Error probability analysis of a MIMO system with noncoherent relaying and MMSE reception

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***Abstract*—This paper deals with symbol error probability (SEP) analysis of a multiple-input multiple-output (MIMO) system with spatial multiplexing, which employs noncoherent (i.e., without channel state information at the relays) amplify-and-forward half-duplex relays and linear minimum mean-square-error (MMSE) equalization at the destination. A closed-form approximation of the SEP is derived, which holds for an arbitrary number of relays placed in different positions. Numerical results show that the obtained SEP approximation is very accurate if the number of relays is greater than two.**

I. INTRODUCTION

COOPERATIVE transmission techniques [1] have attracted considerable research interest, thanks to their capability to improve communication coverage and reliability in wireless channels corrupted by fading. Moreover, cooperative schemes can be conveniently adopted also in multiple-input multiple-output (MIMO) systems, in order to further improve their diversity and/or multiplexing gains [2]. The two relay protocols mostly considered in the literature are decode-and-forward, where the relays decode, re-encode, and retransmit the source messages, and amplify-and-forward (AF), where the relays simply scale the received signal according to a power constraint and forward it to the destination. The AF relaying strategy has gained recently a lot of attention [3]–[5], due to its simplicity and low implementation complexity.

In this letter, we consider a dual-hop AF half-duplex relaying MIMO system with spatial multiplexing, under the assumption that channel state information (CSI) is available at the destination only (so called noncoherent relaying [6]). Dual-hop relaying schemes are relevant whenever the signal received via the direct link is below the noise level, due to deep fading or large obstructions. Moreover, it is widely recognized that complete and reliable CSI can be acquired at the relays with a reasonable overhead only in slowly-changing channels and by recruiting only a few closely located relays.

Performance analysis of dual-hop noncoherent AF relaying MIMO schemes over fading channels has been widely carried out in the literature. To name a few, in [2] closed-form expressions and upper/lower bounds on the ergodic capacity are derived in high signal-to-noise ratio (SNR) conditions, whereas in [7] the outage performance of a dual-hop single-antenna/single-relay system is assessed in closed form.

To the best of our knowledge, a complete study of the symbol error probability (SEP) at the output of a linear minimum mean-square-error (MMSE) receiver, in the case of an arbitrary (possibly large) number of multiple relays placed in different positions, has not been considered yet. An approximate SEP expression for linear zero-forcing (ZF) equalization at the destination has been obtained in [8], but the extension to the MMSE case is not straightforward. We derive in the following a closed-form approximation of the SEP at the output of the MMSE equalizer, which proves to be fairly accurate as long as the number of relays exceeds two.

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II. SIGNAL MODEL

We consider an AF relaying MIMO system employing spatial multiplexing, with N_S source and N_D destination antennas, operating over a wireless channel modeled as an independent and identically distributed (i.i.d.) slowly fading Rayleigh channel. We assume that there is no direct link, and thus communication between source (S) and destination (D) takes place through M intermediate relay nodes, each equipped with N_R antennas, which simply scale their received signals according to a power constraint and forward them to the destination. Relays operate in half-duplex mode, i.e., they cannot transmit and receive simultaneously; in addition, there is no relay-to-relay communication; finally, we assume that source and relays are not aware of channel conditions, and thus CSI is available at the destination only (e.g., by training).

The received signal at D can be written [2] as

$$\mathbf{r} = \sum_{i=1}^{M} \mathbf{H}_2^{(i)} \, \mathbf{z}_\mathrm{R}^{(i)} + \mathbf{w}_\mathrm{D} \tag{1}$$

where $\mathbf{z}_\mathrm{R}^{(i)} \triangleq \sqrt{\alpha_i}\,(\mathbf{H}_1^{(i)}\,\mathbf{s} + \mathbf{w}_\mathrm{R}^{(i)}) \in \mathbb{C}^{N_\mathrm{R}}$ is the signal forwarded by the ith relay towards D, $\alpha_i > 0$ is the scaling factor at the ith relay, $\mathbf{s} \in \mathbb{C}^{N_\mathrm{S}}$ is the transmit symbol vector, $\mathbf{w}_\mathrm{R}^{(i)} \in \mathbb{C}^{N_\mathrm{R}}$ and $\mathbf{w}_\mathrm{D} \in \mathbb{C}^{N_\mathrm{D}}$ represent the noise vectors at the ith relay and D, respectively, $\mathbf{H}_1^{(i)} \in \mathbb{C}^{N_\mathrm{R} \times N_\mathrm{S}}$ and $\mathbf{H}_2^{(i)} \in \mathbb{C}^{N_\mathrm{D} \times N_\mathrm{R}}$ denote the channel matrices of the first hop (S $\to$ ith relay) and second hop (ith relay $\to$ D), respectively.

We assume that: **(a1)** the entries of $\mathbf{s}$ are i.i.d. equiprobable zero-mean circularly symmetric [9] complex random variables

$$\Phi_{\zeta_n}(-u) = \mathbb{E}_{\mathbf{C}_{(-n)}|\mathbf{H}_1}\left[\mathbb{E}_{\mathbf{C}|\mathbf{H}_1\mathbf{C}_{(-n)}}\left(e^{-u\gamma\sum_{i=0}^{N_S-2}\frac{|t_{n,i}|^2}{1+\gamma d_{n,i}^2}}\right)\right] = \mathbb{E}_{\mathbf{C}_{(-n)}|\mathbf{H}_1}\left[\prod_{i=0}^{N_S-2}\mathbb{E}_{\mathbf{C}|\mathbf{H}_1,\mathbf{C}_{(-n)}}\left(e^{-\frac{u\gamma|t_{n,i}|^2}{1+\gamma d_{n,i}^2}}\right)\right]. \quad (6)$$

(RVs) of unit variance, belonging to a quadrature amplitude modulation (QAM) constellation with cardinality Q; **(a2)** $\mathbf{H}_1^{(i)}$ are i.i.d. zero-mean circularly symmetric complex Gaussian (ZMCSCG) RVs, with variance $\sigma_{1,i}^2 \triangleq (d_{\rm SD}/d_{{\rm SR}_i})^\eta$, where $d_{\rm SD}$ is the S-D distance, $d_{{\rm SR}_i}$ is the distance between S and the ith relay, and $\eta \geq 2$ is the path-loss exponent; **(a3)** $\mathbf{H}_2^{(i)}$ are i.i.d. ZMCSCG RVs, with variance $\sigma_{2,i}^2 \triangleq (d_{\rm SD}/d_{{\rm R}_i{\rm D}})^\eta$, where $d_{{\rm R}_i{\rm D}}$ is the distance between the ith relay and D; **(a4)** $\mathbf{w}_{\rm R}^{(i)}$ and $\mathbf{w}_{\rm D}$ are modeled as i.i.d. ZMCSCG RVs of variance $\sigma_{\rm w}^2$. The scaling factor α_i is chosen so as to satisfy the average power constraint $\mathbb{E}[\|\mathbf{z}_{\rm R}^{(i)}\|^2] = N_{\rm R}\, P_i$, where P_i is the power available at the ith relay for each antenna, thus obtaining $\alpha_i = P_i/(N_{\rm S}\, \sigma_{1,i}^2 + \sigma_{\rm w}^2)$.

III. SEP ANALYSIS

Let $\mathbf{C} \triangleq \sum_{i=1}^M \sqrt{\alpha_i}\; \mathbf{H}_2^{(i)}\, \mathbf{H}_1^{(i)} \in \mathbb{C}^{N_{\rm D}\times N_{\rm S}}$ denote the equivalent dual-hop MIMO channel, and define $\mathbf{H}_2 \triangleq [\mathbf{H}_2^{(1)}, \mathbf{H}_2^{(2)}, \ldots, \mathbf{H}_2^{(M)}] \in \mathbb{C}^{N_{\rm D}\times MN_{\rm R}}$. We assume[1] that: **(a5)** ${\rm rank}(\mathbf{C}) = N_{\rm S} \leq N_{\rm D}$ with probability 1, i.e., $\mathbf{C}$ is full-column rank. The received signal (1) at D is subject to MMSE equalization, thus yielding $\mathbf{y} = \mathbf{C}^{\rm H}\,(\mathbf{C}\,\mathbf{C}^{\rm H}+\mathbf{R}_{\mathbf{ww}})^{-1}\,\mathbf{r}$, where $\mathbf{R}_{\mathbf{ww}} \triangleq \mathbb{E}[\mathbf{w}\,\mathbf{w}^{\rm H}|\mathbf{H}_2]$, with $\mathbf{w} \triangleq \sum_{i=1}^M \sqrt{\alpha_i}\; \mathbf{H}_2^{(i)}\, \mathbf{w}_{\rm R}^{(i)} + \mathbf{w}_{\rm D}$, is the correlation matrix of the overall noise at the destination, conditioned on $\mathbf{H}_2$, which can be expressed as

$$\mathbf{R}_{\mathbf{ww}} = \sigma_{\rm w}^2 \left(\sum_{i=1}^M \alpha_i\, \mathbf{H}_2^{(i)}\, \mathbf{H}_2^{(i)H} + \mathbf{I}_{N_{\rm D}}\right). \quad (2)$$

Let $\mathbf{H}_1 \triangleq [\mathbf{H}_1^{(1)\rm T}, \mathbf{H}_1^{(2)\rm T}, \ldots, \mathbf{H}_1^{(M)\rm T}]^{\rm T} \in \mathbb{C}^{MN_{\rm R}\times N_{\rm S}}$. By applying the conditional expectation rule, the error probability ${\rm P}_n(e)$ in detecting the nth entry of $\mathbf{s}$ can be calculated as ${\rm P}_n(e) = \mathbb{E}_{\mathbf{H}_1}\{\mathbb{E}_{\mathbf{H}_2|\mathbf{H}_1}[{\rm P}_n(e|\mathbf{H}_2, \mathbf{H}_1)]\}$, where ${\rm P}_n(e|\mathbf{H}_2, \mathbf{H}_1)$ represents the SEP conditioned on $\mathbf{H}_1$ and $\mathbf{H}_2$. The conditional SEP ${\rm P}_n(e|\mathbf{H}_1) \triangleq \mathbb{E}_{\mathbf{H}_2|\mathbf{H}_1}[{\rm P}_n(e|\mathbf{H}_2, \mathbf{H}_1)]$ can be upper-bounded as (see, e.g., [10]) ${\rm P}_n(e|\mathbf{H}_1) \leq b\, \Phi_{{\rm SINR}_n^{\rm mmse}}(-u)$, with $b \triangleq 2\,(1 - Q^{-1/2})$ and $u \triangleq (3/2)(Q-1)^{-1}$, where $\Phi_{{\rm SINR}_n^{\rm mmse}}(s) \triangleq \mathbb{E}_{\mathbf{H}_2|\mathbf{H}_1}[\exp(s\, {\rm SINR}_n^{\rm mmse})]$ is the moment generating function (MGF), conditioned on $\mathbf{H}_1$, of the RV ${\rm SINR}_n^{\rm mmse}$, representing the signal-to-interference-plus-noise ratio (SINR) at the output of the MMSE equalizer on the nth spatial stream. For fixed values of $\mathbf{H}_1$ and $\mathbf{H}_2$, the SINR on the nth spatial stream is given by

$${\rm SINR}_n^{\rm mmse} = \frac{1}{{\rm MMSE}_n} - 1 = \frac{1}{\{(\mathbf{I}_{N_{\rm S}} + \mathbf{C}^{\rm H}\,\mathbf{R}_{\mathbf{ww}}^{-1}\,\mathbf{C})^{-1}\}_{nn}} - 1 \quad (3)$$

where $\{\mathbf{A}\}_{nn}$ stands for the (n,n) entry of matrix $\mathbf{A}$. Derivation of the exact distribution of ${\rm SINR}_n^{\rm mmse}$, conditioned on $\mathbf{H}_1$, is complicated by the presence in (3) of the noise correlation matrix $\mathbf{R}_{\mathbf{ww}}$. To render the problem mathematically tractable, let us first focus on the term $\mathbf{S}_M \triangleq \sum_{i=1}^M \alpha_i\, \mathbf{H}_2^{(i)}\, \mathbf{H}_2^{(i)\rm H}$ in (2), which is the sum of M complex central Wishart matrices [11], each one having $N_{\rm R}$ degrees of freedom and covariance matrix $\alpha_i\, \sigma_{2,i}^2\, \mathbf{I}_{N_{\rm D}}$. As shown in [8], as the number M of cooperative relays increases, matrix $\mathbf{S}_M$ converges almost surely to its mean $\mathbf{M}_M \triangleq \beta\, N_{\rm R}\, I_{N_{\rm D}}$, with $\beta \triangleq \sum_{i=1}^M \alpha_i\, \sigma_{2,i}^2$; in this case, the noise correlation matrix (2) can be well approximated as $\mathbf{R}_{\mathbf{ww}} \approx \sigma_{\rm w}^2\,(\beta\, N_{\rm R} + 1)\,\mathbf{I}_{N_{\rm D}}$, which, accounting for (3), leads to the following SINR approximation

$${\rm SINR}_n^{\rm mmse} \approx \frac{1}{\{(\mathbf{I}_{N_{\rm S}} + \gamma\,\mathbf{C}^{\rm H}\,\mathbf{C})^{-1}\}_{nn}} - 1 \quad (4)$$

where $\gamma \triangleq 1/(\sigma_{\rm w}^2\, \delta)$, with $\delta \triangleq \beta\, N_{\rm R} + 1$.

Let $\mathbf{C}_{(-n)} \in \mathbb{C}^{N_{\rm D}\times(N_{\rm S}-1)}$ denote the matrix $\mathbf{C}$ with its nth column $\mathbf{c}_n$ removed. We consider the economy-size singular value decomposition $\mathbf{C}_{(-n)} = \mathbf{U}_n\, \mathbf{D}_n\, \mathbf{V}_n^{\rm H}$, where the semi-unitary matrix $\mathbf{U}_n \in \mathbf{C}^{N_{\rm D}\times(N_{\rm S}-1)}$ and the unitary matrix $\mathbf{V}_n \in \mathbf{C}^{(N_{\rm S}-1)\times(N_{\rm S}-1)}$ contain the left and right singular vectors, respectively, associated with the nonzero singular values of $\mathbf{C}_{(-n)}$, gathered in matrix $\mathbf{D}_n \triangleq {\rm diag}(d_{n,0}, d_{n,1}, \ldots, d_{n,N_{\rm S}-2}) \in \mathbb{R}^{(N_{\rm S}-1)\times(N_{\rm S}-1)}$. It can be shown [12] that the SINR at the output of the MMSE receiver can be decomposed into the sum of two statistically independent RVs as ${\rm SINR}_n^{\rm mmse} = \zeta_n + {\rm SINR}_n^{\rm zf}$, where ${\rm SINR}_n^{\rm zf} \approx \gamma/[\beta\{(\mathbf{C}^{\rm H}\,\mathbf{C})^{-1}\}_{nn}]$ is the approximate SINR at the output of the ZF equalizer on the nth spatial stream, whereas $\zeta_n \triangleq \gamma\, \tilde{\mathbf{c}}_n^{\rm H}\, [\mathbf{I}_{N_{\rm S}-1} + \gamma\, \mathbf{D}_n^2]^{-1}\, \tilde{\mathbf{c}}_n$, with $\tilde{\mathbf{c}}_n \triangleq \mathbf{U}_n^{\rm H}\, \mathbf{c}_n$.

Given $\mathbf{H}_1$, the approximate ${\rm SINR}_n^{\rm zf}$ is a Gamma RV [13] with shape parameter $k = N_{\rm D} - N_{\rm S} + 1$ and scale parameter $\theta = \gamma\, \Sigma_n$, with $\Sigma_n \triangleq 1/(\mathbf{R}^{-1})_{nn}$, where $\mathbf{R} \triangleq \widetilde{\mathbf{H}}_1^{\rm H}\,(\mathbf{\Omega}\otimes\mathbf{I}_{N_{\rm D}})\,\widetilde{\mathbf{H}}_1 \in \mathbb{C}^{N_{\rm S}\times N_{\rm S}}$ represents the covariance matrix of each row of $\mathbf{C}$, $\widetilde{\mathbf{H}}_1 \triangleq [{\rm diag}(\sigma_{1,1}^2, \sigma_{1,2}^2, \ldots, \sigma_{1,M}^2)^{-1} \otimes \mathbf{I}_{N_{\rm D}}]\, \mathbf{H}_1$ is the normalized channel matrix, with i.i.d. unit-variance ZMCSCG entries, $\mathbf{\Omega} \triangleq {\rm diag}(\alpha_1\, \sigma_{1,1}^2\, \sigma_{2,1}^2, \alpha_2\, \sigma_{1,2}^2\, \sigma_{2,2}^2, \ldots, \alpha_M\, \sigma_{1,M}^2\, \sigma_{2,M}^2)$, and $\otimes$ denotes Kronecker product [14]. From the statistical independence between ${\rm SINR}_n^{\rm zf}$ and ζ_n, it follows that

$$\begin{aligned}\Phi_{{\rm SINR}_n^{\rm mmse}}(-u) &= \Phi_{\zeta_n}(-u)\, \Phi_{{\rm SINR}_n^{\rm zf}}(-u) \\ &\approx \Phi_{\zeta_n}(-u)\,(1 + u\,\gamma\,\Sigma_n)^{-(N_{\rm D}-N_{\rm S}+1)}\end{aligned} \quad (5)$$

where we approximated $\Phi_{{\rm SINR}_n^{\rm zf}}(-u)$ with the MGF of a Gamma RV. At this point, we need to evaluate the MGF of ζ_n. By writing ζ_n as a function of the entries of $\tilde{\mathbf{c}}_n \triangleq [\tilde{c}_{n,0}, \tilde{c}_{n,1}, \ldots, \tilde{c}_{n,N_{\rm S}-2}]^{\rm T}$ and the singular values $d_{n,i}$, and applying the conditional expectation rule, one obtains the expression of $\Phi_{\zeta_n}(-u)$ reported in (6) at the top of this page. In the last-hand of (6), we have exploited the statistical independence, conditioned on $\mathbf{C}_{(-n)}$, among the circular Gaussian RVs $\tilde{c}_{n,i}$, having mean [12] $\mu_{n,i} \triangleq d_{n,i}\,\{\mathbf{V}_n^{\rm H}\,[\mathbf{R}_{(-n,-n)}]^{-1}\,\mathbf{r}_{n(-n)}\}_i$ and variance Σ_n, where $\mathbf{R}_{(-n,-n)}$ is the covariance matrix $\mathbf{R}$ with its nth row and nth column removed, whereas $\mathbf{r}_{n(-n)}$ denotes the nth column of $\mathbf{R}$ with its nth removed.

[1]This assumption is made to avoid that the MMSE equalizer matrix $\mathbf{C}^{\rm H}\,(\mathbf{C}\,\mathbf{C}^{\rm H} + \mathbf{R}_{\mathbf{ww}})^{-1}$ becomes rank-deficient in the high-SNR regime.

$$
\begin{aligned}
\Phi_{\zeta_n}(-u) &\lesssim \mathbb{E}_{\mathbf{C}_{(-n)}|\mathbf{H}_1}\left[\prod_{i=0}^{N_{\rm S}-2}\left(\frac{d_{n,i}^2}{d_{n,i}^2+u\,\Sigma_n}\right)\right] \lesssim \mathbb{E}_{\mathbf{C}_{(-n)}|\mathbf{H}_1}\left[\frac{\det(\mathbf{D}_n^2)}{\det\left(u\,\Sigma_n\,\mathbf{I}_{N_{\rm S}-1}+\mathbf{D}_n^2\right)}\right] \\
&\lesssim \mathbb{E}_{\mathbf{C}_{(-n)}|\mathbf{H}_1}\left\{\frac{\det\left[\mathbf{R}_{(-n,-n)}^{-1/2}\,\mathbf{C}_{(-n)}^{\rm H}\,\mathbf{C}_{(-n)}\,\mathbf{R}_{(-n,-n)}^{-1/2}\right]}{\det\left[u\,\Sigma_n\,\mathbf{R}_{(-n,-n)}^{-1}+\mathbf{R}_{(-n,-n)}^{-1/2}\,\mathbf{C}_{(-n)}^{\rm H}\,\mathbf{C}_{(-n)}\,\mathbf{R}_{(-n,-n)}^{-1/2}\right]}\right\} \\
&\lesssim \chi^{(N_{\rm S}-1)}\cdot\mathbb{E}_{\mathbf{C}_{(-n)}|\mathbf{H}_1}\left\{\frac{\det\left[\mathbf{R}_{(-n,-n)}^{-1/2}\,\mathbf{C}_{(-n)}^{\rm H}\,\mathbf{C}_{(-n)}\,\mathbf{R}_{(-n,-n)}^{-1/2}\right]}{\det\left[\mathbf{I}_{N_{\rm S}-1}+\chi\,\mathbf{R}_{(-n,-n)}^{-1/2}\,\mathbf{C}_{(-n)}^{\rm H}\,\mathbf{C}_{(-n)}\,\mathbf{R}_{(-n,-n)}^{-1/2}\right]}\right\} \qquad (8)
\end{aligned}
$$

Given $\mathbf{H}_1$ and $\mathbf{C}^{(-n)}$, it turns out that the RV $|\tilde{c}_{n,i}|^2$ has a noncentral Chi-squared distribution [13] with MGF

$$
\begin{aligned}
&\mathbb{E}_{\mathbf{C}|\mathbf{H}_1,\mathbf{C}_{(-n)}}\left[\exp\left(-u\,\gamma\cdot\frac{|t_{n,i}|^2}{1+\gamma\,d_{n,i}^2}\right)\right] \\
&= \left(1+\frac{u\,\gamma\,\Sigma_n}{1+\gamma\,d_{n,i}^2}\right)^{-1}\exp\left(-\frac{|\mu_{n,i}|^2}{\Sigma_n+\frac{(1+\gamma\,d_{n,i}^2)}{u\,\gamma}}\right) \\
&\le \frac{1+\gamma\,d_{n,i}^2}{1+\gamma\,d_{n,i}^2+u\,\gamma\,\Sigma_n} \qquad (7)
\end{aligned}
$$

where the last inequality stems from the fact that $\exp(-a\,x)\le 1$, for any $x\ge 0$ and $a>0$.

Substituting (7) into (6) and recalling that $\mathbf{C}_{(-n)}=\mathbf{U}_n\,\mathbf{D}_n\,\mathbf{V}_n^{\rm H}$, after straightforward calculations, one obtains (8) at the top of this page, which holds for moderate-to-high values of the SNR, where the last term comes from Weyl's inequality [14], with $\chi\triangleq(u\,\Sigma_n\,\lambda_{\rm max})^{-1}$ and $\lambda_{\rm max}$ denoting the largest eigenvalue of $\mathbf{R}_{(-n,-n)}$, conditioned on $\mathbf{H}_1$. At this point, provided that $N_{\rm S}>1$, approximating the expectation in (8) as in [10], one obtains

$$
\begin{aligned}
\Phi_{\zeta_n}(-u) &\lesssim \binom{N_{\rm D}}{N_{\rm S}-1}(N_{\rm S}-1)\underbrace{\int_0^{+\infty}\frac{t^{N_{\rm S}-2}\,e^{-t/\chi}}{(1+t)^{N_{\rm D}+1}}\,{\rm d}t}_{=(N_{\rm S}-2)!\,U(N_{\rm S}-1,N_{\rm S}-N_{\rm D}-1,1/\chi)} \\
&\lesssim \binom{N_{\rm D}}{N_{\rm S}-1}(N_{\rm S}-1)!\,{\rm U}\left(N_{\rm S}-1,N_{\rm S}-N_{\rm D}-1,1/\chi\right) \qquad (9)
\end{aligned}
$$

where ${\rm U}(\cdot,\cdot,\cdot)$ is Tricomi's (confluent hypergeometric) function [13]. Substituting (9) into (5) and using again the approximation for moderate-to-high SNR values, one has

$$
\begin{aligned}
\Phi_{{\rm SINR}_n^{\rm mmse}}(-u) &\lesssim \binom{N_{\rm D}}{N_{\rm S}-1}\frac{N_{\rm S}-1}{(u\gamma\Sigma_n)^{N_{\rm D}-N_{\rm S}+1}} \\
&\times\int_0^{+\infty}\frac{t^{N_{\rm S}-2}e^{-t/\chi}}{(1+t)^{N_{\rm D}+1}}\,{\rm d}t\,. \qquad (10)
\end{aligned}
$$

Thus, accounting for the expression of χ, by substituting (10) into the upper bound on ${\rm P}_n(e|\mathbf{H}_1)$ and averaging with respect to $\mathbf{H}_1$, the SEP on the nth spatial stream assumes the form

$$
\begin{aligned}
{\rm P}_n(e) &\lesssim b\binom{N_{\rm D}}{N_{\rm S}-1}\frac{N_{\rm S}-1}{(u\,\gamma)^{N_{\rm D}-N_{\rm S}+1}} \\
&\times\mathbb{E}_{\mathbf{H}_1}\left[\int_0^{+\infty}\frac{t^{N_{\rm S}-2}\,e^{-ut\lambda_{\rm max}\Sigma_n}}{(1+t)^{N_{\rm D}+1}}\,\Sigma_n^{-(N_{\rm D}-N_{\rm S}+1)}\,{\rm d}t\right]. \qquad (11)
\end{aligned}
$$

To calculate (11), it can be shown [15] that matrix $\mathbf{R}$ has approximately a complex central Wishart distribution with $\nu_0\triangleq N_{\rm R}\left(\sum_{i=1}^M\alpha_i\sigma_{1,i}^2\sigma_{2,i}^2\right)^2/\sum_{i=1}^M(\alpha_i\sigma_{1,i}^2\sigma_{2,i}^2)^2$ degrees of freedom and covariance matrix $\sum_{i=1}^M(\alpha_i\sigma_{1,i}^2\sigma_{2,i}^2)^2/(\sum_{i=1}^M\alpha_i\sigma_{1,i}^2\sigma_{2,i}^2)\,\mathbf{I}_{N_{\rm S}}$.

Therefore, provided that $\nu_0>N_{\rm D}$, it turns out that Σ_n is a Gamma RV with shape parameter $k_0=\nu_0-N_{\rm S}+1$ and scale parameter $\vartheta_0=\sum_{i=1}^M(\alpha_i\sigma_{1,i}^2\sigma_{2,i}^2)^2/(\sum_{i=1}^M\alpha_i\sigma_{1,i}^2\sigma_{2,i}^2)$. In addition, from [11] we know that, for $(N_{\rm S}-1)$ and ν_0 approaching infinity in such a way that $(N_{\rm S}-1)/\nu_0$ tends to a finite limit $\ell\ge 0$, the largest eigenvalue of the complex Wishart matrix $\mathbf{R}_{(-n,-n)}$ converges almost surely to $\lambda_\infty\triangleq 2(N_{\rm S}-1)(1+\sqrt{\ell})^2$; thus, in order to simplify the calculus of the expectation in (11), we replace $\lambda_{\rm max}$ with its limiting (nonrandom) value λ_∞. Relying on Fubini-Tonelli theorem to switch the order of expectation and integration in the expression of ${\rm P}_n(e)$, we obtain the final expression of the SEP reported in (12) at the top of the following page.

Evaluation of (12) entails a computational complexity that is basically dominated by the calculus of the integral, which can be accurately evaluated by using numerical techniques, e.g., Gaussian quadrature methods.[2] Such a computational burden is significantly smaller than the complexity of a Monte Carlo simulation: indeed, since ${\rm P}_n(e)$ might assume extremely low values in the high-SNR region, especially for $M>2$ (see Section IV), a large number of Monte Carlo runs are required to obtain accurate SEP estimates.

Some remarks about (12) are finally in order. First, we highlight that ${\rm P}_n(e)$ turns out to be independent of the symbol index n. Furthermore, it is worth noting that, in the high-SNR regime, the scaling factor α_i can be approximated as $\alpha_i\approx P_i/(N_{\rm S}\,\sigma_{1,i}^2)$, i.e., it becomes independent of γ. Consequently, ϑ_0 in (12) turns out to be independent of γ and, thus, the SEP can be expressed as ${\rm P}(e)\approx G_c\,\gamma^{-d}$, where $d\triangleq N_{\rm D}-N_{\rm S}+1$ is the diversity order of the system, which does not depend on the number M of relays, since no CSI is available, and the coding gain is denoted by

$$
\begin{aligned}
G_c &\triangleq b\binom{N_{\rm D}}{N_{\rm S}-1}\frac{N_{\rm S}-1}{(u\,\vartheta_0)^{N_{\rm D}-N_{\rm S}+1}}\frac{\Gamma(\nu_0-N_{\rm D})}{\Gamma(\nu_0-N_{\rm S}+1)} \\
&\times\int_0^{+\infty}\frac{t^{N_{\rm S}-2}}{(1+t)^{N_{\rm D}+1}(1+u\lambda_\infty\vartheta_0 t)^{\nu_0-N_{\rm D}}}\,{\rm d}t\,. \qquad (13)
\end{aligned}
$$

[2]As an example, the Matlab routine `quadgk` uses Gaussian quadrature by implementing the Gauss-Kronrod method.

$$\mathrm{P}_n(e) \lesssim b \binom{N_\mathrm{D}}{N_\mathrm{S}-1} \frac{N_\mathrm{S}-1}{(u\,\gamma)^{N_\mathrm{D}-N_\mathrm{S}+1}} \int_0^{+\infty} \frac{t^{N_\mathrm{S}-2}}{(1+t)^{N_\mathrm{D}+1}} \mathbb{E}_{\mathbf{H}_1}\left[e^{-ut\lambda_\infty \Sigma_n}\, \Sigma_n^{-(N_\mathrm{D}-N_\mathrm{S}+1)}\right] \mathrm{d}t$$
$$\lesssim b \binom{N_\mathrm{D}}{N_\mathrm{S}-1} \frac{N_\mathrm{S}-1}{(u\,\gamma\,\vartheta_0)^{N_\mathrm{D}-N_\mathrm{S}+1}} \frac{\Gamma(\nu_0-N_\mathrm{D})}{\Gamma(\nu_0-N_\mathrm{S}+1)} \int_0^{+\infty} \frac{t^{N_\mathrm{S}-2}}{(1+t)^{N_\mathrm{D}+1}(1+u\lambda_\infty\vartheta_0 t)^{\nu_0-N_\mathrm{D}}} \mathrm{d}t \qquad (12)$$

IV. Numerical results

We consider a network geometry consisting of a S-D pair spaced by the distance $d_\mathrm{SD}=1$, plus M relaying nodes, which are randomly and independently distributed in a circle of radius $r=0.1$, positioned on the line joining S to D at distance $d_\mathrm{SC}=0.2$ from the source. Source, destination and relays are equipped with $N_\mathrm{S}=2$, $N_\mathrm{D}=3$, and $N_\mathrm{R}=2$ antennas, respectively. The employed modulation is Gray-labeled QPSK ($Q=4$). Both the first-hop and second-hop MIMO channels are generated according to assumptions (a2) and (a3), with $\eta=3$; in addition, all the relays transmit with the same power $P_i=1$, for $i \in \{1,2,\ldots,M\}$ (simulations not reported for brevity show that the ABER does not vary significantly if the relays transmit with slightly different powers.)

In Fig. 1 we plot, for different values of $M \in \{2,3,4\}$, the average bit-error-rate (ABER) of the MMSE receiver. In particular, we compare the ABER values when the SEP $\mathrm{P}_n(e)$ is calculated from the bound (12) [labeled as "MMSE (bound)" in the plots], with the exact ones [labeled as "MMSE (exact)"], evaluated by means of a semi-analytical approach, by averaging $\mathrm{P}_n(e|\mathbf{H}_1,\mathbf{H}_2)$ with respect to $\mathbf{H}_1$ and $\mathbf{H}_2$ over 10^6 Monte Carlo trials. Both curves are reported as a function of SNR $\triangleq 1/\sigma_v^2$, ranging from 0 to 20 dB.

It is apparent from Fig. 1 that the exact performance of the MIMO AF system with MMSE reception, which exhibits in this scenario a diversity order equal to $d=N_\mathrm{D}-N_\mathrm{S}+1=2$, is well predicted by the proposed bound (12), as the SNR increases. The coding gain of the cooperative system rapidly increases as the number of relays M rises. Furthermore, the approximation (12) becomes more and more accurate as the number M of cooperating relays grows, which is in part due to the fact that the accuracy of the approximation (4) improves.

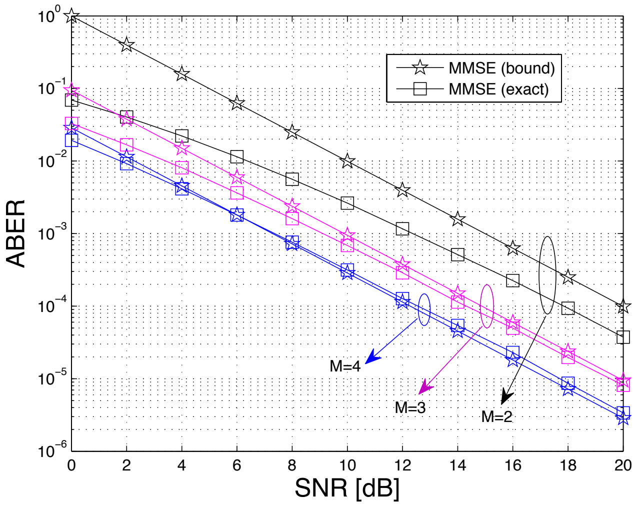

Fig. 1. ABER versus SNR for different values of the number M of relays.

V. Conclusions

In this paper, a tight approximation for the SEP of a dual-hop noncoherent AF MIMO system with multiple relays and MMSE equalization at the destination was calculated, without imposing any restriction on the number of relays or their positions. The validity of the proposed approximation was numerically assessed by showing a remarkably good agreement between theoretical and numerical results in the high-SNR region and for a number of relays greater than two.

References

[1] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity. Part I. System description," *IEEE Trans. Inf. Theory*, vol. 56, no. 5, pp. 2204–2224, Nov. 2010.

[2] S. Jin, M. R. McKay, C. Zhong, and K. K. Wong, "Ergodic capacity analysis of amplify-and-forward MIMO dual-hop systems," *IEEE Trans. Inf. Theory*, vol. 56, no. 5, pp. 2204–2224, May 2010.

[3] F. Verde and A. Scaglione, "Randomized space-time block coding for distributed amplify-and-forward cooperative relays," *Proc. of IEEE Int. Conf. Acoustics, Speech and Signal Process. (ICASSP)*, Dallas, Texas, Mar. 2010, pp. 3030–3033.

[4] F. Verde, "Design of randomized space-time block codes for amplify-and-forward cooperative relaying," *Proc. of 5th Int. Symp. on Commun. Control and Signal Process. (ISCCSP)*, Rome, Italy, May 2012, pp. 1–5.

[5] F. Verde, "Performance analysis of randomized space-time block codes for amplify-and-forward cooperative relaying," *IET Commun.*, vol. 67, pp. 1–16, Dec. 2013.

[6] H. Bolcksei, R.U. Nabar, O. Oyman, and A.J. Paulraj, "Capacity scaling laws in MIMO relay networks," *IEEE Trans. Wireless Commun.*, vol. 5, no. 6, pp. 1433–1444, June 2006.

[7] M.O. Hasna and M.-S. Alouini, "A performance study of dual-hop transmissions with fixed gain relays," *IEEE Trans. Wireless Commun.*, vol. 3, no. 6, pp. 1963–1968, Nov. 2004.

[8] D. Darsena, G. Gelli, F. Melito, and F. Verde, "Performance analysis of amplify-and-forward MIMO-OFDM links with linear ZF equalization," *Proc. of IEEE Int. Conf. on Communications (ICC)*, Budapest, Hungary, June 2013, pp. 296–300.

[9] D. Darsena, G. Gelli, L. Paura, and F. Verde, "Subspace-based blind channel identification of SISO-FIR systems with improper random inputs", *Signal Processing*, vol. 84, no. 11, pp. 2021–2039, Nov. 2004.

[10] F. Verde, D. Darsena, and A. Scaglione, "Cooperative randomized MIMO-OFDM downlink for multicell networks: design and analysis," *IEEE Trans. Signal Process.*, vol. 58, no. 1, pp. 384–402, Jan. 2010.

[11] A. Edelman, *Eigenvalues and Condition Numbers of Random Matrices*. PhD thesis, Department of Mathematics, Massachusetts Institute of Technology, Cambridge, MA, 1989.

[12] P. Li, D. Paul, R. Narasimhan, and J. Cioffi, "On the distribution of SINR for the MMSE MIMO receiver and performance analysis," *IEEE Trans. Inf. Theory*, vol. 52, no. 1, pp. 271–286, Jan. 2006.

[13] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*. New York, 1972.

[14] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge: Cambridge University Press, 1990.

[15] D. Nel and C.V. der Merwe, "A solution to the multivariate Behrens-Fisher problem," *Commun. Stat. – Theory Methods*, vol. 15, no. 2, pp. 3719–3735, 1986.