbf{H} , statistical CSI of the second-hop matrix \mathbf{G} is used in [14], [31] to perform relay/antenna selection for a MIMO AF cooperative network. However, the solutions developed in [14], [31] still exhibit a significant performance degradation compared to designs based on F-CSI.

In this paper, we present new optimization methods for multiple-relay cooperative MIMO WSNs with P-CSI, i.e., knowledge of the instantaneous value of \mathbf{H} and the statistical properties of \mathbf{G} . Our design does not rely on F-CSI as in [7]–[13], [15]–[17], and needs the same amount of P-CSI exploited in [24]–[28], [31]. In this scenario, we consider a relaxed joint minimum-mean-square-error (MMSE) optimization of the source precoder \mathbf{F}_0 , the AF relaying matrices in \mathbf{F} , and the destination equalizer \mathbf{D} , with a power constraint at the source [32] and a *sum-power constraint* at the relays [10]. Specifically, capitalizing on our preliminary results [14], the novel contributions can be summarized as follows:

- 1) We prove that the MMSE-based design attempting to activate all possible antennas of all relays leads to a mathematically intractable optimization problem.
- 2) We provide the proofs of the results reported in [14], by enlightening that single relay selection [14], [24]–[28] is suboptimal in the considered P-CSI scenario.
- 3) We develop a new joint antenna-and-relay selection algorithm, which is shown to significantly outperform the relay/antenna selection approaches [14], [26], [31] in terms of average symbol error probability (ASEP).

The paper is organized as follows. Section II introduces the basic assumptions and discusses their practical implications. The proposed designs are developed in Section III. Section IV reports simulation results in terms of ASEP, whereas Section V draws some conclusions.

II. BASIC ASSUMPTIONS AND PRELIMINARIES

The symbol block \mathbf{b} in (1) is modeled as a circularly symmetric complex random vector, with $\mathbb{E}[\mathbf{b}\mathbf{b}^H] = \mathbf{I}_{N_B}$. The entries of \mathbf{H} and \mathbf{G} are assumed to be unit-variance circularly symmetric complex Gaussian (CSCG) random variables. The noise vectors \mathbf{w} and \mathbf{n} are modeled as mutually independent CSCG random vectors, statistically independent of $(\mathbf{b}, \mathbf{H}, \mathbf{G})$, with $\mathbb{E}[\mathbf{w}\mathbf{w}^H] = \mathbf{I}_{N_C N_R}$ and $\mathbb{E}[\mathbf{n}\mathbf{n}^H] = \mathbf{I}_{N_D}$, respectively.

Hereinafter, we assume that \mathbf{C} in (1) and the following conditional covariance matrix of \mathbf{v} , given \mathbf{G} ,

$$\mathbf{K}_{\mathbf{v}\mathbf{v}} \triangleq \mathbb{E}[\mathbf{v}\mathbf{v}^H | \mathbf{G}] = \mathbf{G}\mathbf{F}\mathbf{F}^H\mathbf{G}^H + \mathbf{I}_{N_D} \quad (5)$$

have been previously acquired at the destination during a

training session. Under such assumptions, it is known (see, e.g., [32]) that, for fixed matrices \mathbf{F}_0 and \mathbf{F} , the matrix \mathbf{D} minimizing the trace of the conditional mean square error (MSE) matrix $\mathbf{E}(\mathbf{F}_0, \mathbf{F}, \mathbf{D}) \triangleq \mathbb{E}[(\hat{\mathbf{b}} - \mathbf{b})(\hat{\mathbf{b}} - \mathbf{b})^H | \mathbf{H}, \mathbf{G}]$, given \mathbf{H} and \mathbf{G} , is the Wiener filter

$$\mathbf{D}_{\text{mmse}} = \mathbf{C}^H(\mathbf{C}\mathbf{C}^H + \mathbf{K}_{\mathbf{v}\mathbf{v}})^{-1}. \quad (6)$$

Optimization of \mathbf{F}_0 and \mathbf{F} is carried out under the assumption that only P-CSI is available at the source and the relays. Specifically, the source and the relays perfectly know the first-hop channel matrix \mathbf{H} , but the i th relay has only knowledge of the second-order statistics (SOS) of its own second-hop channel matrix \mathbf{G}_i . These assumptions are justified since, in some systems, the relays may be able to exchange information among themselves before transmission [33]. In this case, knowledge of \mathbf{H} at the relays is realistic [7]–[14], [20]–[22], [24], [25], [27]–[30]. Moreover, since the SOS of \mathbf{G}_i vary much more slowly than the instantaneous values of \mathbf{G}_i , the feedback overhead from the destination to the relays is significantly reduced, compared to [29].

III. THE PROPOSED P-CSI-BASED DESIGN

To obtain \mathbf{F}_0 and \mathbf{F} , we minimize the statistical average (with respect to \mathbf{G}) of the trace of the following matrix:

$$\mathbf{E}(\mathbf{F}_0, \mathbf{F}) \triangleq \mathbf{E}(\mathbf{F}_0, \mathbf{F}, \mathbf{D}_{\text{mmse}}) = (\mathbf{I}_{N_B} + \mathbf{C}^H \mathbf{K}_{\mathbf{v}\mathbf{v}}^{-1} \mathbf{C})^{-1} \quad (7)$$

under suitable power constraints. To this aim, we assume that: **a1)** \mathbf{F}_0 is full-column rank, i.e., $\text{rank}(\mathbf{F}_0) = N_B \leq N_S$; **a2)** \mathbf{GFH} is full-column rank, i.e., $\text{rank}(\mathbf{GFH}) = N_S \leq N_D$. It is noteworthy that assumption **a2)** necessarily requires that the matrices \mathbf{FH} and \mathbf{H} are full-column rank, i.e., $\text{rank}(\mathbf{FH}) = \text{rank}(\mathbf{H}) = N_S \leq N_C N_R$. Such assumptions ensure that \mathbf{C} is full-column rank as well. Specifically, we consider the following optimization problem:

$$\begin{aligned} & \min_{\mathbf{F}_0, \mathbf{F}} \mathbb{E}_{\mathbf{G}} \left\{ \text{tr} \left[(\mathbf{I}_{N_B} + \mathbf{C}^H \mathbf{K}_{\mathbf{v}\mathbf{v}}^{-1} \mathbf{C})^{-1} \right] | \mathbf{H} \right\} \text{ subject to (s.to)} \\ & \text{tr}(\mathbf{F}_0 \mathbf{F}_0^H) \leq \mathcal{P}_S \quad \text{and} \quad \mathbb{E}_{\mathbf{G}} [\text{tr}(\mathbf{G} \mathbf{F} \mathbf{K}_{\mathbf{z}\mathbf{z}} \mathbf{F}^H \mathbf{G}^H) | \mathbf{H}] \leq \mathcal{P}_D \end{aligned} \quad (8)$$

where $\mathbf{K}_{\mathbf{z}\mathbf{z}} \triangleq \mathbb{E}[\mathbf{z} \mathbf{z}^H | \mathbf{H}] = \mathbf{H} \mathbf{F}_0 \mathbf{F}_0^H \mathbf{H}^H + \mathbf{I}_{N_C N_R}$ is the conditional (given \mathbf{H}) covariance matrix of the vector $\mathbf{z} \in \mathbb{C}^{N_C N_R}$ collecting the signals received by all the relays, with $\mathcal{P}_S > 0$ and $\mathcal{P}_D > 0$ denoting the power threshold at the source and at the destination, respectively. The constraint on the received power at the destination automatically limits the power expenditure at the relays. Since problem (8) is nonconvex, we consider its *relaxed* version:

$$\begin{aligned} & \min_{\mathbf{F}_0, \mathbf{F}} \mathbb{E}_{\mathbf{G}} \left\{ \text{tr} \left[(\mathbf{I}_{N_B} + \mathbf{F}_0^H \mathbf{H}^H \mathbf{F}^H \mathbf{G}^H \mathbf{G} \mathbf{F} \mathbf{H} \mathbf{F}_0)^{-1} \right] | \mathbf{H} \right\} \\ & \text{s.to} \quad \text{tr}(\mathbf{F}_0 \mathbf{F}_0^H) \leq \mathcal{P}_S \quad \text{and} \\ & \quad \quad \mathbb{E}_{\mathbf{G}} [\text{tr}(\mathbf{G} \mathbf{F} \mathbf{F}^H \mathbf{G}^H) | \mathbf{H}] \leq \mathcal{P}_D \end{aligned} \quad (9)$$

where we have used the expression of \mathbf{C} and the inequalities $\text{tr}[(\mathbf{I}_{N_B} + \mathbf{C}^H \mathbf{K}_{\mathbf{v}\mathbf{v}}^{-1} \mathbf{C})^{-1}] \geq \text{tr}[(\mathbf{I}_{N_B} + \mathbf{C}^H \mathbf{C})^{-1}]$ and $\text{tr}(\mathbf{G} \mathbf{F} \mathbf{K}_{\mathbf{z}\mathbf{z}} \mathbf{F}^H \mathbf{G}^H) \leq \text{tr}(\mathbf{G} \mathbf{F} \mathbf{F}^H \mathbf{G}^H) \text{tr}(\mathbf{K}_{\mathbf{z}\mathbf{z}})$ [34], [35].

Closed-form evaluation of the cost function in (9) is cumbersome; however, under **a1)** and **a2)**, it can be observed that²

$$\begin{aligned} & \text{tr} \left[(\mathbf{I}_{N_B} + \mathbf{F}_0^H \mathbf{H}^H \mathbf{F}^H \mathbf{G}^H \mathbf{G} \mathbf{F} \mathbf{H} \mathbf{F}_0)^{-1} \right] \\ & < \text{tr} \left[(\mathbf{F}_0^H \mathbf{H}^H \mathbf{F}^H \mathbf{G}^H \mathbf{G} \mathbf{F} \mathbf{H} \mathbf{F}_0)^{-1} \right] \end{aligned} \quad (10)$$

where the difference between the left- and right-hand sides tends to zero as the minimum eigenvalue of $\mathbf{F}_0^H \mathbf{H}^H \mathbf{F}^H \mathbf{G}^H \mathbf{G} \mathbf{F} \mathbf{H} \mathbf{F}_0$ is significantly larger than one. This happens in the high signal-to-noise ratio (SNR) region, i.e., when \mathcal{P}_S and \mathcal{P}_D are sufficiently large. Relying on (10), we pursue a further relaxation of (8) by replacing $\mathbb{E}_{\mathbf{G}} \left\{ \text{tr} \left[(\mathbf{I}_{N_B} + \mathbf{F}_0^H \mathbf{H}^H \mathbf{F}^H \mathbf{G}^H \mathbf{G} \mathbf{F} \mathbf{H} \mathbf{F}_0)^{-1} \right] | \mathbf{H} \right\}$ in (9) with its upper bound $\mathbb{E}_{\mathbf{G}} \left\{ \text{tr} \left[(\mathbf{F}_0^H \mathbf{H}^H \mathbf{F}^H \mathbf{G}^H \mathbf{G} \mathbf{F} \mathbf{H} \mathbf{F}_0)^{-1} \right] | \mathbf{H} \right\}$, which can be evaluated in closed-form as stated by the following Lemma.

Lemma 1: Let us assume that: **a3)** $N_D > N_B$. Then, under **a1)**, **a2)**, and **a3)**, it results that

$$\mathbb{E}_{\mathbf{G}} \left\{ \text{tr} \left[(\mathbf{F}_0^H \mathbf{H}^H \mathbf{F}^H \mathbf{G}^H \mathbf{G} \mathbf{F} \mathbf{H} \mathbf{F}_0)^{-1} \right] | \mathbf{H} \right\} = \frac{\text{tr}(\mathbf{R}^{-1})}{N_D - N_B} \quad (11)$$

where $\mathbf{R} \triangleq \mathbf{F}_0^H \mathbf{H}^H \mathbf{F}^H \mathbf{F} \mathbf{H} \mathbf{F}_0 \in \mathbb{C}^{N_B \times N_B}$.

Proof: See Appendix A. ■

At this point, evaluation of the expectation in the second constraint of (9) is in order. In this respect, one has

$$\mathbb{E}_{\mathbf{G}} [\text{tr}(\mathbf{F}^H \mathbf{G}^H \mathbf{G} \mathbf{F}) | \mathbf{H}] = \text{tr} [\mathbb{E}_{\mathbf{G}} (\mathbf{G}^H \mathbf{G}) \mathbf{F} \mathbf{F}^H] = \text{tr}(\mathbf{F}^H \mathbf{F}) \quad (12)$$

where we have also used the cyclic property [34] of the trace operator. Therefore, under **a1)**, **a2)**, and **a3)**, the optimization problem (9) can be simplified as follows

$$\begin{aligned} & \min_{\mathbf{F}_0, \mathbf{F}} \text{tr} \left[(\mathbf{F}_0^H \mathbf{H}^H \mathbf{F}^H \mathbf{F} \mathbf{H} \mathbf{F}_0)^{-1} \right] \\ & \text{s.to} \quad \text{tr}(\mathbf{F}_0 \mathbf{F}_0^H) \leq \mathcal{P}_S \quad \text{and} \quad \text{tr}(\mathbf{F}^H \mathbf{F}) \leq \mathcal{P}_D. \end{aligned} \quad (13)$$

At this point, a comment regarding the constraints in (8) and (13) is in order. The constraint $\text{tr}(\mathbf{F}_0 \mathbf{F}_0^H) \leq \mathcal{P}_S$ in (8) and (13) limits the average transmitted power of the source and it is standard in the design of linear MIMO transceivers [32]. Regarding the second constraint in (8), we observe that, given \mathbf{H} and \mathbf{G} , $\mathcal{P}(\mathbf{H}, \mathbf{G}) \triangleq \text{tr}(\mathbf{G} \mathbf{F} \mathbf{K}_{\mathbf{z}\mathbf{z}} \mathbf{F}^H \mathbf{G}^H)$ represents the average received power at the destination. It is noteworthy that $\mathcal{P}(\mathbf{H}, \mathbf{G})$ is typically limited in those scenarios where a target performance has to be achieved and per-node fairness is not of concern [3], [7]. The constraint $\text{tr}(\mathbf{F}^H \mathbf{F}) \leq \mathcal{P}_D$ in (13), which has been obtained by averaging a relaxed version of $\mathcal{P}(\mathbf{H}, \mathbf{G})$ with respect to the probability distribution of \mathbf{G} , fixes a limit on the total average power transmitted by the

²The proof follows easily from the facts [34] that the trace of \mathbf{A} is equal to the sum of its eigenvalues and, if λ is an eigenvalue of a nonsingular matrix \mathbf{A} , then λ^{-1} is an eigenvalue of \mathbf{A}^{-1} .

relays, so-called sum-power constraint [10].³

To solve (13), we use the following Lemma.

Lemma 2: For a positive definite matrix $\mathbf{A} \in \mathbb{C}^{n \times n}$, the following inequality holds:

$$\text{tr}(\mathbf{A}^{-1}) \geq \sum_{\ell=1}^m \frac{1}{\{\mathbf{A}\}_{\ell\ell}} \quad (14)$$

where $\{\mathbf{A}\}_{\ell\ell}$ is the ℓ th diagonal entry of \mathbf{A} and the inequality is achieved if \mathbf{A} is diagonal.

Proof: See [38, p. 65]. ■

As a consequence of Lemma 2, the minimum value of the cost function in (13) is achieved if $\mathbf{F}_0^H \mathbf{A} \mathbf{F}_0$ is diagonal, with $\mathbf{A} \triangleq \mathbf{H}^H \mathbf{F}^H \mathbf{F} \mathbf{H} \in \mathbb{C}^{N_S \times N_S}$. In what follows, we consider three different approaches to achieve the desired diagonalization of $\mathbf{F}_0^H \mathbf{A} \mathbf{F}_0$: the first one is based on the SVD of the composite matrix $\mathbf{H} = [\mathbf{H}_1^T, \mathbf{H}_2^T, \dots, \mathbf{H}_{N_C}^T]^T$ and it results in a (possible) selection of all the relays; the second one relies on the SVDs of the individual matrices $\mathbf{H}_1, \mathbf{H}_2, \dots, \mathbf{H}_{N_C}$, thus leading to a single-relay selection; the last one exploits the SVDs of row-based partitions of \mathbf{H} and it can be interpreted as a joint antenna-and-relay selection scheme.

A. DESIGN BASED ON THE SVD OF THE COMPOSITE FIRST-HOP CHANNEL MATRIX

One can attempt to recruit all the relays in the second hop of the cooperative scheme by diagonalizing $\mathbf{F}_0^H \mathbf{A} \mathbf{F}_0$ through the SVD $\mathbf{H} = \mathbf{U}_h [\mathbf{O}_{N_S \times (N_C N_R - N_S)}, \mathbf{\Lambda}_h]^T \mathbf{V}_h^H$ of \mathbf{H} , where the matrices $\mathbf{U}_h \in \mathbb{C}^{(N_C N_R) \times (N_C N_R)}$ and $\mathbf{V}_h \in \mathbb{C}^{N_S \times N_S}$ are unitary, and $\mathbf{\Lambda}_h \triangleq \text{diag}[\lambda_h(1), \lambda_h(2), \dots, \lambda_h(N_S)]$ gathers the corresponding nonzero singular values arranged in increasing order. By substituting the SVD of \mathbf{H} in \mathbf{A} , it follows by direct inspection that $\mathbf{F}_0^H \mathbf{A} \mathbf{F}_0$ is diagonal if (see, e.g., [39])

$$\mathbf{F}_0 = \mathbf{V}_{h,\text{right}} \mathbf{\Omega}^{1/2} \quad (15)$$

$$\mathbf{F}_i = \mathbf{Q}_i \mathbf{\Delta}_i^{1/2} \mathbf{U}_{h,\text{right},i}^\dagger \quad (16)$$

where $\mathbf{V}_{h,\text{right}} \in \mathbb{C}^{N_S \times N_B}$ contains the N_B rightmost columns from \mathbf{V}_h , the matrices $\mathbf{\Omega} \triangleq \text{diag}[\omega(1), \omega(2), \dots, \omega(N_B)]$ and $\mathbf{\Delta}_i \triangleq \text{diag}[\delta_i(1), \delta_i(2), \dots, \delta_i(N_S)]$ are determined in a second step, for $i \in \{1, 2, \dots, N_C\}$, the arbitrary matrix $\mathbf{Q}_i \in \mathbb{C}^{N_R \times N_S}$ obeys $\mathbf{Q}_i^H \mathbf{Q}_i = \mathbf{I}_{N_S}$, provided that $N_S \leq N_R$, $\mathbf{U}_{h,\text{right}} \triangleq [\mathbf{U}_{h,\text{right},1}^T, \mathbf{U}_{h,\text{right},2}^T, \dots, \mathbf{U}_{h,\text{right},N_C}^T]^T \in \mathbb{C}^{(N_C N_R) \times N_S}$ contains the N_S rightmost columns from \mathbf{U}_h , with the matrix $\mathbf{U}_{h,\text{right},i} \in \mathbb{C}^{N_R \times N_S}$ being full-column rank.

Using (15) and (16), problem (13) ends up to

$$\begin{aligned} \min_{\boldsymbol{\omega}, \{\boldsymbol{\delta}_i\}_{i=1}^{N_C}} f_0(\boldsymbol{\omega}, \{\boldsymbol{\delta}_i\}_{i=1}^{N_C}) \quad \text{s.t.} \quad & \sum_{\ell=1}^{N_B} \omega(\ell) \leq \mathcal{P}_S, \omega(\ell) > 0, \\ \text{and} \quad & \sum_{i=1}^{N_C} \sum_{\ell=1}^{N_S} \delta_i(\ell) (\mathbf{U}_{h,\text{right},i}^H \mathbf{U}_{h,\text{right},i})_{\ell\ell}^{-1} \leq \mathcal{P}_D, \delta_i(\ell) > 0 \end{aligned} \quad (17)$$

³Design with per-relay power constraints can be solved by properly reformulating the problem into an equivalent optimization with a sum-power constraint [36], [37].

where we have defined $\boldsymbol{\omega} \triangleq [\omega(1), \omega(2), \dots, \omega(N_B)]^T \in \mathbb{R}^{N_B}$, $\boldsymbol{\delta}_i \triangleq [\delta_i(1), \delta_i(2), \dots, \delta_i(N_S)] \in \mathbb{R}^{N_S}$, for $i \in \{1, 2, \dots, N_C\}$,

$$f_0(\boldsymbol{\omega}, \{\boldsymbol{\delta}_i\}_{i=1}^{N_C}) \triangleq \sum_{\ell=1}^{N_B} \frac{1}{\omega(\ell) \lambda_h^2(\Delta N + \ell) \sum_{i=1}^{N_C} \delta_i(\Delta N + \ell)} \quad (18)$$

with $\Delta N \triangleq N_S - N_B \geq 0$. All the inequality constraints in (17) are linear. However, it is shown in Appendix B that, when $N_C > 1$, the cost function (18) is the sum of N_B functions that are neither strictly convex nor strictly concave on \mathbb{R}_+^{n+1} . Hence, trying to solve (17) with the available optimization tools leads to poor performance in multiple-relay WSNs.

B. DESIGN BASED ON THE SVD OF THE INDIVIDUAL FIRST-HOP CHANNEL MATRICES

A simple design can be developed by setting $\mathbf{F}_i = \mathbf{O}_{N_R \times N_R}$, for each $i \in \{1, 2, \dots, N_C\} - i^*$. Basically, such a choice leads to a *single-relay selection* scheme [14], which imposes that only one relay (i.e., that for $i = i^*$) is recruited to transmit and all the remaining ones keep silent in the second hop.

Herein, we assume that \mathbf{H}_i is full-column rank, i.e., $\text{rank}(\mathbf{H}_i) = N_S \leq N_R$, for each $i \in \{1, 2, \dots, N_C\}$. Let

$$\mathbf{U}_{h,i} [\mathbf{O}_{N_S \times (N_R - N_S)}, \mathbf{\Lambda}_{h,i}]^T \mathbf{V}_{h,i}^H \quad (19)$$

be the SVD of \mathbf{H}_i , where

$$\mathbf{\Lambda}_{h,i} \triangleq \text{diag}[\lambda_{h,i}(1), \lambda_{h,i}(2), \dots, \lambda_{h,i}(N_S)] \quad (20)$$

contains the singular values of \mathbf{H}_i , arranged in increasing order, and the unitary matrices $\mathbf{U}_{h,i} \in \mathbb{C}^{N_R \times N_R}$ and $\mathbf{V}_{h,i} \in \mathbb{C}^{N_S \times N_S}$ collect the corresponding left and right singular vectors, respectively. In this case, one has $\mathbf{A} = \mathbf{H}_{i^*}^H \mathbf{F}_{i^*}^H \mathbf{F}_{i^*} \mathbf{H}_{i^*}$ and, by substituting the SVD of \mathbf{H}_{i^*} in this matrix equation, one has that the diagonalization of $\mathbf{F}_0^H \mathbf{A} \mathbf{F}_0$ is ensured by

$$\mathbf{F}_0 = \mathbf{V}_{h,i^*,\text{right}} \mathbf{\Omega}^{1/2} \quad (21)$$

$$\mathbf{F}_{i^*} = \mathbf{Q}_{i^*} \mathbf{\Delta}_i^{1/2} \mathbf{U}_{h,i^*,\text{right}}^H \quad (22)$$

where $\mathbf{U}_{h,i^*,\text{right}} \in \mathbb{C}^{N_R \times N_S}$ and $\mathbf{V}_{h,i^*,\text{right}} \in \mathbb{C}^{N_S \times N_B}$ contain the N_S and N_B rightmost columns from \mathbf{U}_{h,i^*} and \mathbf{V}_{h,i^*} , respectively, $\mathbf{Q}_{i^*} \in \mathbb{C}^{N_R \times N_S}$ is an arbitrary matrix obeying $\mathbf{Q}_{i^*}^H \mathbf{Q}_{i^*} = \mathbf{I}_{N_S}$, $\mathbf{\Omega}$ has been defined in Subsection III-A, and $\mathbf{\Delta}_i \triangleq \text{diag}[\delta(1), \delta(2), \dots, \delta(N_S)]$. To fully specify the solution of (13) in the case of single-relay selection, optimization of $\boldsymbol{\Omega}$, $\mathbf{\Delta}$, and i^* is accomplished in two steps.

First, for a given $i^* \in \{1, 2, \dots, N_C\}$, by substituting (21) and (22) in (13), one obtains the scalar optimization problem with linear inequality constraints:

$$\begin{aligned} \min_{\boldsymbol{\omega}, \boldsymbol{\delta}} f_1(i^*, \boldsymbol{\omega}, \boldsymbol{\delta}) \quad \text{s.t.} \quad & \sum_{\ell=1}^{N_B} \omega(\ell) \leq \mathcal{P}_S, \omega(\ell) > 0, \\ \text{and} \quad & \sum_{\ell=1}^{N_S} \delta(\ell) \leq \mathcal{P}_D, \delta(\ell) > 0 \end{aligned} \quad (23)$$

with

$$f_1(i^*, \boldsymbol{\omega}, \boldsymbol{\delta}) \triangleq \sum_{\ell=N_S-N_B+1}^{N_S} \frac{1}{\lambda_{h,i^*}^2(\ell) \omega(\ell) \delta(\ell)} \quad (24)$$

where $\boldsymbol{\omega}$ has been previously defined in Subsection III-A and $\boldsymbol{\delta} \triangleq [\delta(1), \delta(2), \dots, \delta(N_S)]^T \in \mathbb{R}^{N_S}$. Since $f_1(i^*, \boldsymbol{\omega}, \boldsymbol{\delta})$ is a convex function (see Appendix B), the optimization problem (23) is convex and, thus, its solution $\boldsymbol{\omega}_{\text{opt}}(i^*)$ and $\boldsymbol{\delta}_{\text{opt}}(i^*)$ can be found by using efficient numerical techniques [40]. For instance, if one resorts to interior point methods, convergence arbitrarily close to the optimal solution is achieved in a number of iterations that is proportional to the logarithm of the problem dimension [41], with a complexity per iteration dictated by the cost M of computing a Newton direction [42].

Second, the optimal value i_{opt} of i^* is obtained as

$$i_{\text{opt}} \triangleq \arg \min_{i^* \in \{1, 2, \dots, N_C\}} f_1(i^*, \boldsymbol{\omega}_{\text{opt}}(i^*), \boldsymbol{\delta}_{\text{opt}}(i^*)) \quad (25)$$

which allows one to single out the best relay among the N_C available ones. The solution of (25) can be obtained by solving (23) for each $i^* \in \{1, 2, \dots, N_C\}$, with an overall complexity $\mathcal{O}[N_C M \log(N_B + N_S)]$.

In the SISO configuration, i.e., when $N_B = N_S = N_R = N_D = 1$, and when there is no precoding at the source, i.e., $\mathbf{F}_0 \equiv \mathbf{f}_0 = \sqrt{\mathcal{P}_S}$, one gets $\mathbf{F}_{i^*} = \text{diag}(0, \dots, 0, f_{i^*}, 0, \dots, 0)$ and (25) boils down to $i_{\text{opt}} \triangleq \arg \max_{i^* \in \{1, 2, \dots, N_C\}} \{|h_i|^2\}$, with h_i denoting the channel coefficients between the source and the i th relay. According to [43], such a scheme has a full diversity order equal to N_C . However, as we will see in Section IV, such a design suffers from a diversity loss in a MIMO setting, i.e., when $N_S, N_R, N_D > 1$.

C. DESIGN BASED ON THE SVD OF ROW-BASED PARTITIONS OF THE COMPOSITE FIRST-HOP CHANNEL MATRIX

In the considered cooperative MIMO WSN, there are N_C relays equipped with N_R antennas, which amounts to a total number of $N_C N_R$ distributed antennas. Here, we propose to choose the best $N_B = N_S$ antennas out of the $N_C N_R$ ones.⁴ Such antennas can either be physically located on a single relay, or be spatially distributed over different relays, thus accomplishing a joint antenna-and-relay selection scheme.

Let $\mathcal{S} \triangleq \{(n, i), \forall n \in \{1, 2, \dots, N_R\}, \forall i \in \{1, 2, \dots, N_C\}\}$ collect all the $N_C N_R$ antenna elements in the network, with the generic (ordered) pair (n, i) uniquely identifying the n th antenna located on the i th relay. The number of distinct subsets of \mathcal{S} that have exactly N_B elements is given by the binomial coefficient $Q \triangleq \binom{N_C N_R}{N_B}$.⁵ By excluding the trivial choice \emptyset and the degenerate case \mathcal{S} (discussed in Subsection III-A), we denote with

$$\mathcal{S}^{(q)} \triangleq \left\{ (n_1^{(q)}, i_1^{(q)}), (n_2^{(q)}, i_2^{(q)}), \dots, (n_{N_B}^{(q)}, i_{N_B}^{(q)}) \right\} \quad (26)$$

the selected subset of \mathcal{S} , with $q \in \{1, 2, \dots, Q-2\}$, obeying

⁴Our design can be simply extended to the case $N_S \geq N_B$.

⁵The empty set \emptyset and the set \mathcal{S} are considered as subsets of \mathcal{S} as well.

$\mathcal{S}^{(q_1)} \neq \mathcal{S}^{(q_2)}$ for $q_1 \neq q_2$. Additionally, we use the notation $N_i^{(q)} \in \{0, 1, \dots, N_B\}$ to indicate the number of pairs of $\mathcal{S}^{(q)}$ having the same second entry: in other words, $N_i^{(q)}$ represents the number of antennas activated on the i th relay according to the q th selection. It results that $\sum_{i=1}^{N_C} N_i^{(q)} = N_B$.

The selected antennas generate a first-hop channel matrix

$$\mathbf{H}^{(q)} \triangleq [(\mathbf{H}_1^{(q)})^T, (\mathbf{H}_2^{(q)})^T, \dots, (\mathbf{H}_{N_C}^{(q)})^T]^T \in \mathbb{C}^{N_B \times N_B} \quad (27)$$

and a relaying matrix

$$\mathbf{F}^{(q)} \triangleq \text{diag}(\mathbf{F}_1^{(q)}, \mathbf{F}_2^{(q)}, \dots, \mathbf{F}_{N_C}^{(q)}) \in \mathbb{C}^{N_B \times N_B} \quad (28)$$

with $\mathbf{H}_i^{(q)} \in \mathbb{C}^{N_i^{(q)} \times N_B}$ and $\mathbf{F}_i^{(q)} \in \mathbb{C}^{N_i^{(q)} \times N_i^{(q)}}$. By convention, if $N_i^{(q)} = 0$, then $\mathbf{H}_i^{(q)}$ and $\mathbf{F}_i^{(q)}$ are empty matrices.

With reference to the q th selection, we formulate a new optimization problem, for $q \in \{1, 2, \dots, Q-2\}$, which is formally obtained from (13) by replacing \mathbf{H} and \mathbf{F} with $\mathbf{H}^{(q)}$ and $\mathbf{F}^{(q)}$, respectively, whose cost function achieves its minimum value if $\mathbf{F}_0^H \mathbf{A}^{(q)} \mathbf{F}_0$ is diagonal (see Lemma 2), with $\mathbf{A}^{(q)} \triangleq (\mathbf{H}^{(q)})^H (\mathbf{F}^{(q)})^H \mathbf{F}^{(q)} \mathbf{H}^{(q)}$. For $i \in \{1, 2, \dots, N_C\}$, let $\mathbf{H}_i^{(q)} = \mathbf{U}_{h,i}^{(q)} [\mathbf{O}_{N_i^{(q)} \times (N_B - N_i^{(q)})}, \boldsymbol{\Lambda}_{h,i}^{(q)}] (\mathbf{V}_{h,i}^{(q)})^H$ be the SVD of the (nonempty) matrix $\mathbf{H}_i^{(q)}$, which is assumed to be full-row rank, i.e., $\text{rank}(\mathbf{H}_i^{(q)}) = N_i^{(q)}$, where $\mathbf{U}_{h,i}^{(q)} \in \mathbb{C}^{N_i^{(q)} \times N_i^{(q)}}$ and $\mathbf{V}_{h,i}^{(q)} \in \mathbb{C}^{N_B \times N_B}$ are unitary, and the diagonal matrix $\boldsymbol{\Lambda}_{h,i}^{(q)} \triangleq \text{diag}[\lambda_{h,i}^{(q)}(1), \lambda_{h,i}^{(q)}(2), \dots, \lambda_{h,i}^{(q)}(N_i^{(q)})]$ collects the corresponding nonzero singular values arranged in increasing order. In this case, the diagonalization of $\mathbf{F}_0^H \mathbf{A}^{(q)} \mathbf{F}_0$ can be obtained by resorting to the following structures

$$\mathbf{F}_0 = [(\mathbf{V}_{h,\text{right}}^{(q)})^H]^{-1} \boldsymbol{\Omega}^{1/2} \quad (29)$$

$$\mathbf{F}_i^{(q)} = \mathbf{Q}_i \boldsymbol{\Delta}_i^{1/2} (\mathbf{U}_{h,i}^{(q)})^H \quad (30)$$

where

$$\mathbf{V}_{h,\text{right}}^{(q)} \triangleq [\mathbf{V}_{h,\text{right},1}^{(q)}, \mathbf{V}_{h,\text{right},2}^{(q)}, \dots, \mathbf{V}_{h,\text{right},N_C}^{(q)}] \in \mathbb{C}^{N_B \times N_B} \quad (31)$$

with $\mathbf{V}_{h,\text{right},i}^{(q)} \in \mathbb{C}^{N_B \times N_i^{(q)}}$ gathering the $N_i^{(q)}$ rightmost columns from $\mathbf{V}_{h,i}^{(q)}$, $\mathbf{Q}_i \in \mathbb{C}^{N_i^{(q)} \times N_i^{(q)}}$ is an arbitrary unitary matrix, $\boldsymbol{\Omega}$ and $\boldsymbol{\Delta}_i$ have been defined in Subsection III-A.

To optimize $\boldsymbol{\Omega}$, $\boldsymbol{\Delta}_i$, and q , we resort to a two-step procedure as in the previous subsection. By substituting (29)–(30) in (13) (with $\mathbf{H}^{(q)}$ and $\mathbf{F}^{(q)}$ in lieu of \mathbf{H} and \mathbf{F} , respectively), for a given $q \in \{1, 2, \dots, Q-2\}$, one gets the convex optimization problem (see Appendix B) with linear inequality constraints:

$$\min_{\boldsymbol{\omega}, \{\boldsymbol{\delta}_i\}_{i=1}^{N_C}} f_2(q, \boldsymbol{\omega}, \{\boldsymbol{\delta}_i\}_{i=1}^{N_C}) \quad \text{s.t.} \\ \sum_{\ell=1}^{N_B} \omega(\ell) \left[(\mathbf{V}_{h,\text{right}}^{(q)})^H \mathbf{V}_{h,\text{right}}^{(q)} \right]_{\ell\ell}^{-1} \leq \mathcal{P}_S, \omega(\ell) > 0,$$

$$\text{and} \quad \sum_{i=1}^{N_C} \sum_{\ell=1}^{N_S} \delta_i(\ell) \leq \mathcal{P}_D, \delta_i(\ell) > 0 \quad (32)$$

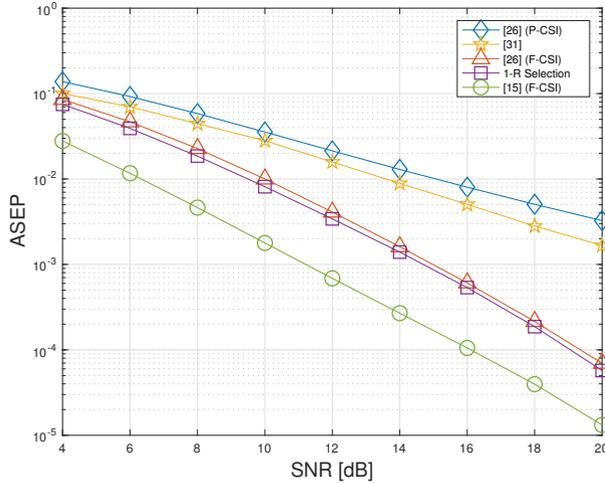


Figure 1: ASEP versus SNR (Example 1: $N = 1$ and $N_C = 2$).

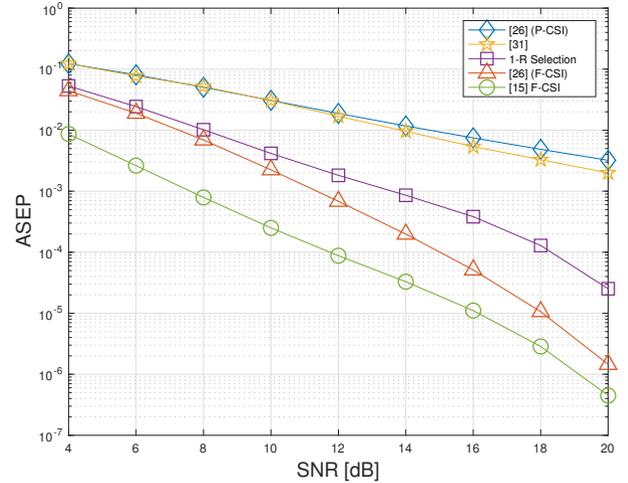


Figure 2: ASEP versus SNR (Example 1: $N = 1$ and $N_C = 3$).

with

$$f_2(q, \mathbf{w}, \{\boldsymbol{\delta}_i\}_{i=1}^{N_C}) \triangleq \sum_{i=1}^{N_C} \sum_{\ell=1}^{N_B} \frac{1}{[\lambda_{n,i}^{(q)}(\ell)]^2 \omega_i(\ell) \delta_i(\ell)} \quad (33)$$

where $\omega_i(\ell) \triangleq \omega(\sum_{m=1}^{i-1} N_m^{(q)} + \ell)$, whereas \mathbf{w} and $\boldsymbol{\delta}_i$ have been defined in Subsection III-A. Similarly to problem (23), the solution $\mathbf{w}_{\text{opt}}(q)$ and $\{\boldsymbol{\delta}_{i,\text{opt}}(q)\}_{i=1}^{N_C}$ of (32) can be found by using, e.g., interior point methods [40]. Finally, the best value q_{opt} of q is found by solving

$$q_{\text{opt}} \triangleq \arg \min_{q \in \{1, 2, \dots, Q-2\}} f_2(q, \mathbf{w}_{\text{opt}}(q), \{\boldsymbol{\delta}_{i,\text{opt}}(q)\}_{i=1}^{N_C}) \quad (34)$$

which determines the best N_B -dimensional subset of the available $N_C N_R$ antennas. The solution of (34) can be obtained by solving (32) for each $q \in \{1, 2, \dots, Q-2\}$, with an overall complexity $\mathcal{O}[(Q-2)M \log(N_B + N_B N_C)]$, which is larger than that required to select the best relay (see Subsection III-B), especially for large number of relays.

When $N_B = N_S = N_R = 1$, the optimization problems (23)-(25) and (32)-(34) yield the same solution and, thus, the design (32)-(34) exhibits full diversity order N_C , too. However, we will show in the next section that, when $N_B, N_S, N_R > 1$ (MIMO WSN), the proposed joint antenna-and-relay selection scheme ensures a significant performance improvement with respect to single-relay selection, in terms of both diversity order and coding gain.

IV. NUMERICAL RESULTS

In this section, to assess the performance of the considered P-CSI designs, we present the results of Monte Carlo computer simulations, aimed at evaluating the ASEP of the corresponding cooperative systems, transmitting quadrature phase-shift-keying (QPSK) symbols. We set $N \triangleq N_B = N_S = N_R = N_D$ in all the forthcoming examples, with $N \in \{1, 2, 3\}$. We also assume that $\mathcal{P}_S = \mathcal{P}_D = \mathcal{P}_{\text{tot}}$. Consequently, the SNR is defined as $\text{SNR} \triangleq \mathcal{P}_{\text{tot}}$, which measures the per-antenna

link quality of both the first- and second-hop transmissions. Besides the single-relay selection method described in Subsection III-B, referred to as “1-R Selection”, and the joint antenna-and-relay selection scheme developed in Subsection III-C, referred to as “JAR Selection”, we also report the performance of [26, CSI Assumptions I and II] in the case of single-antenna nodes (i.e., $N = 1$) and that of [31] for both single- and multiple-antennas nodes (i.e., $N \in \{2, 3\}$). As a reference lower bound, we additionally include in all the plots the ASEP curves of the F-CSI design proposed in [15], whose design relies on the additional knowledge of the i th second-hop channel matrix \mathbf{G}_i at the i th relay, for $i \in \{1, 2, \dots, N_C\}$. This F-CSI method exhibits a theoretical diversity order equal to $N_C N_R - N_B + 1$ [15].

The ASEP has been evaluated by carrying out 10^3 independent Monte Carlo trials, with each run using independent sets of channel realizations and noise, and an independent record of 10^6 source symbols.

A. EXAMPLE 1: SINGLE-ANTENNA NODES

We report in Figs. 1 and 2 the ASEP performance of the considered schemes as a function of the SNR, for single-antenna nodes (i.e., $N = 1$) and two different values of the number of relays $N_C \in \{2, 3\}$. We would like to remember that, in the case of $N = 1$, the two approaches “1-R Selection” and “JAR Selection” are equivalent and, thus, only the performance of the “1-R Selection” method are reported.

Results clearly show that no diversity is achieved by [26] (CSI Assumption II corresponding to P-CSI) and [31], irrespective of the number of relays. On the other hand, the “1-R Selection” scheme exhibits the same diversity order of the F-CSI methods proposed in [26] (CSI Assumption I) and [15], which linearly increases with N_C . This fact allows the “1-R Selection” design to significantly outperform both [26] (P-CSI) and [31], which rely on the same amount of CSI. Remarkably, the “1-R Selection” scheme performs comparably to [26] (F-CSI) in the case of $N_C = 2$ relays. Compared to

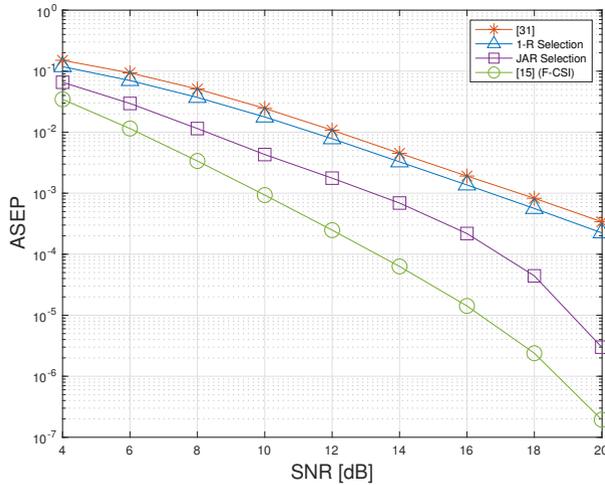


Figure 3: ASEP versus SNR (Example 2: $N = 2$ and $N_C = 2$).

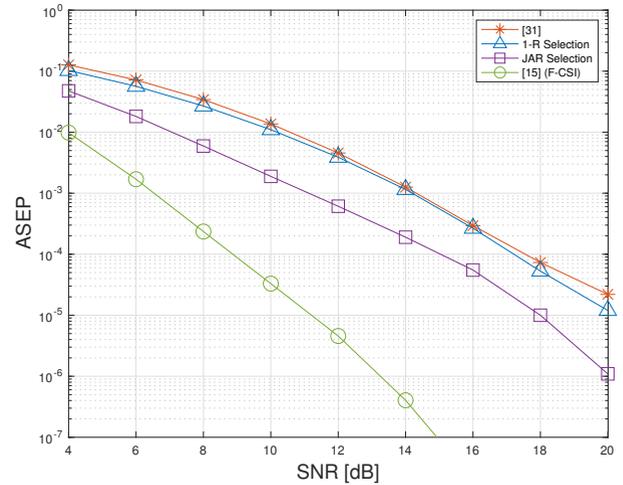


Figure 5: ASEP versus SNR (Example 2: $N = 3$ and $N_C = 2$).

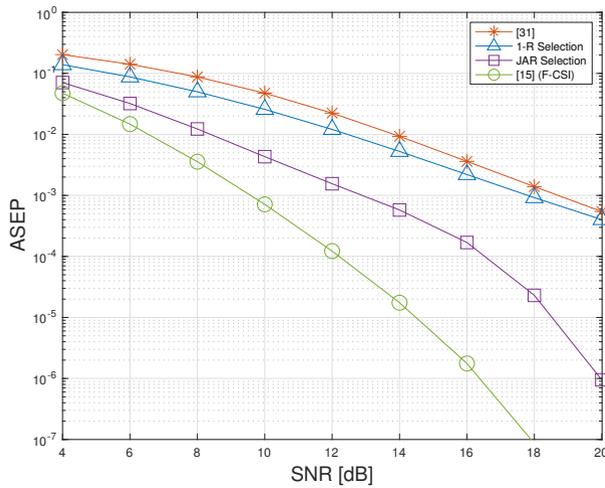


Figure 4: ASEP versus SNR (Example 2: $N = 2$ and $N_C = 3$).

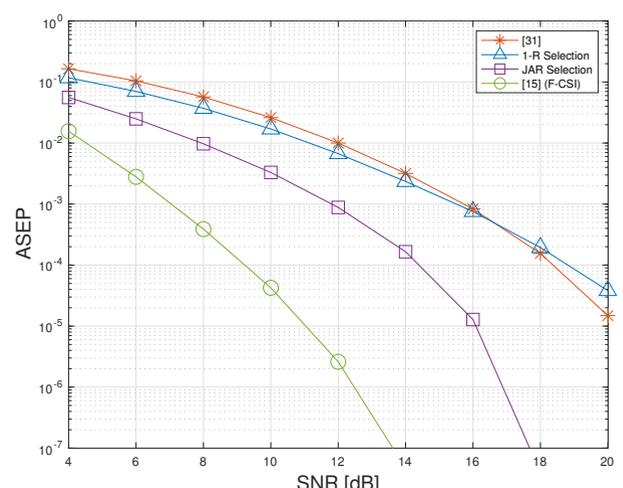


Figure 6: ASEP versus SNR (Example 2: $N = 3$ and $N_C = 3$).

single-relay selection, the performance improvement of the F-CSI approaches – arising from the additional instantaneous knowledge of the second-hop matrix \mathbf{G} – becomes more and more apparent when the number of relays N_C increases.

B. EXAMPLE 2: MULTIPLE-ANTENNA NODES

Figs. 3, 4, 5, and 6 show the ASEP performance of the considered designs as a function of the SNR, for two different multi-antenna configurations $N \in \{2, 3\}$ and two different values of the number of relays $N_C \in \{2, 3\}$, respectively.

It is apparent from these plots that, in a multi-antenna deployment, the “1-R Selection” approach and [31] perform comparably, both exhibiting a diversity loss with respect to the F-CSI design [15]. As claimed, especially in the high SNR regime, the proposed “JAR Selection” design significantly outperforms both the “1-R Selection” scheme and [31], under the same amount of P-CSI. Such a performance gap remarkably scales up as the number of antennas at the nodes increases from $N = 2$ to $N = 3$. Interestingly, the diversity order of the “JAR Selection” scheme increases with

N_C , as in [15] which, however, requires F-CSI.

V. CONCLUSIONS

We studied the problem of designing multi-relay AF cooperative WSNs, based on the knowledge of the instantaneous values of the first-hop MIMO channel matrix and statistical characterization of the second-hop one (partial CSI scenario). In this case, antenna/relay selection schemes arise necessarily to formulate mathematically tractable design problems, which can be solved by using standard convex optimization tools. We have shown that, in a MIMO setting, the selection of the best relay is suboptimal and large performance improvements can be obtained by selecting the best antennas distributed over multiple relays. Numerical simulations have shown that the proposed joint antenna-and-relay selection approach significantly outperforms existing schemes, which exploit the same amount of P-CSI.

APPENDIX A PROOF OF LEMMA 1

Preliminarily, we remember that $\mathbf{C} = \mathbf{G}\mathbf{F}\mathbf{H}\mathbf{F}_0$ is full-column rank if **a1)** and **a2)** hold. It can be shown (see, e.g., [44]) that, conditioned on \mathbf{H} , the k th diagonal entry $\{(\mathbf{C}^H\mathbf{C})^{-1}\}_{kk}$ of the matrix $(\mathbf{C}^H\mathbf{C})^{-1}$ follows an inverse-Gamma distribution, with shape parameter $\alpha \triangleq N_D - N_B + 1$ and scale parameter $\beta_k \triangleq 1/\{\mathbf{R}^{-1}\}_{kk}$, where \mathbf{R} is defined in the lemma statement. Thus, the probability density function of the random variable $\{(\mathbf{C}^H\mathbf{C})^{-1}\}_{kk}$, given \mathbf{H} , reads as

$$p_k(x) = \frac{1}{\Gamma(\alpha)\beta_k^\alpha} x^{-\alpha-1} e^{-\frac{1}{x\beta_k}} \quad (35)$$

where the gamma function $\Gamma(\alpha) = (\alpha - 1)!$ since $N_D - N_B$ is a non-negative integer number [45]. Therefore, one has

$$\begin{aligned} \mathbb{E}_{\mathbf{G}} \left[\text{tr}(\mathbf{C}^H\mathbf{C})^{-1} \mid \mathbf{H} \right] &= \sum_{k=1}^{N_B} \mathbb{E}_{\mathbf{G}} \left[\left\{ (\mathbf{C}^H\mathbf{C})^{-1} \right\}_{kk} \mid \mathbf{H} \right] \\ &= \frac{1}{\Gamma(\alpha)} \sum_{k=1}^{N_B} \frac{1}{\beta_k^\alpha} \left(\lim_{\delta \rightarrow 0} \int_{\delta}^{+\infty} x^{-\alpha} e^{-\frac{1}{x\beta_k}} dx \right). \end{aligned} \quad (36)$$

After some calculations, eq. (36) can be rewritten as

$$\begin{aligned} \mathbb{E}_{\mathbf{G}} \left[\text{tr}(\mathbf{C}^H\mathbf{C})^{-1} \mid \mathbf{H} \right] &= \sum_{k=1}^{N_B} \frac{\beta_k^{-1}}{\Gamma(\alpha)} \lim_{\delta \rightarrow 0} \gamma(\alpha - 1, (\delta\beta_k)^{-1}) \\ &= \frac{\Gamma(\alpha - 1)}{\Gamma(\alpha)} \sum_{k=1}^{N_B} \beta_k^{-1} = \frac{1}{\alpha - 1} \sum_{k=1}^{N_B} \{\mathbf{R}^{-1}\}_{kk} = \frac{\text{tr}(\mathbf{R}^{-1})}{N_D - N_B} \end{aligned} \quad (37)$$

where we have exploited the definition of the incomplete gamma function $\gamma(s, x) \triangleq \int_0^x t^{s-1} e^{-t} dt$ [45] and its asymptotic property $\Gamma(s) = \lim_{x \rightarrow +\infty} \gamma(s, x)$.

APPENDIX B HESSIAN OF THE COST FUNCTION (15)

Let us check convexity of a generic summand of the cost function (18). To this end, it is sufficient to study the multivariate function

$$f(x, y_1, y_2, \dots, y_n) \triangleq \frac{1}{Ax(y_1 + y_2 + \dots + y_n)} \quad (38)$$

with $A > 0$, $x > 0$, and $y_i > 0$, for each $i \in \{1, 2, \dots, n\}$. The domain of f is therefore given by \mathbb{R}_+^{n+1} , which is a convex set. The function f is twice differentiable over its domain. It is noteworthy that, when $n = 1$, the function (38) ends up to a generic summand of (23) or (32).

Let us calculate the Hessian matrix $\nabla^2 f \in \mathbb{R}^{(n+1) \times (n+1)}$, whose entries are the second-order partial derivatives of f at $(x, y_1, y_2, \dots, y_n) \in \mathbb{R}_+^{n+1}$, i.e.,

$$\{\nabla^2 f\}_{ij} = \begin{cases} \frac{\partial^2}{\partial x^2} f, & \text{for } i = j = 1; \\ \frac{\partial^2}{\partial x \partial y_j} f, & \text{for } i = 1 \text{ and } j \in \{2, 3, \dots, n\} \\ \frac{\partial^2}{\partial y_i \partial y_j} f, & \text{for } j = 1 \text{ and } i \in \{2, 3, \dots, n\}; \\ \frac{\partial^2}{\partial y_i \partial y_j} f, & \text{for } i, j \in \{2, 3, \dots, n\}. \end{cases} \quad (39)$$

We recall that the function f is convex [concave] if and only if the Hessian matrix $\nabla^2 f$ is positive [negative] semidefinite

for all the points belonging to its domain.

Using standard calculus concepts, it can be verified that

$$\frac{\partial^2}{\partial x^2} f = \frac{2}{Ax^3(y_1 + y_2 + \dots + y_n)} \quad (40)$$

$$\frac{\partial^2}{\partial x \partial y_j} f = \frac{1}{Ax^2(y_1 + y_2 + \dots + y_n)^2} \quad (41)$$

$$\frac{\partial^2}{\partial y_i \partial y_j} f = \frac{2}{Ax(y_1 + y_2 + \dots + y_n)^3}. \quad (42)$$

We note that all the entries of $\nabla^2 f$ are nonnegative on \mathbb{R}_+^{n+1} . In the particular case of $n = 1$, it is readily seen that the determinant of $\nabla^2 f \in \mathbb{R}^{2 \times 2}$ is given by

$$\det(\nabla^2 f) = \frac{3}{A^2 x^4 y_1^4} > 0 \quad (43)$$

which shows that, when $n = 1$, f is a strictly convex function on \mathbb{R}_+^2 . Therefore, since the sum of convex functions is convex [40], the cost functions (23) or (32) are convex.

On the other hand, when $n > 1$, by resorting to the Laplacian determinant expansion by minors, it results that

$$\det(\nabla^2 f) = \sum_{j=1}^{n+1} (-1)^{j+1} \{\nabla^2 f\}_{1j} \mathbf{M}_{1j} \quad (44)$$

where $\mathbf{M}_{1j} \in \mathbb{R}^{n \times n}$ is a so-called minor of $\nabla^2 f$, obtained by taking the determinant of $\nabla^2 f$ with row 1 and column j crossed out. It can be verified that \mathbf{M}_{1j} is zero, for each $j \in \{1, 2, \dots, n+1\}$. Thus, the determinant of $\nabla^2 f$ is zero at each point belonging to the domain of f if $n > 1$. This is sufficient to infer that $\nabla^2 f$ is neither positive nor negative definitive, which implies in its turn that, when $n > 1$, f is neither strictly convex nor strictly concave on \mathbb{R}_+^{n+1} .

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