Adaptive Minimum Variance Equalization with Interference Suppression Capabilities

Giacinto Gelli and Francesco Verde

Abstract

We propose an adaptive minimum variance equalizer, which is able to recover the transmitted symbols even in the presence of severe co-channel interference (CCI) or adjacent-channel interference (ACI). Under the assumption that the interference exhibits a different symbol rate from the desired signal, the proposed equalizer can mitigate CCI/ACI without requiring knowledge of the channel impulse response of the interference.

I. Introduction

Recently, the problem of synthesizing equalization techniques aimed at simultaneously reducing the harmful effects of intersymbol interference (ISI), co-channel interference (CCI) and/or adjacent-channel interference (ACI) in high-speed digital communications systems has been considered [1]. In this work, the equalizer structure is derived under the assumption that the desired and the interference signals exhibit the same symbol rate, which leads to time-invariant structures. Moreover, besides the knowledge of the channel impulse response (CIR) of the desired user, in [1] it is also required that the CIR of the interfering signal is known, which is not a realistic assumption in many cases. However, we have shown in [2], [3] that, when the desired and interference signals exhibit different symbol rates, knowledge of the interference CIR can be avoided by exploiting the cyclostationarity of the CCI and/or ACI, which allowed us to synthesize *linear periodically time-varying* (LPTV) equalizers with remarkable interference suppression capabilities. Unfortunately, adaptive implementations of the minimum mean-square error (MMSE) equalizers [2], [3] are not straightforward, since they correspond to the optimization of a time-varying cost function.

In this letter, we propose an adaptive implementation of the LPTV-MMSE solution adopted in [3], which is found by resorting to Fourier-series representation (FSR). To this aim, we exploit the minimum variance distortionless response (MVDR) optimization criterion, which has been successfully employed, in the context of code-division multiple-access systems, for deriving the well-known minimum output energy (MOE) adaptive multiuser detector [4]. However, the nonstationary nature of the CCI and/or ACI prevents the application to our problem of a simple *time-invariant* equalization technique based on the MOE algorithm. Instead, we first formulate the MMSE LPTV equalization problem in terms of a *time-varying* MVDR optimization criterion. Then, we further show that the optimal LPTV-MVDR equalizer can be reformulated as the solution of a linearly constrained minimum variance (LCMV) optimization problem [5] with multiple linear constraints. Finally, by using the generalized sidelobe canceller (GSC) [6] scheme and applying the well-known recursive least-square (RLS) algorithm, we obtain a fast and accurate adaptive implementation of the considered LPTV-MVDR equalizer.

II. System model and LPTV-MVDR equalization

Let $s(i)$ and $s_I(i)$ denote the symbols of the desired and interference user, respectively. At the transmitter, $s(i)$ and $s_I(i)$ are linearly modulated with different signaling periods T_U and T_I, and transmitted over linear time-invariant (LTI) channels with CIRs $c_a(t)$ and $c_{I,a}(t)$, respectively. The complex envelope of the received signal can be expressed as $r_a(t) = u_a(t) + i_a(t) + w_a(t)$, where

$$u_a(t) = \sum_{i=-\infty}^{\infty} s(i)\, c_a(t - iT_U)\,, \tag{1}$$

$$i_a(t) = \sum_{i=-\infty}^{\infty} s_I(i)\, c_{I,a}(t - iT_I)\, e^{j2\pi f_I t} \tag{2}$$

The authors are with Dipartimento di Ingegneria Elettronica e delle Telecomunicazioni, Università degli Studi Federico II di Napoli, via Claudio 21, I-80125 Napoli, Italy. Tel.: +39-0817683121, Fax : +39-0817683149, E-mail: {gelli,f.verde}@unina.it

are the desired signal and the ACI/CCI, respectively, f_I is the frequency offset of the interference, and, finally, $w_a(t)$ denotes thermal noise. We will assume that: (i) $s(i)$ and $s_I(i)$ are mutually independent zero-mean and independent identically-distributed; (ii) $w_a(t)$ is a wide-sense stationary zero-mean complex circular process, independent from $s(i)$ and $s_I(i)$; (iii) $c_a(t)$ has finite support $[0, L_c T_U)$. The signal $r_a(t)$ is then oversampled at rate N/T_U (with $N > 1$ denoting the oversampling factor). After stacking N consecutive samples of $r_a(t)$ (taken within the kth symbol interval of the desired user), the observations can be regarded as a N-column vector

$$\boldsymbol{r}(k) = \sum_{i=0}^{L_c-1} \boldsymbol{c}(i)\, s(k-i) + \boldsymbol{i}(k) + \boldsymbol{w}(k)\,, \tag{3}$$

where $\boldsymbol{c}(k) \triangleq [\, c_a(kT_U),\, c_a(kT_U + \frac{T_U}{N}),\, \ldots,\, c_a(kT_U + \frac{N-1}{N} T_U)\,]^T$ is the signal channel vector, with T denoting transpose, and $\boldsymbol{i}(k)$ and $\boldsymbol{w}(k)$ are the interference and noise vectors, respectively.

In vector form, the output $\hat{s}(k)$ of a linear equalizer, spanning L_e symbols in the past, can be written as $\hat{s}(k) = \boldsymbol{b}^H(k)\, \boldsymbol{z}(k)$, where $\boldsymbol{b}(k)$ and $\boldsymbol{z}(k) \triangleq [\, \boldsymbol{r}^T(k), \boldsymbol{r}^T(k-1), \ldots, \boldsymbol{r}^T(k - L_e + 1)\,]^T$ are both (NL_e)-column vectors, and H denotes hermitian transpose. It is shown in [2] that, due to the presence of the interference, the statistical correlation matrix $\boldsymbol{R_{zz}}(k) \triangleq E[\boldsymbol{z}(k)\, \boldsymbol{z}^H(k)]$ of the input vector $\boldsymbol{z}(k)$, with $E[\cdot]$ denoting statistical averaging, can be periodically time-varying (PTV) or almost periodically time-varying (APTV) in k, depending on the ratio T_U/T_I. Thus, a conventional LTI filter is expected to perform very poorly in the presence of high-level interference, which led us to consider a time-varying equalizer $\boldsymbol{b}(k)$. To solve for $\boldsymbol{b}(k)$, we resort here to the MVDR criterion [4], that is, we minimize the equalizer output variance $E[|\hat{s}(k)|^2]$ for each value of k, subject to a linear response constraint, which has the effect of preserving the desired symbol $s(k)$. This can be formulated mathematically as

$$\min_{\boldsymbol{b}(k)} \left\{ \boldsymbol{b}^H(k)\, \boldsymbol{R_{zz}}(k)\, \boldsymbol{b}(k) \right\} \quad \text{subject to} \quad \boldsymbol{b}^H(k)\, \boldsymbol{c}_0 = 1\,, \tag{4}$$

where the (NL_e)-column vector $\boldsymbol{c}_0 \triangleq [\boldsymbol{c}^T(0), 0, 0, \ldots, 0]^T$ depends on the signal CIR. Using the method of Lagrange multipliers, it is not difficult to show that the potential solutions of the constrained optimization problem (4) are all vectors $\boldsymbol{b}(k)$ satisfying the linear system $\boldsymbol{R_{zz}}(k)\, \boldsymbol{b}(k) - \lambda(k)\, \boldsymbol{c}_0 = \boldsymbol{0}$, where $\lambda(k)$ is the Lagrange multiplier. To proceed further, although our derivation can be generalized to the APTV case, we concentrate on the case where $T_U/T_I = p_1/P$ is a rational number (with p_1 and P co-prime integers), which implies that $\boldsymbol{R_{zz}}(k)$ is simply periodic in k with period P [2]. In this case, if the periodic functions $\boldsymbol{R_{zz}}(k)$, $\boldsymbol{b}(k)$, and $\lambda(k)$ are expanded in terms of their discrete Fourier series and substituted in the above linear system and in the constraint (4), by straightforward calculations one obtains

$$\frac{1}{P} \sum_{\ell=0}^{P-1} \sum_{m=0}^{P-1} \boldsymbol{\mathcal{R}_{zz}}(\ell - m)_P\, \boldsymbol{\beta}(m)\, e^{j\frac{2\pi}{P}\ell k} = \sum_{\ell=0}^{P-1} \Lambda(\ell)\, \boldsymbol{c}_0\, e^{j\frac{2\pi}{P}\ell k} \tag{5}$$

subject to

$$\frac{1}{P} \sum_{\ell=0}^{P-1} \boldsymbol{\beta}^H(\ell)\, \boldsymbol{c}_0\, e^{j\frac{2\pi}{P}\ell k} = 1\,, \tag{6}$$

where $\boldsymbol{\mathcal{R}_{zz}}(\ell)$, $\boldsymbol{\beta}(l)$, and $\Lambda(\ell)$ denote the ℓth Fourier coefficient of $\boldsymbol{R_{zz}}(k)$, $\boldsymbol{b}(k)$ and $\lambda(k)$, respectively, and $(\cdot)_P$ denotes modulo-P operation. Since identities (5) and (6) hold for all values of k, we can equate factors of corresponding exponential terms, obtaining thus, for $\ell = 0, 1, \ldots, P-1$, the following P linear systems:

$$\widetilde{\boldsymbol{\mathcal{R}}}_{\boldsymbol{zz}}(\ell)\, \boldsymbol{\psi} = \Lambda(\ell)\, \boldsymbol{c}_0 \quad \text{subject to} \quad \boldsymbol{C}_0^H\, \boldsymbol{\psi} = \boldsymbol{j}\,, \tag{7}$$

where $\widetilde{\boldsymbol{\mathcal{R}}}_{\boldsymbol{zz}}(\ell) \triangleq \frac{1}{P}\, [\boldsymbol{\mathcal{R}_{zz}}(\ell)_P, \ldots, \boldsymbol{\mathcal{R}_{zz}}(\ell - P + 1)_P]$, $\boldsymbol{\psi} \triangleq [\boldsymbol{\beta}^T(0), \ldots, \boldsymbol{\beta}^T(P-1)]^T$, and $\boldsymbol{C}_0 \triangleq \boldsymbol{I} \otimes \boldsymbol{c}_0$, with $\boldsymbol{I}$ and $\otimes$ denoting the $P \times P$ identity matrix and the Kronecker product, respectively, and, finally, $\boldsymbol{j} \triangleq [\, P, 0, \ldots, 0\,]^T$ is the P-column response vector.

The systems in (7) can be rewritten as a single system $\boldsymbol{\Phi_{zz}}\, \boldsymbol{\psi} = \boldsymbol{C}_0\, \boldsymbol{\lambda}$, whose solution is $\boldsymbol{\psi_{\lambda}} = \boldsymbol{\Phi}_{\boldsymbol{zz}}^{-1}\, \boldsymbol{C}_0\, \boldsymbol{\lambda}$, where $\boldsymbol{\Phi_{zz}} \triangleq [\widetilde{\boldsymbol{\mathcal{R}}}_{\boldsymbol{zz}}^T(0), \ldots, \widetilde{\boldsymbol{\mathcal{R}}}_{\boldsymbol{zz}}^T(P-1)]^T$ and $\boldsymbol{\lambda} \triangleq [\Lambda(0), \ldots, \Lambda(P-1)]^T$. By imposing that $\boldsymbol{\psi_{\lambda}}$ satisfies the constraints

(7), the FSR-based solution for (4) is given by

$$\boldsymbol{\psi}_{\mathrm{LCMV}} = \boldsymbol{\Phi}_{\boldsymbol{zz}}^{-1} \boldsymbol{C}_0 \ \left(\boldsymbol{C}_0^H \boldsymbol{\Phi}_{\boldsymbol{zz}}^{-1} \boldsymbol{C}_0\right)^{-1} \boldsymbol{j} \, . \tag{8}$$

Note that the optimum weight vector $\boldsymbol{\psi}_{\mathrm{LCMV}}$ can also be regarded as resulting from minimization of the time-averaged output power $\langle E[|\hat{s}(k)|^2] \rangle$ subject to the P linear constraints expressed by $\boldsymbol{C}_0^H \boldsymbol{\psi} = \boldsymbol{j}$. In other words, we have turned the LPTV-MVDR problem (4) with a single constraint into an equivalent time-averaged LCMV optimization problem with multiple constraints, which has the same mathematical form as that considered in [5] in the framework of array processing. Such an equivalence allows one to decompose the weight vector $\boldsymbol{\psi}$, according to the GSC scheme [6], into two orthogonal components, as $\boldsymbol{\psi} = \boldsymbol{\psi}_0 - \boldsymbol{\Pi}_0 \boldsymbol{\psi}_a$, where the quiescent vector is given by $\boldsymbol{\psi}_0 = \boldsymbol{C}_0 (\boldsymbol{C}_0^H \boldsymbol{C}_0)^{-1} \boldsymbol{j}$ and the $(NPL_e) \times (NL_e - 1)P$ signal blocking matrix $\boldsymbol{\Pi}_0$ is chosen such as its columns form a basis for the null space of $\boldsymbol{C}_0$ [6]. Finally, the optimal solution of the data-dependent component $\boldsymbol{\psi}_a$ is given by $\boldsymbol{\psi}_a = \boldsymbol{\Xi}_{\boldsymbol{zz}}^{-1} \boldsymbol{\Pi}_0^H \boldsymbol{\Phi}_{\boldsymbol{zz}} \boldsymbol{\psi}_0$, with $\boldsymbol{\Xi}_{\boldsymbol{zz}} \stackrel{\triangle}{=} \boldsymbol{\Pi}_0^H \boldsymbol{\Phi}_{\boldsymbol{zz}} \boldsymbol{\Pi}_0$. In [6], the component $\boldsymbol{\psi}_a$ is adaptively estimated from the data using the normalized least-mean square (LMS) algorithm. However, in general, LMS-based algorithms exhibit relatively slow convergence which, moreover, is heavily dependent on the eigenvalue spread of $\boldsymbol{\Phi}_{\boldsymbol{zz}}$. To overcome this drawback, in the simulations reported in the next section, we consider a version of the RLS algorithm tailored to the GSC case [7], which exhibits much faster convergence with a tolerable increase in complexity.

III. Numerical results

In this section, the performances of the proposed adaptive equalizers are compared with those of the following receivers: (i) the batch version of the LPTV-MVDR receiver (8); (ii) the batch version of the conventional LTI-MVDR. The desired signal constellation is BPSK, the interfering signal is a QPSK CCI with $f_I = 0$, the ratio T_U/T_I is $3/4$, i.e., $P = 4$. The channels $c_a(t)$ and $c_{I,a}(t)$ are modeled as in [3], with $L_c = 5$ for the signal channel. The thermal noise is a complex circular Gaussian process, and the signal-to-noise ratio (SNR) is set to 30 dB. The oversampling factor is set to $N = 3$ and the equalizer length is set to $L_e = 5$ for all considered equalizers. As performance measure, we choose the time-averaged signal-to-interference-plus-noise ratio (TA-SINR) at the output of the considered equalizers, averaged over 100 Monte Carlo trials.

Figure 1 shows the TA-SINR normalized to the corresponding value obtained with the batch version of the LPTV-MVDR receiver, as a function of the number of iterations and for different SIR values. Remarkably, the performances of the RLS version of the proposed equalizer are the same for all the different SIR values (the RLS curves are practically indistinguishable), whereas the performances of the LMS version are heavily dependent on SIR values.

Figure 2 reports the steady-state performances as a function of SIR, evaluated over 2000 symbols and neglecting the first 400 ones. The curves show that the performances of the RLS version of the proposed equalizer are quite robust to SIR variations and follow closely those of its batch counterpart LPTV-MVDR, whereas those of the LMS one exhibit an increasing degradation with decreasing SIR. Note, moreover, the performance advantage of all LPTV receivers over the LTI one, whose performances are unacceptable for moderate-to-low values of SIR.

References

[1] B.R. Peterson and D.D. Falconer, "Suppression of Adjacent-Channel, Cochannel, and Intersymbol Interference by Equalizers and Linear Combiners," *IEEE Trans. Commun.*, vol. 42, pp. 3109–3118, Dec. 1994.

[2] G. Gelli and F. Verde, "Interference-Resistant LPTV-MMSE Equalization," in *Proc. of X European Signal Processing Conference*, Tampere (Finland), 2000, pp. 1629-1632.

[3] G. Gelli and F. Verde, "FSR-LPTV filtering for blind equalization and interference rejection," in *Proc. of 2nd Int. Symposium on Image and Signal Processing and Analysis*, Pula (Croatia), 2001, pp. 616-621.

[4] M. Honig, U. Madhow, and S. Verdù, "Blind Adaptive Multiuser Detection," *IEEE Trans. Inform. Theory*, vol. 41, pp. 944–960, July 1995.

[5] O.L. Frost, "An Algorithm for Linearly Constrained Adaptive Array Processing ," *Proc. of IEEE*, vol. 60, pp. 926–935, Aug. 1972.

[6] L.J. Griffiths and C.W. Jim, "An Alternative Approach to Linearly Constrained Adaptive Beamforming," *IEEE Trans. Antennas and Propagat.*, vol. 30, pp. 27–34, Jan. 1982.

[7] S. Yu and J. Lee, "Adaptive Array Beamforming Based on an Efficient Technique," *IEEE Trans. Antennas and Propagat.*, vol. 44, pp. 1094–1101, Aug. 1996.

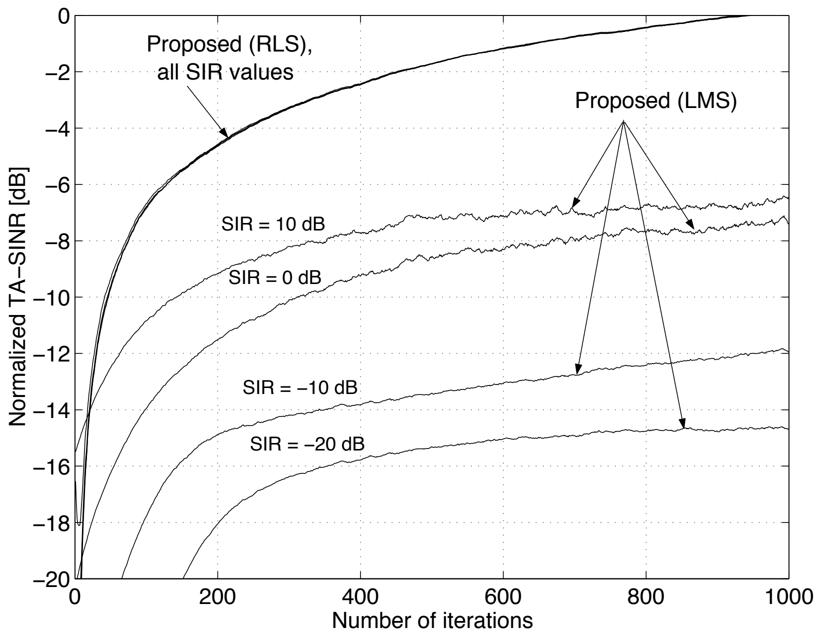

Fig. 1. Normalized TA-SINR versus number of iterations for the adaptive equalizers (SIR equal to $-20, -10, 0$, and 10 dB).

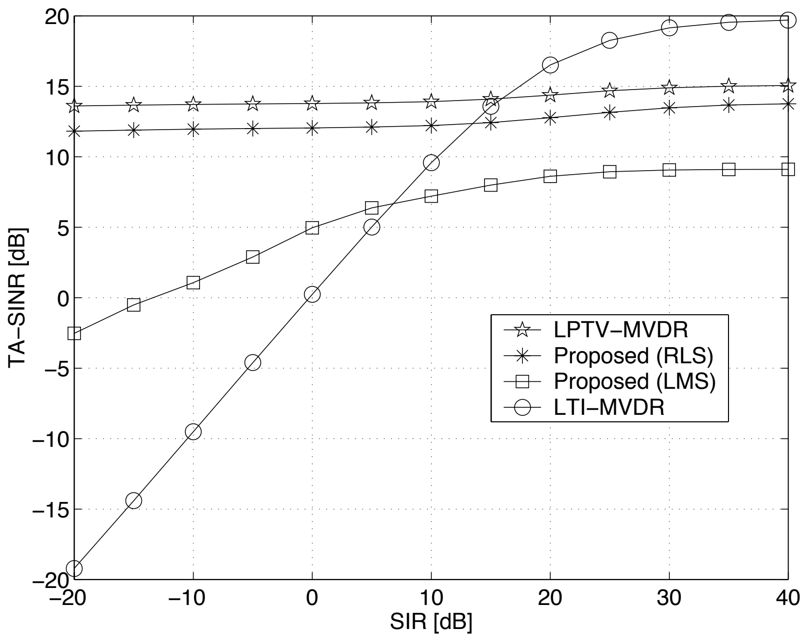

Fig. 2. TA-SINR versus SIR for the different equalizers.