NBI-Resistant Zero-Forcing Equalizers for OFDM Systems

Donatella Darsena, Giacinto Gelli, Luigi Paura, and Francesco Verde

***Abstract*— The synthesis and the performance analysis of interference-resistant finite-impulse response (FIR) zero-forcing receivers for orthogonal frequency-division multiplexing (OFDM) systems is considered. Specifically, by judiciously exploiting the redundancy contained in the cyclic prefix, a FIR-ZF minimum output-energy equalizer is proposed, which generalizes and outperforms existing receivers. Furthermore, the narrowband interference (NBI) rejection capabilities of the proposed equalizer are analyzed both theoretically and experimentally.**

***Index Terms*—**

I. Introduction and Preliminaries

LET us consider an orthogonal frequency-division multiplexing (OFDM) system with M subcarriers and a cyclic prefix (CP) of length $L_{\text{cp}} \ll M$, whose input information stream $s(n)$ is modeled as a zero-mean sequence of independent and identically distributed (i.i.d.) complex random variables, with variance $\sigma_s^2 \triangleq \text{E}[|s(n)|^2]$ and symbol period T, with $\text{E}[\cdot]$ denoting ensemble averaging. A block $\boldsymbol{s}(n) \triangleq [s(n\,M), s(n\,M+1), \ldots, s(n\,M+M-1)]^T$ of M consecutive symbols, with $(\cdot)^T$ denoting transpose, is subject to standard OFDM precoding $\boldsymbol{u}(n) \triangleq \mathbf{T}_0\, \boldsymbol{s}(n)$, with $\mathbf{T}_0 \triangleq \mathbf{T}_{\text{cp}}\, \mathbf{W}_{\text{IDFT}} \in \mathbb{C}^{P \times M}$, where $\mathbf{W}_{\text{IDFT}} \in \mathbb{C}^{M \times M}$ denotes the unitary inverse discrete Fourier transform (IDFT) and $\mathbf{T}_{\text{cp}} \triangleq [\mathbf{I}_{\text{cp}}^T, \mathbf{I}_M] \in \mathbb{R}^{P \times M}$, with $P \triangleq M + L_{\text{cp}}$, accounts for the CP insertion, whereas $\mathbf{I}_M$ denotes the $M \times M$ identity matrix and $\mathbf{I}_{\text{cp}} \in \mathbb{R}^{L_{\text{cp}} \times M}$ is obtained from $\mathbf{I}_M$ by picking its last L_{cp} rows. Vector $\boldsymbol{u}(n)$ undergoes parallel-to-serial conversion, and the resulting sequence feeds a digital-to-analog converter, having impulse response $\phi_c(t)$ and operating at rate $1/T_c = P/T$. The transmitted signal $u_c(t)$ propagates through a linear time-invariant channel with impulse response $g_c(t)$; the received baseband signal is given by $\widetilde{r}_c(t) = u_c(t) \star g_c(t) \star \psi_c(t) + \widetilde{\jmath}_c(t) + \widetilde{w}_c(t)$, where $\star$ denotes linear convolution, $\psi_c(t)$ is the impulse response of the receiver filter, $\widetilde{\jmath}_c(t)$ and $\widetilde{w}_c(t)$ account for the narrowband interference (NBI) and thermal noise, respectively, at the output of the receiver filter. After sampling $\widetilde{r}_c(t)$ at the time instants $t_{n,\ell} \triangleq n\,T + \ell\,T_c$, with $n \in \mathbb{Z}$ and $\ell \in \{0, 1, \ldots, P-1\}$, and gathering the samples $\widetilde{r}_\ell(n) \triangleq \widetilde{r}_c(t_{n,\ell})$ into the vector $\widetilde{\boldsymbol{r}}(n) \triangleq [\widetilde{r}_0(n), \widetilde{r}_1(n), \ldots, \widetilde{r}_{P-1}(n)]^T \in \mathbb{C}^P$, one has (see, e.g., [1])

$$\widetilde{\boldsymbol{r}}(n) = \widetilde{\mathbf{H}}_0\, \mathbf{T}_0\, \boldsymbol{s}(n) + \widetilde{\mathbf{H}}_1\, \mathbf{T}_0\, \boldsymbol{s}(n-1) + \widetilde{\boldsymbol{\jmath}}(n) + \widetilde{\boldsymbol{w}}(n)\ , \quad (1)$$

where $\widetilde{\boldsymbol{\jmath}}(n) \in \mathbb{C}^P$ and $\widetilde{\boldsymbol{w}}(n) \in \mathbb{C}^P$ are defined similarly to $\widetilde{\boldsymbol{r}}(n)$, whereas $\widetilde{\mathbf{H}}_0$ and $\widetilde{\mathbf{H}}_1 \in \mathbb{C}^{P \times P}$ are Toeplitz lower- and upper-triangular channel matrices, whose first column and row are given by $[h(0), h(1), \ldots, h(L_h), 0, \ldots, 0]^T$ and $[0, \ldots, 0, h(L_h), h(L_h - 1), \ldots, h(1)]$, whereas $h(m) \triangleq [\phi_c(t) \star g_c(t) \star \psi_c(t)]_{t=mT_c}$ is a causal finite impulse response (FIR) filter of order[1] $L_h < L_{\text{cp}}$. For the sake of analysis, it is assumed[2] that the noise samples $\widetilde{w}(n) \triangleq \widetilde{w}_c(n\,T_c)$ are zero-mean wide-sense stationary uncorrelated complex random variables, with variance $\sigma_w^2 \triangleq \text{E}[|\widetilde{w}(n)|^2]$, independent of both $s(n)$ and $\widetilde{\jmath}_c(t)$.

The presence of the NBI, which arises in both wireless (e.g., overlay systems or systems transmitting in non-licensed bands) and wireline (e.g., crosstalk or radio-frequency interference) broadband communications, degrades significantly the performance of the conventional zero-forcing (ZF) receiver [1]. To synthesize NBI-resistant FIR-ZF equalizers, a sensible approach, known as receiver windowing (see, e.g., [2]), consists of additionally processing a suitable portion, so-called unconsumed, of the CP. However, the receiver windowing approaches proposed so far do not fully exploit the redundancy contained in the unconsumed portion of the CP. The contribution of this letter is twofold. First, we derive the *canonical* structure of the FIR-ZF receiver that elaborates the unconsumed portion of the CP and fully exploits all the available degrees of freedom for NBI suppression, by resorting to the *minimum output-energy* (MOE) optimization criterion [3]. Second, by modeling the NBI as a tone interferer, which is common practice [4] for the analysis of NBI-contaminated spread-spectrum systems, we theoretically assess the NBI rejection capabilities of the proposed equalizer, and derive in particular the conditions that assure perfect NBI suppression.

II. FIR Zero-Forcing MOE Equalization

Initially, we neglect the disturbance (NBI-plus-noise) and employ a linear FIR equalizer $\widetilde{\boldsymbol{\mathcal{G}}} \in \mathbb{C}^{M \times P}$, whose input-output relation is $\boldsymbol{y}(n) = \widetilde{\boldsymbol{\mathcal{G}}}\, \widetilde{\boldsymbol{r}}(n)$. From eq. (1), it is shown that ZF equalization is obtained by imposing $\widetilde{\boldsymbol{\mathcal{G}}}\, \widetilde{\mathbf{H}}_0 \mathbf{T}_0 = \mathbf{I}_M$ and $\widetilde{\boldsymbol{\mathcal{G}}}\, \widetilde{\mathbf{H}}_1 \mathbf{T}_0 = \mathbf{O}_{M \times M}$, where $\mathbf{O}_{M \times M}$ denotes the $M \times M$ zero matrix. Let us first consider the second ZF condition, which assures elimination of the *interblock interference* (IBI). By exploiting the Toeplitz upper-triangular structure of $\widetilde{\mathbf{H}}_1$, it results [1] that $\widetilde{\boldsymbol{\mathcal{G}}} = \boldsymbol{\mathcal{G}}\, \mathbf{R}_{L_h}$, where $\mathbf{R}_{L_h} \triangleq [\mathbf{O}_{N \times L_h}, \mathbf{I}_N] \in \mathbb{R}^{N \times P}$ and $\boldsymbol{\mathcal{G}} \in \mathbb{C}^{M \times N}$ is arbitrary, with $N \triangleq P - L_h$,

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[1]If L_h exceeds L_{cp}, one can resort to a preventive channel-shortening equalization to fulfill the assumption $L_h < L_{\text{cp}}$.

[2]This can be accomplished by imposing that the receiver matched filter $\psi_c(t)$ has a root-raised-cosine spectrum.

i.e., perfect IBI cancellation is obtained by discarding the first L_h samples of $\widetilde{r}(n)$. Accounting for the IBI-free structure of $\mathcal{G}$, the first ZF condition, which assures elimination of the *intercarrier interference* (ICI), becomes $\mathcal{G}\,\mathbf{H}_0\mathbf{T}_0 = \mathbf{I}_M$, where $\mathbf{H}_0 \triangleq \mathbf{R}_{L_h}\,\widetilde{\mathbf{H}}_0 \in \mathbb{C}^{N\times P}$. Since $L_{\text{cp}} > L_h$, implying thus $N > M$, the matrix equation $\mathcal{G}\,\mathbf{H}_0\mathbf{T}_0 = \mathbf{I}_M$ admits solutions [5] if and only if (iff) $\mathbf{H}_0\mathbf{T}_0 = \mathbf{H}_0\mathbf{T}_{\text{cp}}\mathbf{W}_{\text{IDFT}}$ is a full-column rank matrix, i.e., $\text{rank}(\mathbf{H}_0\mathbf{T}_0) = M$. Observe now that $\mathbf{H}_0\mathbf{T}_{\text{cp}}$ can be conveniently factorized as $\mathbf{H}_0\mathbf{T}_{\text{cp}} = \mathbf{\Phi}\,\mathbf{H}_{\text{circ}}$, where $\mathbf{\Phi} \triangleq [\mathbf{I}_\phi^T, \mathbf{I}_M]^T \in \mathbb{R}^{N\times M}$, with $\mathbf{I}_\phi \in \mathbb{R}^{(N-M)\times M}$ obtained from $\mathbf{I}_M$ by picking its last $N-M$ rows, and $\mathbf{H}_{\text{circ}} \in \mathbb{C}^{M\times M}$ is the circulant matrix having as first column the vector $[h(0), \ldots, h(L_h), 0, \ldots, 0]^T$. Furthermore, $\mathbf{H}_{\text{circ}}$ can be decomposed [1] as $\mathbf{H}_{\text{circ}} = \mathbf{W}_{\text{IDFT}}\,\boldsymbol{\mathcal{H}}\,\mathbf{W}_{\text{DFT}}$, where $\boldsymbol{\mathcal{H}} \triangleq \text{diag}[H(1), H(e^{j2\pi/M}), \ldots, H(e^{j2\pi(M-1)/M})]$, with $H(z) \triangleq \sum_{n=0}^{L_h} h(n)\,z^{-n}$ denoting the channel transfer function, and $\mathbf{W}_{\text{DFT}} = \mathbf{W}_{\text{IDFT}}^{-1}$. Since $\mathbf{\Phi}$ is full-column rank, it follows that $\text{rank}(\mathbf{H}_0\mathbf{T}_0) = \text{rank}(\mathbf{\Phi}\,\mathbf{W}_{\text{IDFT}}\,\boldsymbol{\mathcal{H}}) = M$ iff $H(z)$ has no zero located at $\{e^{j2\pi m/M}\}_{m=0}^{M-1}$, which is the standard OFDM equalizability condition [1].

Assuming that $\mathbf{H}_0\mathbf{T}_0$ is full-column rank, the general solution of the ICI-free equation can be decomposed [5] as $\mathcal{G}_{\text{zf}} = \mathcal{G}_{\text{zf}}^{(\text{f})} + \mathcal{G}_{\text{zf}}^{(\text{a})}$, where $\mathcal{G}_{\text{zf}}^{(\text{f})} = (\mathbf{H}_0\mathbf{T}_0)^\dagger = \boldsymbol{\mathcal{H}}^{-1}\mathbf{W}_{\text{DFT}}\,\mathbf{\Phi}^\dagger$, with $\mathbf{\Phi}^\dagger = (\mathbf{\Phi}^T\mathbf{\Phi})^{-1}\mathbf{\Phi}^T$ denoting the Moore-Penrose inverse of $\mathbf{\Phi}$, represents the *minimum-norm* solution of the ICI-free equation, whereas $\mathcal{G}_{\text{zf}}^{(\text{a})}$ is the general solution of the homogeneous equation $\mathcal{G}_{\text{zf}}^{(\text{a})}\,\mathbf{H}_0\mathbf{T}_0 = \mathbf{O}_{M\times M}$. Observe that $\mathcal{G}_{\text{zf}}^{(\text{a})}$ is the general solution of $\mathcal{G}_{\text{zf}}^{(\text{a})}\,\mathbf{H}_0\mathbf{T}_0 = \mathbf{O}_{M\times M}$ iff the column space $\mathcal{R}[(\mathcal{G}_{\text{zf}}^{(\text{a})})^H]$ of its conjugate transpose $(\mathcal{G}_{\text{zf}}^{(\text{a})})^H$ is contained in the orthogonal complement $\mathcal{R}^\perp(\mathbf{H}_0\mathbf{T}_0)$ of the column space of $\mathbf{H}_0\mathbf{T}_0$. Since $\mathbf{H}_0\mathbf{T}_0$ is full-column rank, it follows that $\mathcal{R}^\perp(\mathbf{H}_0\mathbf{T}_0)$ has dimension $N-M$ and, thus, after straightforward algebraic manipulations, $\mathcal{G}_{\text{zf}}^{(\text{a})}$ can be expressed as $\mathcal{G}_{\text{zf}}^{(\text{a})} = \boldsymbol{\mathcal{Y}}\,\mathbf{\Pi}$, where $\boldsymbol{\mathcal{Y}} \in \mathbb{C}^{M\times(N-M)}$ is arbitrary and can be further optimized, whereas the *signal blocking matrix* $\mathbf{\Pi} \in \mathbb{C}^{(N-M)\times N}$ satisfies the equation $\mathbf{\Pi}\,\mathbf{H}_0\mathbf{T}_0 = \mathbf{\Pi}\,\mathbf{\Phi}\,\mathbf{W}_{\text{IDFT}}\,\boldsymbol{\mathcal{H}} = \mathbf{O}_{(N-M)\times M}$, whose particular solution is $\mathbf{\Pi} = [\mathbf{I}_{N-M}, \mathbf{O}_{(N-M)\times(2M-N)}, -\mathbf{I}_{N-M}]$. Thus, we can state that the *canonical* structure of a FIR-ZF receiver is

$$\mathcal{G}_{\text{zf}} = \mathcal{G}_{\text{zf}}^{(\text{f})} + \boldsymbol{\mathcal{Y}}\,\mathbf{\Pi}\,. \tag{2}$$

When the dimension of $\mathcal{R}^\perp(\mathbf{H}_0\mathbf{T}_0)$ is zero, i.e., $N = M$ or, equivalently, $L_{\text{cp}} = L_h$, one has $\mathcal{G}_{\text{zf}}^{(\text{a})} = \mathbf{O}_{M\times N}$ and, consequently, the ZF solution (2) turns out to be unique, degenerating into the well-known conventional OFDM receiver $\mathcal{G}_{\text{zf-conv}} \triangleq \boldsymbol{\mathcal{H}}^{-1}\mathbf{W}_{\text{DFT}}$; in other words, when the CP is entirely removed, it is *not* possible to further optimize the receiver for rejecting the NBI. Interestingly, a disturbance-resistant FIR-ZF equalizer can be synthesized by removing only a portion $L_h < L_{\text{cp}}$ of the CP[3]; in this case, the dimension $D \triangleq N-M = L_{\text{cp}}-L_h$ of $\mathcal{R}^\perp(\mathbf{H}_0\mathbf{T}_0)$ represents the number of *available degrees of freedom* for disturbance suppression.

We propose now to resort to a simple and effective optimization criterion which assures satisfactory disturbance suppression, *without* requiring knowledge of the statistics of both the NBI and noise. Accounting for eq. (1), the output $\boldsymbol{y}_{\text{zf}}(n)$ of the canonical ZF receiver (2) can be explicitly written as $\boldsymbol{y}_{\text{zf}}(n) = \mathcal{G}_{\text{zf}}\,\boldsymbol{r}(n) = \boldsymbol{s}(n) + \mathcal{G}_{\text{zf}}\,\boldsymbol{d}(n)$, where $\boldsymbol{r}(n) \triangleq \mathbf{R}_{L_h}\,\widetilde{\boldsymbol{r}}(n) \in \mathbb{C}^N$ and $\boldsymbol{d}(n) \triangleq \boldsymbol{\jmath}(n) + \boldsymbol{w}(n) \in \mathbb{C}^N$, with $\boldsymbol{\jmath}(n) \triangleq \mathbf{R}_{L_h}\,\widetilde{\boldsymbol{\jmath}}(n) \in \mathbb{C}^N$ and $\boldsymbol{w}(n) \triangleq \mathbf{R}_{L_h}\,\widetilde{\boldsymbol{w}}(n) \in \mathbb{C}^N$. To mitigate the disturbance, the proposed strategy adopts the MOE criterion [3], namely, the energy at the equalizer output is minimized by solving, with respect to $\boldsymbol{\mathcal{Y}}$, the minimization problem $\min_{\boldsymbol{\mathcal{Y}}} \text{E}[\|\boldsymbol{y}_{\text{zf}}(n)\|^2] = \min_{\boldsymbol{\mathcal{Y}}} \text{trace}[\mathcal{G}_{\text{zf}}\,\mathbf{R}_{\boldsymbol{rr}}\,\mathcal{G}_{\text{zf}}^H]$, where $\mathbf{R}_{\boldsymbol{rr}} \triangleq \text{E}[\boldsymbol{r}(n)\,\boldsymbol{r}^H(n)]$ is the autocorrelation matrix of $\boldsymbol{r}(n)$, while $\|\cdot\|$ and $\text{trace}[\cdot]$ represent the Frobenius norm and the trace operator, respectively. The solution of this MOE problem is (see, e.g., [3])

$$\boldsymbol{\mathcal{Y}}_{\text{moe}} = -\mathcal{G}_{\text{zf}}^{(\text{f})}\,\mathbf{R}_{\boldsymbol{rr}}\,\mathbf{\Pi}^H(\mathbf{\Pi}\,\mathbf{R}_{\boldsymbol{rr}}\,\mathbf{\Pi}^H)^{-1}\,. \tag{3}$$

By substituting eq. (3) in eq. (2), the proposed MOE receiver is characterized by

$$\mathcal{G}_{\text{zf-moe}} = \mathcal{G}_{\text{zf}}^{(\text{f})} - \mathcal{G}_{\text{zf}}^{(\text{f})}\,\mathbf{R}_{\boldsymbol{rr}}\,\mathbf{\Pi}^H(\mathbf{\Pi}\,\mathbf{R}_{\boldsymbol{rr}}\,\mathbf{\Pi}^H)^{-1}\,\mathbf{\Pi}\,. \tag{4}$$

III. Interference Rejection Analysis

We consider a tone NBI, whose baseband model [4] is $\widetilde{\jmath}_c(t) = \alpha_I\,e^{j(2\pi f_I t+\theta)}$, where $\alpha_I \in \mathbb{R}$ is a deterministic amplitude, f_I represents the frequency offset from the carrier frequency, and θ is a random variable uniformly distributed in $[0, 2\pi)$. In this case, the autocorrelation matrix of $\boldsymbol{\jmath}(n)$ has the form $\mathbf{R}_{\boldsymbol{\jmath\jmath}} \triangleq \text{E}[\boldsymbol{\jmath}(n)\,\boldsymbol{\jmath}^H(n)] = \alpha_I^2\boldsymbol{\xi}_I\,\boldsymbol{\xi}_I^H$, where $\boldsymbol{\xi}_I \triangleq [1, e^{j2\pi\gamma_I}, \ldots, e^{j2\pi(N-1)\gamma_I}]^T \in \mathbb{C}^N$, with $\gamma_I \triangleq f_I\,T_c$. Without loss of generality, we consider only values of $\gamma_I \in [0, 1)$ by setting $\gamma_I = \delta_I/M$, with $\delta_I \in [0, M)$. Hence, the autocorrelation matrix of the disturbance $\boldsymbol{d}(n)$ is given by $\mathbf{R}_{\boldsymbol{dd}} \triangleq \text{E}[\boldsymbol{d}(n)\,\boldsymbol{d}^H(n)] = \alpha_I^2\boldsymbol{\xi}_I\,\boldsymbol{\xi}_I^H + \sigma_w^2\,\mathbf{I}_N$ and, due to the signal blocking property of $\mathbf{\Pi}$, it results that $\mathbf{\Pi}\,\mathbf{R}_{\boldsymbol{rr}} = \mathbf{\Pi}\,\mathbf{R}_{\boldsymbol{dd}}$, which implies that eq. (3) can also be expressed as $\boldsymbol{\mathcal{Y}}_{\text{moe}} = -\mathcal{G}_{\text{zf}}^{(\text{f})}\,\mathbf{R}_{\boldsymbol{dd}}\,\mathbf{\Pi}^H(\mathbf{\Pi}\,\mathbf{R}_{\boldsymbol{dd}}\,\mathbf{\Pi}^H)^{-1}$. As a performance measure, we employ the signal-to-interference-plus-noise ratio (SINR) at the equalizer output, which is defined as $\text{SINR}(\mathcal{G}_{\text{zf-moe}}) \triangleq \text{E}[\|\boldsymbol{s}(n)\|^2]/\text{E}[\|\mathcal{G}_{\text{zf-moe}}\,\boldsymbol{d}(n)\|^2] = (M\,\sigma_s^2)/\mathcal{P}_d(\mathcal{G}_{\text{zf-moe}})$, where the disturbance output-power $\mathcal{P}_d(\mathcal{G}_{\text{zf-moe}}) \triangleq \text{trace}(\mathcal{G}_{\text{zf-moe}}\,\mathbf{R}_{\boldsymbol{dd}}\,\mathcal{G}_{\text{zf-moe}}^H)$ depends also on the noise variance σ_w^2.

Since the performance of interference-contaminated OFDM systems is mainly limited by the NBI, we evaluate $\mathcal{P}_d(\mathcal{G}_{\text{zf-moe}})$ in the high signal-to-noise ratio (SNR) region, i.e., as σ_w^2 approaches zero. We preliminarily observe that, accounting for eq. (4), after some algebra, one has

$$\mathcal{P}_d(\mathcal{G}_{\text{zf-moe}}) = \text{trace}[\mathcal{G}_{\text{zf}}^{(\text{f})}\,\mathbf{R}_{\boldsymbol{dd}}\,(\mathcal{G}_{\text{zf}}^{(\text{f})})^H] - \text{trace}[\mathcal{G}_{\text{zf}}^{(\text{f})}\,\mathbf{R}_{\boldsymbol{dd}}\,\mathbf{\Pi}^T(\mathbf{\Pi}\,\mathbf{R}_{\boldsymbol{dd}}\,\mathbf{\Pi}^T)^{-1}\mathbf{\Pi}\,\mathbf{R}_{\boldsymbol{dd}}\,(\mathcal{G}_{\text{zf}}^{(\text{f})})^H]\,. \tag{5}$$

When $\sigma_w^2 \to 0$, evaluation of $\mathcal{P}_d(\mathcal{G}_{\text{zf-moe}})$ is complicated by the fact that $\mathbf{R}_{\boldsymbol{dd}}$ becomes singular and, thus, the inverse $(\mathbf{\Pi}\,\mathbf{R}_{\boldsymbol{dd}}\,\mathbf{\Pi}^T)^{-1}$ does not exist. By using the properties of generalized inverses [5], it can be proved that $\lim_{\sigma_w^2\to 0}[\mathbf{R}_{\boldsymbol{dd}}\,\mathbf{\Pi}^T(\mathbf{\Pi}\,\mathbf{R}_{\boldsymbol{dd}}\,\mathbf{\Pi}^T)^{-1}] = \boldsymbol{\xi}_I\,(\mathbf{\Pi}\,\boldsymbol{\xi}_I)^\dagger$. Accounting for the expression of $\mathbf{R}_{\boldsymbol{dd}}$, it follows that

$$\lim_{\sigma_w^2\to 0} \mathcal{P}_d(\mathcal{G}_{\text{zf-moe}}) = \alpha_I^2\,\Big\{\|\mathcal{G}_{\text{zf}}^{(\text{f})}\,\boldsymbol{\xi}_I\|^2 - \text{trace}[\mathcal{G}_{\text{zf}}^{(\text{f})}\,\boldsymbol{\xi}_I\,(\mathbf{\Pi}\,\boldsymbol{\xi}_I)^\dagger\,\mathbf{\Pi}\,\boldsymbol{\xi}_I\,\boldsymbol{\xi}_I^H\,(\mathcal{G}_{\text{zf}}^{(\text{f})})^H]\Big\}\,. \tag{6}$$

[3]For the sake of clarity, it is assumed the exact knowledge of L_h. In practice, when only an upper bound $L \geq L_h$ is available, one must resort to a suboptimal solution by discarding the first L samples of $\widetilde{r}(n)$.

It is interesting to note that, when the CP is completely removed ($N = M$) and, thus, the proposed equalizer degenerates into the conventional receiver $\boldsymbol{\mathcal{G}}_{\text{zf-conv}}$, eq. (6) reduces to

$$\lim_{\sigma_w^2 \to 0} \mathcal{P}_d(\boldsymbol{\mathcal{G}}_{\text{zf-conv}}) = \alpha_I^2 \, \|\boldsymbol{\mathcal{H}}^{-1} \mathbf{W}_{\text{DFT}} \, \boldsymbol{\xi}_I\|^2 \, . \tag{7}$$

Since $\boldsymbol{\mathcal{H}}^{-1} \mathbf{W}_{\text{DFT}}$ is nonsingular, the NBI contribution cannot be nullified for any $\boldsymbol{\xi}_I$, i.e., as previously stated, the conventional receiver does not exhibit any interference suppression capability. This can be clearly evidenced by considering an ideal channel, i.e., $h(m)$ is the Kronecker delta: in this case, from eq. (7), one has $\|\boldsymbol{\mathcal{H}}^{-1} \mathbf{W}_{\text{DFT}} \, \boldsymbol{\xi}_I\|^2 = \|\mathbf{W}_{\text{DFT}} \, \boldsymbol{\xi}_I\|^2 = M$; consequently, the output SINR turns out to be independent of the spectral position δ_I of the NBI and assumes the value σ_s^2/α_I^2, which represents the signal-to-interference ratio (SIR) at the equalizer input. In contrast, for a frequency-selective channel, the SINR explicitly depends on the values of the channel transfer function $H(z)$ at the points $\{e^{j 2\pi m/M}\}_{m=0}^{M-1}$ and on the spectral position δ_I of the NBI. Specifically, from eq. (7), the effects of the interference are particularly deleterious if the NBI is located in the neighborhood of those subcarriers where $|H(z)|$ assumes the smallest values.

Returning to the case of partial CP removal ($N > M$), we provide an explicit expression of $(\boldsymbol{\Pi} \, \boldsymbol{\xi}_I)^\dagger$. By exploiting the structure of $\boldsymbol{\Pi}$, we obtain $\boldsymbol{\Pi} \, \boldsymbol{\xi}_I = (1 - e^{j 2\pi \delta_I}) \, \boldsymbol{\zeta}_I$, with $\boldsymbol{\zeta}_I \triangleq [1, e^{j 2\pi \gamma_I}, \ldots, e^{j 2\pi (L_{\text{cp}} - L_h - 1)\gamma_I}]^T \in \mathbb{C}^{L_{\text{cp}} - L_h}$. When the NBI is located between two consecutive subcarriers, i.e., δ_I is not an integer number, it results that $\boldsymbol{\xi}_I^H \boldsymbol{\Pi}^T \boldsymbol{\Pi} \, \boldsymbol{\xi}_I$ is a nonzero scalar number and, thus, $(\boldsymbol{\Pi} \, \boldsymbol{\xi}_I)^\dagger = (\boldsymbol{\xi}_I^H \boldsymbol{\Pi}^T \boldsymbol{\Pi} \, \boldsymbol{\xi}_I)^{-1} \, \boldsymbol{\xi}_I^H \boldsymbol{\Pi}^T$ which, substituted in eq. (6), yields $\lim_{\sigma_w^2 \to 0} \mathcal{P}_d(\boldsymbol{\mathcal{G}}_{\text{zf-moe}}) = 0$, for any value of the interference amplitude α_I and for any FIR channel. In other words, if the NBI is placed between two consecutive subcarriers, the proposed equalizer is capable of nullifying the disturbance in the high SNR region. On the other hand, when the NBI is located exactly on a subcarrier, i.e., δ_I is an integer number, it results that $\boldsymbol{\Pi} \, \boldsymbol{\xi}_I = \mathbf{0}$, implying thus $(\boldsymbol{\Pi} \, \boldsymbol{\xi}_I)^\dagger = \mathbf{0}^T$; loosely speaking, the matrix $\boldsymbol{\Pi}$ blocks not only the desired OFDM symbol but also the NBI. Furthermore, noting that in the latter case $\boldsymbol{\xi}_I = [1, 1, \ldots, 1]^T \in \mathbb{R}^N$ and accounting for eqs. (6) and (7), one has $\lim_{\sigma_w^2 \to 0} \mathcal{P}_d(\boldsymbol{\mathcal{G}}_{\text{zf-moe}}) = \lim_{\sigma_w^2 \to 0} \mathcal{P}_d(\boldsymbol{\mathcal{G}}_{\text{zf-conv}})$. Hence, when the NBI is located exactly on a subcarrier, the proposed receiver cannot completely suppress the NBI, even in the absence of noise and when the channel is ideal, and exhibits the same SINR performance as the conventional receiver.

To numerically corroborate our analysis and assess the performances of the proposed equalizer[4] in a noisy environment, we considered an OFDM system employing $M = 32$ subcarriers and a CP of length $L_{\text{cp}} = 8$, which operates over the same fourth-order FIR channel of [1], with zeros at $(1.2, -1.2, j\,0.7, -j\,0.7)$. In addition to the tone model (T-NBI) for the NBI, we employed a nonnull-bandwidth zero-mean NBI signal (NT-NBI), whose autocorrelation function is $r_{\text{NBI}}(m) \triangleq \text{E}[\widetilde{\jmath}_c(n\,T_c)\,\widetilde{\jmath}_c^*((n-m)\,T_c)] = \alpha_I^2 \, a^{|m|} \, e^{j(2\pi/M)\delta_I m}$, with $a = 0.99$ (corresponding to a 3-dB bandwidth $\approx 0.05/M$). The SNR $\triangleq \sigma_s^2/\sigma_w^2$ and the SIR are set to 20 and 10 dB, respectively. Fig. 1 reports the SINR performance of the proposed equalizer as a function of $\delta_I \in [0, 4]$ [similar results can be observed for $\delta_I \in (4, 32)$], in comparison with that of the conventional receiver and of the equalizer of Redfern [2]. Results show that our analysis accurately predicts also the performance in the presence of noise and for the NT-NBI model. In particular, it is shown that, for both the NBI models, the proposed receiver significantly outperforms (up to 10 dB) the other receivers when the NBI is placed between two consecutive subcarriers, whereas it achieves the same SINR performances when the NBI is located exactly on a subcarrier. Moreover, the SINR values of the conventional receiver are unsatisfactory if the NBI is placed in the neighborhood of those subcarriers (for instance, $0 \leq \delta_I \leq 1$) where $|H(z)|$ assumes the smallest values. In conclusion, the large performance gain achievable by the proposed receiver strongly motivates further research aimed at reducing its computational complexity, which is larger than that of the receiver of [2], in order to devise suboptimal low-complexity structures.

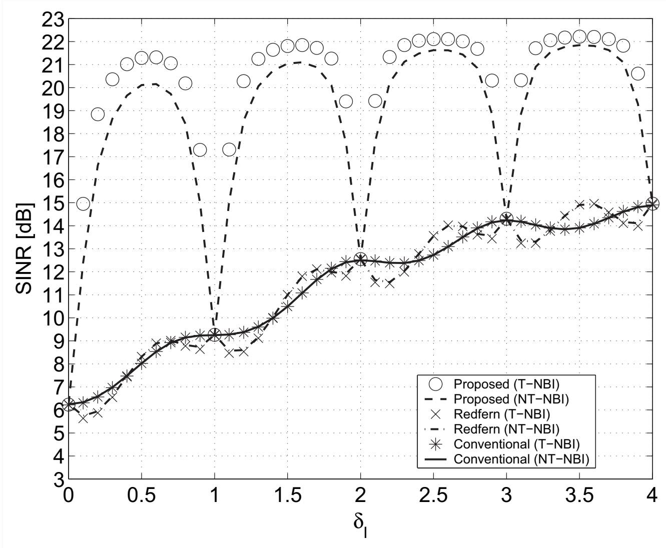

Fig. 1. SINR versus δ_I (SNR = 20 dB and SIR = 10 dB).

[4]We assumed perfect knowledge of the autocorrelation matrix $\mathbf{R}_{\boldsymbol{rr}}$ required for the synthesis of the proposed receiver.

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