Blind FSR-LPTV equalization and interference rejection

Giacinto Gelli and Francesco Verde

Abstract—**We address the problem of synthesizing blind channel identification and equalization methods for digital communications systems, aimed at counteracting the presence of co-channel or adjacent-channel interference. Owing to the presence of the interfering signal, the minimum mean-square error equalizer turns out to be linear periodically time-varying (LPTV), which is implemented by resorting to its Fourier series representation (FSR). Moreover, by exploiting the cyclic conjugate second-order statistics of the channel output, we propose a new weighted subspace-based channel identification method, which is asymptotically immune to the presence of high-level interference. Computer simulation results confirm the effectiveness of the proposed identification/equalization technique.**

Keywords— **Equalizers, time-varying filters, channel identification, cyclostationary signals.**

I. Introduction

THE increasing demand for high-speed digital communications is rapidly saturating the scarce available bandwidth resources, both in wired and wireless systems. Traditional equalization techniques, aimed at counteracting the degrading effects of intersymbol interference (ISI), cannot cope satisfactorily with severe *co-channel interference* (CCI) and *adjacent-channel interference* (ACI). Recently, the problem of channel equalization in the presence of CCI and/or ACI has been considered in [1], where the desired and the interfering signal are assumed to exhibit the same symbol rate, and hence the receiver structure turns out to be linear time-invariant (LTI). However, the situation where the desired and interference signals exhibit different symbol rates arises in many scenarios (e.g., crosstalk in wired applications, multi-rate or overlay systems in wireless ones). In these cases, the optimal equalization structures are *linear periodically time-varying* (LPTV) or *linear almost periodically time-varying* (LAPTV) [2], [3].

Work on LPTV equalization has been carried out in [4], [5], [6], [7]. A non-blind minimum mean-square error (MMSE) equalizer has been proposed in [4], [7], which requires knowledge of the desired-signal channel, but not of the interfering one. Instead, blind techniques have been proposed in [5], [6]: in the former, the estimation of the desired-signal channel is performed on the basis of the different circularity and/or cyclostationarity properties of the desired and interfering signal; in the latter, equalization is based on the constant modulus properties of the desired signal, without requiring an explicit channel identification step.

Most blind channel equalization techniques are able to recover the transmitted symbols up to an arbitrary complex factor, which introduces symbol constellation scaling/rotation. Both the blind approaches of [5], [6], being based on time-sequenced representation (TSR) [8], introduce different symbol constellation scalings/rotations, one for each polyphase component of the filter, whose compensation requires non-trivial algorithms. In this paper, to overcome such a drawback, we reformulate the blind equalization problem in terms of the Fourier series representation (FSR) [2] of the LPTV equalizer. The major benefit of the FSR approach is that the blind channel identification problem can be expressed in terms of a *single* optimization criterion, which naturally leads to a *single* channel estimate; therefore, the problem of different symbol rotations is inherently avoided.

The authors are with Dipartimento di Ingegneria Elettronica e delle Telecomunicazioni, Università degli Studi Federico II di Napoli, via Claudio 21, I-80125 Napoli, Italy. Tel.: +39-0817683121, Fax : +39-0817683149, E-mail: {gelli, f.verde}@unina.it

II. Mathematical model

Let $s(i)$ and $s_I(i)$ denote the symbols of the desired and interference user, respectively. At the transmitter, $s(i)$ and $s_I(i)$ are linearly modulated with different signaling periods T_U and T_I, and transmitted over LTI channels with impulse responses $c_a(t)$ and $c_{I,a}(t)$, respectively. The complex envelope of the received signal can be expressed as $r_a(t) = u_a(t) + i_a(t) + w_a(t)$, where

$$u_a(t) \;=\; \sum_{i=-\infty}^{\infty} s(i)\, c_a(t - iT_U)\,, \tag{1}$$

$$i_a(t) \;=\; \sum_{i=-\infty}^{\infty} s_I(i)\, c_{I,a}(t - iT_I)\, e^{j2\pi f_I t} \tag{2}$$

are the desired signal and the ACI or CCI, respectively, f_I is the frequency offset of the interference, and, finally, $w_a(t)$ denotes thermal noise. By employing an oversampling factor $N > 1$, the sampling instants for the received signal $r_a(t)$ can be expressed as $t_{k\ell} = kT_U + \ell\,\frac{T_U}{N}$, with $\ell = 0, 1, \ldots, N-1$ and $k \in \mathbb{Z}$; thus, the *polyphase decomposition* [9] of the received signal with respect to N is given by

$$r^{(\ell)}(k) \;\triangleq\; r_a(t_{k\ell}) = \sum_{i=-\infty}^{\infty} s(i)\, c^{(\ell)}(k-i) \;+\; i^{(\ell)}(k) + w^{(\ell)}(k)\,, \tag{3}$$

where $c^{(\ell)}(k) \triangleq c_a(t_{k\ell})$, $i^{(\ell)}(k) \triangleq i_a(t_{k\ell})$ and $w^{(\ell)}(k) \triangleq w_a(t_{k\ell})$ denote the *polyphase components* of the signal channel impulse response, interference and noise, respectively. The following assumptions will be considered throughout the paper: A1) $s(i)$ and $s_I(i)$ are mutually independent zero-mean and independent identically-distributed sequences; A2) $w_a(t)$ is a wide-sense stationary (WSS) zero-mean complex circular process, which is independent from $s(i)$ and $s_I(i)$; A3) the signal channel impulse response $c_a(t)$ has finite support $[\,0, L_c T_U)$. Under A3, the oversampled received signal can be represented in a compact vector model at the symbol rate as

$$\boldsymbol{r}(k) = \sum_{i=0}^{L_c-1} \boldsymbol{c}(i)\, s(k-i) + \boldsymbol{i}(k) + \boldsymbol{w}(k)\,, \tag{4}$$

where $\boldsymbol{r}(k) \triangleq [\, r^{(0)}(k),\, r^{(1)}(k), \ldots, r^{(N-1)}(k)\,]^T$, with T denoting transpose, whereas $\boldsymbol{c}(k)$, $\boldsymbol{i}(k)$ and $\boldsymbol{w}(k)$ are similarly defined. Let us focus on the properties of the (statistical) correlation matrix $\boldsymbol{R_{rr}}(k,m) \triangleq E[\boldsymbol{r}(k)\, \boldsymbol{r}^H(k-m)]$, where $E[\cdot]$ denotes statistical averaging and the superscript H denotes hermitian (conjugate) transpose, which will be extensively exploited in the sequel. On the basis of (4), and accounting for A1 and A2, one has

$$\begin{aligned} \boldsymbol{R_{rr}}(k,m) \;&=\; \sigma_s^2 \sum_{i=0}^{L_c-1} \boldsymbol{c}(i)\, \boldsymbol{c}^H(i-m) \\ &+\;\; \boldsymbol{R_{ii}}(k,m) + \boldsymbol{R_{ww}}(m)\,, \end{aligned} \tag{5}$$

where $\sigma_s^2 \triangleq E[\,|s(k)|^2\,]$, $\boldsymbol{R_{ii}}(k,m) \triangleq E[\boldsymbol{i}(k)\, \boldsymbol{i}^H(k-m)]$ and $\boldsymbol{R_{ww}}(m) \triangleq E[\boldsymbol{w}(k)\, \boldsymbol{w}^H(k-m)]$. Note that the contribution of the desired signal and noise to (5) depends on m but not on k, whereas the contribution of the interference might depend on k. In fact, it can be shown [10] that the matrix $\boldsymbol{R_{ii}}(k,m)$ is possibly time-varying in k, depending on the ratio T_U/T_I: when T_U/T_I is integer, $\boldsymbol{R_{ii}}(k,m)$ does not depend on k; when $T_U/T_I = p_1/P$ is a rational number (with p_1 and P co-prime integers), $\boldsymbol{R_{ii}}(k,m)$ is periodic in k with period P; when T_U/T_I is not a rational number, $\boldsymbol{R_{ii}}(k,m)$ is an *almost periodic* [11] function of k.

III. Fourier series representation of the LPTV-MMSE equalizer

The output $\hat{s}(k)$ of a fractionally-spaced [12] linear time-varying equalizer spanning L_e symbols into the past can be written in vector form as

$$\hat{s}(k) = \boldsymbol{b}^H(k)\, \boldsymbol{z}(k)\,, \tag{6}$$

where $\boldsymbol{z}(k) \triangleq [\,\boldsymbol{r}^T(k), \boldsymbol{r}^T(k-1), \ldots, \boldsymbol{r}^T(k-L_e+1)\,]^T$ and $\boldsymbol{b}(k)$ are both NL_e-column vectors. To single out $\boldsymbol{b}(k)$, we minimize the mean-square error $\mathrm{MSE}(k) \triangleq E[|\hat{s}(k)-s(k)|^2]$ for each value of k, by applying the *orthogonality principle* [2], thus obtaining

$$\boldsymbol{b}_{\mathrm{mmse}}(k) = \sigma_s^2\, \boldsymbol{R_{zz}^{-1}}(k)\, \boldsymbol{c}_0\,, \tag{7}$$

where $\boldsymbol{R_{zz}}(k) \triangleq E[\boldsymbol{z}(k)\, \boldsymbol{z}^H(k)]$ and $\boldsymbol{c}_0 \triangleq [\boldsymbol{c}^T(0), 0, 0, \ldots, 0]^T$ is the (NL_e)-dimensional desired-channel vector, which is assumed to be known or estimated in this section (algorithms for estimating $\boldsymbol{c}_0$ are proposed in Section 4.) Note that in the MMSE criterion we have arbitrarily chosen to recover the zero-delay symbol $s(k)$, even though this might not be the optimal choice in general; indeed, it is widely recognized (see e.g. [12]) that the choice of the equalization delay has a significant impact on performances. However, our derivation can be straightforwardly extended to the case of non-zero delay, which would allow one to tackle the interesting issue of choosing the optimal equalization delay.[1]

Accounting for the structure of $\boldsymbol{z}(k)$, it can be shown [10] that $\boldsymbol{R_{zz}}(k)$ exhibits the same time-varying properties with k of $\boldsymbol{R_{rr}}(k,m)$, which are governed, as previously discussed, by the ratio T_U/T_I. Therefore, as evidenced in (7), the optimal weight vector depends on k, that is, the optimal equalizer can be LPTV or LAPTV. In the sequel, we will focus our attention on the case where T_U/T_I is a rational number, which implies that the correlation matrix $\boldsymbol{R_{zz}}(k)$ and its inverse are simply periodic in k with (known)[2] period P; this in turn implies that the optimal vector $\boldsymbol{b}_{\mathrm{mmse}}(k)$ is periodically time-varying with the same period P. In this case, as proposed in [4], [5], the MMSE-LPTV equalizer (7) can be implemented by resorting to the time-sequenced representation (TSR) [8]. More precisely, denoting with $\boldsymbol{R}_{\boldsymbol{zz}}^{(h)} \triangleq \boldsymbol{R_{zz}}(kP+h)$ and $\boldsymbol{b}_{\mathrm{mmse}}^{(h)} \triangleq \boldsymbol{b}_{\mathrm{mmse}}(kP+h)$, for $h = 0, 1, \ldots, P-1$, the hth polyphase component [9] with respect to P of the correlation matrix $\boldsymbol{R_{zz}}(k)$ and of $\boldsymbol{b}_{\mathrm{mmse}}(k)$, respectively, at each value of k the hth filter $\boldsymbol{b}_{\mathrm{mmse}}^{(h)} = \sigma_s^2\, [\boldsymbol{R}_{\boldsymbol{zz}}^{(h)}]^{-1}\, \boldsymbol{c}_0$ must be used, with $h = (k)_P$, where $(\cdot)_P$ denotes modulo-P operation.

The MMSE-LPTV equalizer (7) can also be implemented by resorting to the Fourier series representation (FSR) [2], which is the frequency-time dual of the TSR. Indeed, since $\boldsymbol{R_{zz}}(k)$ and $\boldsymbol{b}_{\mathrm{mmse}}(k)$ are periodic functions in k of the same period P, they can be expanded in terms of their discrete Fourier series (DFS) and substituted in (7); by straightforward calculations, the MMSE solution can be written in terms of the Fourier coefficients as

$$\frac{1}{P} \sum_{\ell=0}^{P-1} \left\{ \sum_{m=0}^{P-1} \boldsymbol{\mathcal{R}_{zz}}(\ell-m)_P\, \boldsymbol{\beta}(m) \right\} e^{j\frac{2\pi}{P}\ell k} = P\, \sigma_s^2\, \boldsymbol{c}_0\,, \tag{8}$$

where $\boldsymbol{\mathcal{R}_{zz}}(m)$ and $\boldsymbol{\beta}(m)$ denote the mth Fourier coefficient of $\boldsymbol{R_{zz}}(k)$ and $\boldsymbol{b}_{\mathrm{mmse}}(k)$, respectively. Since identity (8) holds for all values of k, and the complex exponentials in (8) are linearly independent functions, we can equate factors of corresponding exponential terms, obtaining thus the following P systems of linear equations:

$$\begin{aligned} \frac{1}{P} \sum_{m=0}^{P-1} \boldsymbol{\mathcal{R}_{zz}}(\ell-m)_P\, \boldsymbol{\beta}(m) \;&= \\ &\begin{cases} P\, \sigma_s^2\, \boldsymbol{c}_0\,, & \text{for } \;\ell = 0\,; \\ \boldsymbol{0}\,, & \text{for } \;\ell = 1, 2, \ldots, P-1\,, \end{cases} \end{aligned} \tag{9}$$

where $\boldsymbol{0}$ denotes the (NL_e)-column zero vector.[3] In matrix form, (9) can be written as

$$\widetilde{\boldsymbol{\mathcal{R}}}_{\boldsymbol{zz}}(\ell)\, \boldsymbol{\psi} = \begin{cases} P\, \sigma_s^2\, \boldsymbol{c}_0\,, & \text{for } \;\ell = 0\,; \\ \boldsymbol{0}\,, & \text{for } \;\ell = 1, 2, \ldots, P-1\,, \end{cases} \tag{10}$$

where $\widetilde{\boldsymbol{\mathcal{R}}}_{\boldsymbol{zz}}(\ell) \triangleq \frac{1}{P}\, [\boldsymbol{\mathcal{R}_{zz}}(\ell)_P, \ldots, \boldsymbol{\mathcal{R}_{zz}}(\ell-P+1)_P]$, and $\boldsymbol{\psi} \triangleq [\boldsymbol{\beta}^T(0), \boldsymbol{\beta}^T(1), \ldots, \boldsymbol{\beta}^T(P-1)]^T$. Finally, equations (10)

[1] It is worth observing that such a choice is not a trivial one, since the optimal delay will depend not only on the desired-signal channel, but also on the interfering one, as well as on the value of the ratio T_U/T_I.

[2] In this paper, we assume that P is known *a priori*. However, in principle, the unknown cycles $\alpha = \ell/P$, for $\ell = 0, 1, \ldots, P-1$, present in the second-order statistics of $\boldsymbol{r}(k)$, could be detected by using suitable statistical tests (see [13] and references therein).

[3] For the sake of notational simplicity, we have chosen here and in the following not to indicate the dimensions of the zero/identity vectors or matrix, which can be easily deduced from context.

can be compacted in a single linear system $\boldsymbol{\Phi}_{\boldsymbol{zz}}\, \boldsymbol{\psi} = \sigma_s^2\, \boldsymbol{D}\, \boldsymbol{c}_0$, whose solution is

$$\boldsymbol{\psi}_{\text{mmse}} = \sigma_s^2\, \boldsymbol{\Phi}_{\boldsymbol{zz}}^{-1}\, \boldsymbol{D}\, \boldsymbol{c}_0 \ , \tag{11}$$

where $\boldsymbol{\Phi}_{\boldsymbol{zz}} \triangleq [\widetilde{\boldsymbol{\mathcal{R}}}_{\boldsymbol{zz}}^T(0), \widetilde{\boldsymbol{\mathcal{R}}}_{\boldsymbol{zz}}^T(1), \ldots, \widetilde{\boldsymbol{\mathcal{R}}}_{\boldsymbol{zz}}^T(P-1)]^T$ and $\boldsymbol{D} \triangleq P\,[\boldsymbol{I}, \boldsymbol{O}, \ldots, \boldsymbol{O}]^T$ are block matrices, whose dimensions are $(NPL_e) \times (NPL_e)$ and $(NPL_e) \times (NL_e)$, respectively, with $\boldsymbol{I}$ and $\boldsymbol{O}$ denoting the $(NL_e) \times (NL_e)$ identity and zero matrix, respectively. It turns out [10] that $\boldsymbol{\Phi}_{\boldsymbol{zz}}$ is a *block circulant* matrix, which can be estimated directly from frequency-shifted versions of the vector process $\boldsymbol{z}(k)$. More precisely, defining the (NPL_e)-column vector $\widetilde{\boldsymbol{z}}(k) \triangleq \boldsymbol{\zeta}(k) \otimes \boldsymbol{z}(k)$, with $\boldsymbol{\zeta}(k) \triangleq [1, e^{-j\frac{2\pi}{P}k}, \ldots, e^{-j\frac{2\pi}{P}(P-1)k}]^T$ and $\otimes$ denoting the Kronecker product, the matrix $\boldsymbol{\Phi}_{\boldsymbol{zz}}$ coincides with the time-averaged statistical correlation matrix of $\widetilde{\boldsymbol{z}}(k)$, that is, $\boldsymbol{\Phi}_{\boldsymbol{zz}} = \boldsymbol{R}_{\widetilde{\boldsymbol{z}}\widetilde{\boldsymbol{z}}} \triangleq \langle E[\widetilde{\boldsymbol{z}}(k)\widetilde{\boldsymbol{z}}^H(k)]\rangle$, where $\langle\cdot\rangle$ denotes infinite-time temporal averaging. Therefore, $\boldsymbol{\Phi}_{\boldsymbol{zz}}$ can be directly estimated from the data, using batch or adaptive algorithms [7].

Interestingly, the vector $\boldsymbol{\psi}_{\text{mmse}}$ given by (11) can also be regarded as the solution of a different optimization criterion, i.e., the minimization of the time-averaged mean-square error $\text{TAMSE} \triangleq \langle E[|\hat{s}(k) - s(k)|^2]\rangle$. In fact, by substituting the DFS of $\boldsymbol{b}_{\text{mmse}}(k)$ in (6), the output $\hat{s}(k)$ can be expressed directly as a function of $\boldsymbol{\psi}$ and $\widetilde{\boldsymbol{z}}(k)$ as $\hat{s}(k) = \frac{1}{P}\,\boldsymbol{\psi}^H\,\widetilde{\boldsymbol{z}}(k)$ and therefore, the minimum TAMSE solution is straightforwardly given by (11).

As a final remark about complexity, when batch algorithms are employed to estimate the filter coefficients, computational complexity is dominated by sample correlation matrix inversion. Thus, the FSR-LPTV equalizer exhibits the highest complexity, of order $O(N^3P^3L_e^3)$ (corresponding to inversion of a single (NPL_e)-dimensional matrix); the TSR-LPTV equalizer exhibits intermediate complexity, of order $P\,O(N^3L_e^3)$ (corresponding to P inversions of (NL_e)-dimensional matrices); finally, the LTI equalizer, based on a time-averaged estimate of $\boldsymbol{R}_{\boldsymbol{zz}}(k)$, exhibits the lowest complexity, of order $O(N^3L_e^3)$ (corresponding to inversion of a single (NL_e)-dimensional matrix). However, according also for performance issues, the FSR-based solution offers an interesting tradeoff between performance and complexity, since: (i) in the considered non-stationary environment, a conventional LTI equalizer suffers a severe performance degradation when the signal-to-interference ratio (SIR) is low (see simulations results in [4], [7]); (ii) although the FSR- and TSR-LPTV equalizers are *theoretically* equivalent, the latter introduces different symbol constellation scalings/rotations in a blind setting; (iii) finally, as shown in [7], the FSR-LPTV equalizer can be adaptively implemented by using a simple and effective recursive least square (RLS) algorithm, which assures a substantial reduction of the computational load without sacrificing convergence speed.

IV. Blind channel identification

Subspace-based identification methods [14] estimate the vector $\boldsymbol{c} \triangleq [\boldsymbol{c}^T(0), \ldots, \boldsymbol{c}^T(L_c-1)]^T$ (and hence $\boldsymbol{c}_0$) by resorting to eigendecomposition of the correlation matrix $\boldsymbol{R}_{\boldsymbol{zz}}(k)$, provided that: A4) the z-transforms of the impulse responses $c^{(\ell)}(k)$, for $\ell = 0, 1, \ldots, N-1$, have no common zeros; A5) $NL_e > (L_c + L_e - 1)$. However, there are two main difficulties in extending such an approach to our identification problem, due to the presence of the interference; to begin with, the disturbance (interference plus noise) is colored and its statistical correlation matrix is not known *a priori*; moreover, $\boldsymbol{R}_{\boldsymbol{zz}}(k)$ is time-varying, which complicates estimation of the correlation matrix from received data. To overcome the previous drawbacks, we propose here a blind identification procedure based on the eigenstructure of the time-averaged conjugate (statistical) correlation matrix (TA-CCM) $\boldsymbol{\Phi}_{\boldsymbol{zz}^*} = \boldsymbol{R}_{\widetilde{\boldsymbol{z}}\widetilde{\boldsymbol{z}}^*} \triangleq \langle E[\widetilde{\boldsymbol{z}}(k)\widetilde{\boldsymbol{z}}^T(k)]\rangle$. Indeed, let us rewrite $\widetilde{\boldsymbol{z}}(k)$ as

$$\widetilde{\boldsymbol{z}}(k) = \widetilde{\boldsymbol{\mathcal{H}}}(\boldsymbol{c})\,\widetilde{\boldsymbol{s}}(k) + \widetilde{\boldsymbol{\jmath}}(k) + \widetilde{\boldsymbol{v}}(k) \ , \tag{12}$$

where $\widetilde{\boldsymbol{\mathcal{H}}}(\boldsymbol{c}) \triangleq \boldsymbol{I} \otimes \boldsymbol{\mathcal{H}}(\boldsymbol{c})$, with $\boldsymbol{I}$ and $\boldsymbol{\mathcal{H}}(\boldsymbol{c})$ denoting the $P \times P$ identity matrix and the $(NL_e) \times (L_c + L_e - 1)$ *Sylvester matrix* associated with the channel vector $\boldsymbol{c}$ (see [14] for details), respectively; and, finally, $\widetilde{\boldsymbol{s}}(k) \triangleq \boldsymbol{\zeta}(k) \otimes \boldsymbol{s}(k)$, $\widetilde{\boldsymbol{\jmath}}(k) \triangleq \boldsymbol{\zeta}(k) \otimes \boldsymbol{j}(k)$, $\widetilde{\boldsymbol{v}}(k) \triangleq \boldsymbol{\zeta}(k) \otimes \boldsymbol{v}(k)$, with

$$\boldsymbol{s}(k) \triangleq [\, s(k),\, s(k-1),\, \ldots,\, s(k-L_e-L_c+2)\,]^T \ , \tag{13}$$

$$\boldsymbol{j}(k) \triangleq [\, \boldsymbol{i}^T(k),\, \boldsymbol{i}^T(k-1),\, \ldots,\, \boldsymbol{i}^T(k-L_e+1)\,]^T \ , \tag{14}$$

$$\boldsymbol{v}(k) \triangleq [\, \boldsymbol{w}^T(k),\, \boldsymbol{w}^T(k-1),\, \ldots,\, \boldsymbol{w}^T(k-L_e+1)\,]^T. \tag{15}$$

By accounting for A1 and A2, one has

$$\boldsymbol{\Phi}_{\boldsymbol{zz}^*} = \widetilde{\boldsymbol{\mathcal{H}}}(\boldsymbol{c})\,\boldsymbol{\Phi}_{\boldsymbol{ss}^*}\,\widetilde{\boldsymbol{\mathcal{H}}}^T(\boldsymbol{c}) + \boldsymbol{\Phi}_{\boldsymbol{jj}^*} \ , \tag{16}$$

where we have defined the matrices $\boldsymbol{\Phi}_{\boldsymbol{ss}^*} \triangleq \langle\, E[\widetilde{\boldsymbol{s}}(k)\,\widetilde{\boldsymbol{s}}^T(k)]\,\rangle$ and $\boldsymbol{\Phi}_{\boldsymbol{jj}^*} \triangleq \langle\, E[\widetilde{\boldsymbol{\jmath}}(k)\,\widetilde{\boldsymbol{\jmath}}^T(k)]\,\rangle$. Note that $\boldsymbol{\Phi}_{\boldsymbol{ss}^*}$, according to the definition of $\widetilde{\boldsymbol{s}}(k)$, can be partitioned in blocks, where the (p,q)th block, for $p, q = 0, 1, \ldots, P-1$, is given by $\{\boldsymbol{\Phi}_{\boldsymbol{ss}^*}\}_{(p,q)} = \langle E[\boldsymbol{s}(k)\,\boldsymbol{s}^T(k)]\, e^{-j\frac{2\pi}{P}(p+q)k}\rangle$, which represents the cyclic conjugate correlation matrix (CCCM) [2] of the vector $\boldsymbol{s}(k)$ at cycle frequency $\beta = (p+q)/P$. Moreover, accounting for A1, it results that $E[\boldsymbol{s}(k)\,\boldsymbol{s}^T(k)] = \gamma\,\boldsymbol{I}$, with $\gamma \triangleq E[s^2(k)]$, and thus the (p,q)th block can be written as

$$\{\boldsymbol{\Phi}_{\boldsymbol{ss}^*}\}_{(p,q)} = \gamma\,\langle e^{-j\frac{2\pi}{P}(p+q)k}\rangle\,\boldsymbol{I} = \gamma\,\delta_{(p+q)_P}\,\boldsymbol{I} \ , \tag{17}$$

where δ_p is the Kronecker delta. Therefore, as long as $\gamma \neq 0$, i.e., the desired symbol sequence is non-circular, the square matrix $\boldsymbol{\Phi}_{\boldsymbol{ss}^*}$ turns out to be full rank. As to $\boldsymbol{\Phi}_{\boldsymbol{jj}^*}$, it can be partitioned in blocks in the same manner, according to the definition of $\widetilde{\boldsymbol{\jmath}}(k)$, where the (p,q)th block, for $p, q = 0, 1, \ldots, P-1$, is given by $\{\boldsymbol{\Phi}_{\boldsymbol{jj}^*}\}_{(p,q)} = \langle\, E[\boldsymbol{j}(k)\,\boldsymbol{j}^T(k)]\, e^{-j\frac{2\pi}{P}(p+q)k}\,\rangle$, which represents the CCCM of the vector $\boldsymbol{j}(k)$ at cycle frequency $\beta = (p+q)/P$. The TA-CCM $\boldsymbol{\Phi}_{\boldsymbol{jj}^*}$ in (16) turns out to be identically zero [10] (i) when the interference is circular, i.e., when $\gamma_I \triangleq E[s_I^2(k)] = 0$; (ii) even though the interference is non-circular, provided that $2f_IT_U \neq h + \frac{\ell}{P}$, for $h, \ell \in \mathbb{Z}$. The latter condition is almost always verified in practice if the interference is adjacent-channel, whereas it does not surely hold for $f_I = 0$, in which case we are forced to assume that the interference is circular. In both cases, the contribution of the interference vanishes, and (16) reduces to

$$\boldsymbol{\Phi}_{\boldsymbol{z}\boldsymbol{z}^*} = \widetilde{\boldsymbol{\mathcal{H}}}(\boldsymbol{c})\,\boldsymbol{\Phi}_{\boldsymbol{s}\boldsymbol{s}^*}\,\widetilde{\boldsymbol{\mathcal{H}}}^T(\boldsymbol{c})\,. \quad (18)$$

Observe that, under assumptions A4 and A5, $\widetilde{\boldsymbol{\mathcal{H}}}(\boldsymbol{c})$ has full column rank $d \triangleq P(L_c + L_e - 1)$, since it is composed by P diagonal blocks $\boldsymbol{\mathcal{H}}(\boldsymbol{c})$, each of full column rank $L_c + L_e - 1$ [14]. This property, together with the observation that $\boldsymbol{\Phi}_{\boldsymbol{s}\boldsymbol{s}^*}$ is full rank, implies that $\boldsymbol{\Phi}_{\boldsymbol{z}\boldsymbol{z}^*}$ is of rank d. Therefore, if we consider the singular value decomposition (SVD) of $\boldsymbol{\Phi}_{\boldsymbol{z}\boldsymbol{z}^*}$, the $NPL_e - d$ smallest singular values are identically zero. Thus, if $\boldsymbol{u}_m$ and $\boldsymbol{v}_m$ denote the mth left and right singular vectors of $\boldsymbol{\Phi}_{\boldsymbol{z}\boldsymbol{z}^*}$, respectively, by partitioning $\boldsymbol{u}_m$ and $\boldsymbol{v}_m$, according to the structure of $\widetilde{\boldsymbol{\mathcal{H}}}(\boldsymbol{c})$, in P distinct (NL_e)-column vectors $\boldsymbol{u}_m = [\boldsymbol{u}_{m1}^T, \boldsymbol{u}_{m2}^T, \ldots, \boldsymbol{u}_{mP}^T]^T$ and $\boldsymbol{v}_m = [\boldsymbol{v}_{m1}^T, \boldsymbol{v}_{m2}^T, \ldots, \boldsymbol{v}_{mP}^T]^T$, one has

$$\boldsymbol{\Pi}_m\,\boldsymbol{\mathcal{H}}(\boldsymbol{c}) = \boldsymbol{O}\,, \quad (19)$$

for $m = d+1, d+2, \ldots, NPL_e$, with $\boldsymbol{\Pi}_m \triangleq [\boldsymbol{U}_m^*, \boldsymbol{V}_m]^T$, where we have defined the $(NL_e) \times P$ matrices $\boldsymbol{U}_m \triangleq [\boldsymbol{u}_{m1}, \ldots, \boldsymbol{u}_{mP}]$ and $\boldsymbol{V}_m \triangleq [\boldsymbol{v}_{m1}, \ldots, \boldsymbol{v}_{mP}]$. Reasoning as in [14], it can be shown that the systems (19), under A4 and A5, allow one to derive the vector $\boldsymbol{c}$ up to a complex scalar factor.

In practice, the TA-CCM $\boldsymbol{\Phi}_{\boldsymbol{z}\boldsymbol{z}^*}$ is estimated from received data and, therefore, only sample estimates $\widehat{\boldsymbol{\Pi}}_m$ of $\boldsymbol{\Pi}_m$ are available. Thus, the matrices $\widehat{\boldsymbol{\Pi}}_m$ and the channel matrix $\boldsymbol{\mathcal{H}}(\boldsymbol{c})$ will be only approximately orthogonal, that is, $\mathrm{vec}\,[\,\widehat{\boldsymbol{\Pi}}_m\,\boldsymbol{\mathcal{H}}(\boldsymbol{c})\,] = \boldsymbol{\epsilon}_m$, for $m = d+1, d+2, \ldots, NPL_e$, where $\boldsymbol{\epsilon}_m$ is the residual and the column vectorization operator $\mathrm{vec}\,[\boldsymbol{A}]$ associates with any matrix $\boldsymbol{A}$ the vector formed by stacking its columns. Thus, we propose to estimate the channel $\boldsymbol{c}$ by minimizing a weighted linear combinations of the squared Euclidean norms of the residuals $\boldsymbol{\epsilon}_m$, that is, the cost function

$$J(\boldsymbol{c}) = \sum_{m=d+1}^{NPL_e} w_m\,\boldsymbol{\epsilon}_m^H\,\boldsymbol{\epsilon}_m \quad \text{subject to} \quad \|\boldsymbol{c}\| = 1\,, \quad (20)$$

where the coefficients w_m are positive scalar numbers. Since the residual vector $\boldsymbol{\epsilon}_m$ can be expressed as $\boldsymbol{\epsilon}_m = \widehat{\boldsymbol{D}}_m\,\boldsymbol{c}$, with $\widehat{\boldsymbol{D}}_m$ depending on $\widehat{\boldsymbol{\Pi}}_m$, the cost function (20) can be written as a quadratic form in $\boldsymbol{c}$

$$J(\boldsymbol{c}) = \boldsymbol{c}^H\,\widehat{\boldsymbol{D}}^H\,(\boldsymbol{W} \otimes \boldsymbol{I})\,\widehat{\boldsymbol{D}}\,\boldsymbol{c} \quad \text{subject to} \quad \|\boldsymbol{c}\| = 1\,, \quad (21)$$

where we refer again to [14] for the detailed expression of matrices $\widehat{\boldsymbol{D}}_m$ and we have defined the data-dependent matrix $\widehat{\boldsymbol{D}} \triangleq [\widehat{\boldsymbol{D}}_{d+1}^T, \widehat{\boldsymbol{D}}_{d+2}^T, \ldots, \widehat{\boldsymbol{D}}_{NPL_e}^T]^T$ and the weighting matrix $\boldsymbol{W} \triangleq \mathrm{diag}\,[w_{d+1}, w_{d+2}, \ldots, w_{NPL_e}]$, with $\mathrm{diag}\,[\cdot]$ denoting the diagonal matrix operator. Using standard eigenstructure concepts, the solution $\widehat{\boldsymbol{c}}$ of the constrained minimization problem (21) is found by computing the eigenvector associated with the smallest eigenvalue of the matrix $\widehat{\boldsymbol{D}}^H\,(\boldsymbol{W} \otimes \boldsymbol{I})\,\widehat{\boldsymbol{D}}$.

As to the choice of the weighting matrix $\boldsymbol{W}$, the matrices $\widehat{\boldsymbol{\Pi}}_m$ are obtained in practice from the SVD of the sample estimate $\widehat{\boldsymbol{\Phi}}_{\boldsymbol{z}\boldsymbol{z}^*}$. Due to the finite sample size, the estimated matrix $\widehat{\boldsymbol{\Phi}}_{\boldsymbol{z}\boldsymbol{z}^*}$ is typically of full rank, thus, its $NPL_e - d$ smallest singular values are no longer zero, and the corresponding singular vectors $\widehat{\boldsymbol{u}}_m$ and $\widehat{\boldsymbol{v}}_m$ are perturbed versions of the true ones. It should be observed that, in the unweighted case (corresponding to $\boldsymbol{W} = \boldsymbol{I}$), all the singular vectors $\widehat{\boldsymbol{u}}_m$ and $\widehat{\boldsymbol{v}}_m$, corresponding to the $NPL_e - d$ smallest singular values of $\widehat{\boldsymbol{\Phi}}_{\boldsymbol{z}\boldsymbol{z}^*}$, will contribute equally to the cost function (20). The simulation results in Section 5 show that this unweighted version suffers a severe performance degradation when the signal-to-interference ratio (SIR) is low. In order to improve the robustness against interference, we suggest a simple and effective weighting strategy, based on the following observation. The loss of identification accuracy is due to perturbation in the estimated singular vectors $\widehat{\boldsymbol{u}}_m$ and $\widehat{\boldsymbol{v}}_m$, which increases with decreasing values of SIR and/or sample size. Intuitively, a simple indicator of the amount of perturbation in $\widehat{\boldsymbol{u}}_m$ and $\widehat{\boldsymbol{v}}_m$ is the corresponding singular value $\widehat{\mu}_m$. Therefore, by choosing $\boldsymbol{W} = \mathrm{diag}\,[\widehat{\mu}_{d+1}^{-1}, \widehat{\mu}_{d+2}^{-1}, \ldots, \widehat{\mu}_{NPL_e}^{-1}]$, the estimated singular vectors in (20) are weighted according to their relative estimation accuracy.

V. Simulation results

In this section, we present the results of Monte Carlo computer simulations, carried out over 500 independent trials, aimed at assessing the performance of the proposed FSR-LPTV equalizer, both in its non-blind (NB) and blind (B) versions. Moreover, the effectiveness of the proposed subspace-based identification algorithms is separately investigated. In all the experiments, the following common simulation setting is assumed. The desired signal constellation is BPSK, the ratio T_U/T_I is $3/4$ (i.e., the period of the considered LPTV equalizers is $P = 4$), and the oversampling factor is set to $N = 3$. The interfering signal is a QPSK CCI[4], with $f_I = 0$, which implies that $E[s_I^2(k)] = 0$. The channels $c_a(t)$ and $c_{I,a}(t)$ are modeled as truncated two-ray multipath channel, whose impulse response is, for $t \in [0, T_0)$,

$$p(t) = a_1\,e^{j2\pi\xi_1}\,g(t-\tau_1) + a_2\,e^{j2\pi\xi_2}\,g(t-\tau_2)\,, \quad (22)$$

where $g(t)$ is a Nyquist-shaped pulse with 35 % excess bandwidth, and $a_1 = 1\,[1]$, $a_2 = 0.8\,[0.5]$, $\xi_1 = 0.15\,[0.09]$, $\xi_2 = 0.6\,[0.5]$, $\tau_1 = 0.25T_U\,[0]$, $\tau_2 = T_U\,[T_I]$, and $T_0 = 5T_U\,[5T_I]$ for the desired signal [for the interference, respectively], which results in $L_c = 5$ for the signal channel. The thermal noise is modeled as a complex circular Gaussian process, and the signal-to-noise ratio (SNR) at the equalizer input is defined as

$$\mathrm{SNR} \triangleq \frac{\sigma_s^2\,\sum_{n=0}^{L_c-1}\|\,\boldsymbol{c}(n)\,\|^2}{E[\,\|\boldsymbol{w}(k)\|^2\,]}\,, \quad (23)$$

which is set to 30 dB in all the experiments, whereas the signal-to-interference ratio (SIR) at the equalizer input is defined as

$$\mathrm{SIR} \triangleq \frac{\sigma_s^2\,\sum_{n=0}^{L_c-1}\|\,\boldsymbol{c}(n)\,\|^2}{\langle E[\,\|\boldsymbol{i}(k)\|^2\,]\rangle}\,. \quad (24)$$

To assess the identification accuracy, we employ the magnitude of the sample correlation coefficient [2] ρ between the estimated

[4]We have also carried out simulations for a BPSK ACI, whose results are not reported here, since they are very similar to the CCI case.

channel vector $\widehat{\boldsymbol{c}}$ and the true one $\boldsymbol{c}$, whereas for evaluating the equalization performance, we adopted the time-averaged signal-to-noise-plus-interference ratio (TA-SINR) at the output of the FSR-LPTV equalizer; both measures are especially useful in a blind setting, since they are insensitive to complex scaling of the estimated channel vector.

A) Identification performance as a function of the window length: In this experiment, we investigated the performance of both the unweighted (Fig. 1) and weighted versions (Fig. 2) of the proposed identification algorithm as a function of the window length L_e used for identification, for a sample size of $K = 2000$ symbols, and for different values of SIR. For the unweighted version, the magnitude of the correlation coefficient increases monotonically with L_e for the highest value of SIR, whereas, for SIR $= 0, -20$ dB, there is a marked performance degradation as L_e increases. This effect is due to the experimental evidence that the dynamic range of the smallest singular values is an increasing function of the dimension NPL_e of the estimated matrix $\widehat{\boldsymbol{\Phi}}_{\boldsymbol{z}\boldsymbol{z}^*}$, and ultimately increases with L_e. The behavior of the unweighted case should be compared with the results for the weighted version: the performance for the higher values of SIR is not significantly improved by weighting, whereas the major benefits can be appreciated for SIR $= 0, -20$ dB, where now the performance increases with increasing L_e.

B) Identification performance versus SIR: For a fixed sample size of $K = 2000$ symbols and $L_e = 5$, we evaluated the identification performance of the proposed method as a function of SIR, in comparison with that of the following methods: (i) the TI-CCM method [5], based on the SVD of the TA-CCM of the received vector $\boldsymbol{z}(k)$; (ii) the TSR-CCM method [5], based on different SVDs of the TA-CCM, one for each polyphase component of the TA-CCM. The values of $|\rho|$ for the latter are obtained as average values over the different polyphase components. Results of Fig. 3 show that the weighted method exhibits better performances for all values of SIR, whereas the TI-CCM suffers a severe performance degradation for low values of SIR, since in this case the conjugate correlation matrix of $\boldsymbol{z}(k)$ is strongly time-varying and, hence, cannot be well approximated by its time-averaged version; finally, the unweighted method and TSR-CCM perform comparably. It should be observed that, although the weighting technique could be applied in principle also to the TSR-CCM, the time-sequenced nature of such a method provides P channel estimates, one for each polyphase component, which (ideally) differ from the true one by different complex scalars; thus, using such estimates in a TSR-based equalization scheme would entail a time-varying scaling/rotation effect on the equalized constellation, which cannot be compensated with simple differential techniques.

C) Blind equalization performance versus SIR: Finally, in Fig. 4 we present a comparative study of the performance of different blind equalizers as a function of SIR, for a sample size $K = 2000$ and $L_e = 5$. The following equalizers are considered: (i) non-blind FSR-LPTV receiver (referred to as NB-FSR in the plots); (ii) blind FSR-LPTV receiver, with unweighted channel identification (referred to as uB-FSR in the plots); (iii) blind FSR-LPTV receiver, with weighted channel identification (referred to as wB-FSR in the plots); (iv) blind FSR-LPTV receiver, with TI-CCM-based channel identification (referred to as B-TI-FSR in the plots). Results for the NB-FSR receiver show that, for the considered scenario, the choice of a zero equalization delay, although not assured to be optimal, provides satisfactory performances, mainly because the vector $\boldsymbol{c}(0)$ contains a significant portion of the desired-signal channel energy; moreover, the performances of the wB-FSR receiver are within a few dB from those of the NB-FSR, whereas the unweighted uB-FSR exhibits a remarkable performance degradation for low values of SIR, and finally the B-TI-FSR exhibits acceptable performances only for the highest values of SIR. The satisfactory performances of the proposed wB-FSR and uB-FSR equalizers are corroborated by the symbol constellation of Fig. 5, for a value of SIR equal to -5 dB. In particular, note the close similarity between the constellation pattern at the output of wB-FSR and NB-FSR equalizers, apart from a phase rotation, which can be easily compensated by using differential techniques.

REFERENCES

[1] B. R. Peterson and D. D. Falconer, "Suppression of adjacent-channel, cochannel, and intersymbol interference by equalizers and linear combiners," *IEEE Trans. Commun.*, vol. 42, pp. 3109–3118, Dec. 1994.

[2] W.A. Gardner, *Introduction to Random Processes*. New York: McGraw-Hill, 1990.

[3] J.H. Reed and T.C. Hsia, "The performance of time-dependent adaptive filters for interference rejection," *IEEE Trans. Acoust. Speech, Signal Processing*, vol. 38, pp. 1373–1385, Aug. 1990.

[4] G. Gelli and F. Verde, "Interference-Resistant LPTV-MMSE Equalization," in *Proc. of X Eur. Signal Processing Conf.*, Tampere, Finland, pp. 1629-1632, 2000.

[5] G. Gelli and F. Verde, "Blind LPTV joint equalization and interference suppression," in *Proc. of 2000 Int. Conf. on Acoustic, Speech, and Signal Processing*, Instanbul, Turkey, pp. 2753-2756, 2000.

[6] G. Gelli and F. Verde, "Two-stage interference-resistant adaptive periodically time-varying CMA blind equalization," *IEEE Trans. Signal Processing*, vol. 50, pp. 662-672, Mar. 2002.

[7] G. Gelli and F. Verde, "Adaptive minimum variance equalization with interference suppression capabilities," *IEEE Commun. Letters* , vol. 5, pp. 491-493, Dec. 2001.

[8] E.R. Ferrara Jr. and B. Widrow, "The time-sequenced adaptive filter," *IEEE Trans. Acoust. Speech, Signal Processing*, vol. 29, pp. 4–14, June 1981.

[9] P.P. Vaidyanathan, *Multirate Systems and Filter Banks*. New Jersey: Prentice Hall, 1993.

[10] F. Verde, "Tecniche di egualizzazione e soppressione di interferenza per sistemi digitali ad elevata capacità," Tesi di Dottorato di Ricerca, Università degli Studi Federico II, Napoli, Nov. 2001 (in italian).

[11] C. Corduneanu, *Almost Periodic Functions*. New York: Interscience (Wiley), 1968.

[12] C.B. Papadias and D.T.M. Slock, "Fractionally spaced equalization of linear polyphase channels and related blind techniques based on multichannel linear prediction," *IEEE Trans. Signal Processing*, vol. 47, pp. 641–653, Mar. 1997.

[13] A.V. Dandawaté and G.B. Giannakis, "Statistical tests for presence of cyclostationarity," *IEEE Trans. Signal Processing*, vol. 42, pp. 2355–2369, Sep. 1994.

[14] K. Abed-Meraim, J.-F. Cardoso, A.Y. Gorokhov, P. Loubaton and E. Moulines, "On subspace methods for blind identification of single-input multiple-output FIR systems," *IEEE Trans. Signal Processing*, vol. 45, pp. 42–55, Jan. 1997.