A time-frequency synchronization algorithm for MC-CDMA systems in LMDS applications

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We propose a semi-blind algorithm for tracking the synchronization parameters (time delays and frequency offsets) of multiple users in MC-CDMA systems targeted for broadband LMDS applications. Simulation results show that the proposed algorithm assures accurate estimates with modest computational complexity and is robust against the presence of multiple-access interference.

Introduction: Due to crowding of the existing frequency bands, broadband fixed wireless access systems operating between 24 and 48 GHz, often referred to as *local multipoint distribution systems (LMDS)* [1], have become recently the object of intense standardization activities [2]. Since the subscriber antenna in a LMDS system is a narrowbeam directional antenna pointed toward the serving base station (BS), LMDS networks are essentially free of multipath propagation [3]. In such multipath-free scenario, the combination of *multicarrier* modulation and code-division multiple-access (CDMA) systems offer several advantages (reduced symbol rate, handled of heterogeneous multimedia traffic and increased system capacity). Among the drawbacks, besides sensitivity to non-linear distortion, multicarrier techniques are extremely prone to frequency-offset errors, which arise because of unavoidable mismatches between transmitting and receiving RF sections, and to the lack of synchronization among the subscriber data frames at the BS, which is due to the different propagation delays in the uplink. In this asynchronous scenario, acquisition and tracking of synchronization parameters at the BS for all the users often require insertion of pilot symbols across both time and frequency [4]. In this letter, we propose a simple and accurate algorithm for tracking synchronization parameters in asynchronous MC-CDMA systems. The proposed algorithm is semi-blind since it requires only training sequences for frequency-offset estimation, and performs delay estimation blindly, by exploiting the code properties.

The MC-CDMA system model: Let us consider the uplink of a MC-CDMA system with N subcarriers and J users, all transmitting asynchronously at the same symbol rate $1/T$. The symbol d_{jm}, transmitted by the jth user in the mth symbol interval, is first copied N times, then is spread in the frequency domain by the code $\boldsymbol{c}_j = [c_j(0), c_j(1), \ldots, c_j(N-1)]$ and, finally, is transformed by N-point IDFT, obtaining $s_{jm}(q) = d_{jm} \cdot \mathrm{IDFT}[c_j(k)]$, with $q = 0, 1, \ldots, N-1$. After insertion of a cyclic prefix (CP) of length L, one obtains the *extended* sequence

$$\tilde{s}_{jm}(q) = \frac{1}{N}\, d_{jm} \sum_{k=0}^{N-1} c_j(k)\, e^{i\frac{2\pi}{N}kq} \;, \tag{1}$$

for $q = -L, \ldots, 0, 1, \ldots, N-1$, where $\{\tilde{s}_{jm}(q)\}_{q=0}^{N-1} = \{s_{jm}(q)\}_{q=0}^{N-1}$. The Q-sequence $\tilde{s}_{jm}(q)$, with $Q \stackrel{\triangle}{=} L + N$, is first parallel-to-serial converted, then is filtered by the D/A device, which operates at rate $1/T_c = Q/T$ and, finally, is upconverted to RF and transmitted through the channel. In the considered LMDS operation scenario, we will reasonably assume that the channel is non-selective both in time and in frequency. Thus, the received complex envelope is given by[1]

$$r(t) = \sum_{j=1}^{J} h_j \sum_{m=-\infty}^{+\infty} \sum_{q=-L}^{N-1} \tilde{s}_{jm}(q)\, \psi_{D/A}(t - q\,T_c - m\,T - \tau_j)\, e^{i2\pi\theta_j t} \;, \tag{2}$$

where $\psi_{D/A}(t)$ denotes the impulse response of the D/A converter and, with reference to the jth user, h_j is the complex gain (amplitude plus phase) of the channel (which is assumed to be constant within

[1]To keep notation (reasonably) simple, in the sequel we will omit the noise in our derivations.

the observation interval), $|\tau_j| \leq T/2$ accounts for the combined effect of transmission delay and asynchronism among users, and θ_j denotes the carrier frequency offset.

At the receiver, the signal $r(t)$ is first filtered in the A/D device by $\psi_{A/D}(t)$ and then sampled at rate $f_c = 1/T_c$, obtaining hence

$$r_n(k) = \sum_{j=1}^{J} h_j \sum_{m=-\infty}^{+\infty} \sum_{q=-L}^{N-1} \tilde{s}_{jm}(q)\, \phi_{\theta_j}\left[(k-q)T_c + (n-m)\,T - \tau_j\right]\, e^{i2\pi\theta_j(nT+kT_c)}\,, \qquad (3)$$

for $n \in \mathbb{Z}$ and $k = -L, \ldots, 0, 1, \ldots, N-1$, where $\phi_{\theta_j}(t) \triangleq \psi_{D/A}(t) * [\psi_{A/D}(t)\, e^{-i2\pi\theta_j t}]$ is the overall A/D and D/A impulse response, with $*$ denoting linear convolution. Note that, since the D/A and A/D filters have bandwidth of the order of f_c, when $|\theta_j| \ll f_c$, we can neglect the frequency shift in the definition of $\phi_{\theta_j}(t)$, writing $\phi_{\theta_j}(t) \approx \phi(t) \triangleq \psi_{D/A}(t) * \psi_{A/D}(t)$. Moreover, if we assume that $\phi(t)$ has finite memory, that is $\phi(t) \neq 0$ for $t \in [0, L_f\, T_c)$, and the CP length L is such that $L > L_f + f_c \max_j(\tau_j)$, then the intersymbol interference (ISI) can be perfectly suppressed by removing the first L samples $\{r_n(k)\}_{k=-L}^{-1}$. The remaining samples $\{r_n(k)\}_{k=0}^{N-1}$ are thus given by

$$r_n(k) = \sum_{j=1}^{J} h_j\, e^{i2\pi(\theta_j T)n} \sum_{q=-L}^{N-1} \tilde{s}_{jm}(q)\, g_{\tau_j}(k-q)\, e^{i2\pi(\theta_j T_c)k}\,, \qquad (4)$$

where $g_{\tau_j}(k) \triangleq \phi(kT_c - \tau_j)$. We observe that, since $\tilde{s}_{jm}(q)$ is the periodic extension of $s_{jm}(q)$, then $\sum_{q=-L}^{N-1} \tilde{s}_{jm}(q)\, g_{\tau_j}(k-q) = s_{jm}(k) \circledast g_{\tau_j}(k)$, where $\circledast$ denotes *circular convolution* over N points. Thus, by taking the DFT of $\{r_n(k)\}_{k=0}^{N-1}$ and applying DFT properties, one obtains

$$v_n(\ell) = \mathrm{DFT}[r_n(k)] \approx \sum_{j=1}^{J} \tilde{h}_{\tau_j}(\ell)\, c_j(\ell)\, d_{jm}\, e^{i2\pi(\theta_j T)n}\,, \qquad (5)$$

where $\tilde{h}_{\tau_j}(\ell) \triangleq h_j\, G_{\tau_j}(\ell)$ is the *equivalent channel*, with $G_{\tau_j}(\ell)$ representing the N-point DFT of $g_{\tau_j}(k)$; finally, by assuming that $|\theta_j| \ll \Delta f$, with $\Delta f \triangleq 1/(N\, T_c)$, we have approximated the $\mathrm{DFT}[e^{i2\pi\theta_j T_c k}]$ by $N\, \delta(\ell)$.

The proposed algorithm: Let $\Phi(f)$ denote the Fourier transform of $\phi(t)$; if $\Phi(f) \neq 0$ for $|f| \leq f_p$ and $\Phi(f) \approx 0$ for $|f| \geq f_s$, and if $f_c > f_p + f_s$, the Fourier transform $G_{\tau_j}(\nu)$ of $g_{\tau_j}(k)$ can be approximately written as $G_{\tau_j}(\nu) \approx f_c \, \Phi(\nu \, f_c) \, e^{-i2\pi\nu \, f_c \, \tau_j}$, for $|\nu| \leq \nu_p \triangleq f_p/(2f_c)$ and, thus, $\tilde{h}_{\tau_j}(\ell) \approx f_c \, \Phi(\ell\Delta f) \, e^{-i2\pi\ell\Delta f \, \tau_j}$, for $|\ell| \leq N \, \nu_p$. By substituting back in (5), and rearranging indexes, we have simply

$$y_n[\ell] \triangleq \frac{v_n[\ell]}{f_c\Phi(\ell\Delta f)} = \sum_{j=1}^{J} h_j \, c_j[\ell] \, d_{jn} \, e^{-i2\pi\mu_j[\ell]} \, e^{i2\pi\lambda_j n} \, , \tag{6}$$

where $[\cdot]$ denotes modulo-N operation; we have introduced the *normalized* synchronization parameters $\lambda_j \triangleq \theta_j T$ and $\mu_j \triangleq \tau_j \Delta f$; and, finally, since $\Phi(f)$ is known at the receiver, we have compensated its effects by defining $y_n[\ell]$. Based on the above model, we perform frequency-offset estimation by observing a single subcarrier ℓ. Let us consider, without restriction, the contribution of the first user ($j = 1$) in (6). Thus, we can estimate λ_1, and hence θ_1, by cross-correlating (in n) $y_n[\ell]$ with $d_{1n} \, e^{i2\pi\nu n}$, and finding the maximum of the correlation magnitude with respect to ν. This requires a training sequence d_{1n} of length M, which can be accommodated in a suitable header of the frame. The proposed estimator for θ_1 is then given by

$$\hat{\theta}_1 = \frac{1}{T} \, \arg \max_{|\nu| \leq 0.5} \left| \sum_{n=0}^{M-1} y_n[\ell] \, d_{1n}^* \, e^{-i2\pi\nu n} \right| \, , \tag{7}$$

which can be implemented by means of a simple 1D-FFT. The range of the proposed estimator is $|\theta_1| \leq 1/(2T)$, which is more than adequate for tracking purposes. Note that the contribution of the MAI to the estimator (7) essentially depends on the zero-delay frequency-shifted cross-correlation among the training sequences. Thus, the degradation due to the MAI can be reduced by an appropriate choice of the training sequences. A dual reasoning is followed to carry out delay estimation, which is performed by looking at a single symbol n in the frame. Indeed, we can estimate μ_1 by cross-correlating (in ℓ) $y_n[\ell]$ with $c_1[\ell]\, e^{-i2\pi\mu[\ell]}$ and finding the maximum of the correlation magnitude with respect to ν. Note that in this case training sequences are not required since we exploit the knowledge of the spreading codes. The proposed estimator is

$$\hat{\tau}_1 = N\, T_c \ \arg \max_{|\nu| \leq 0.5} \left| \sum_{|\ell| \leq N\nu_p} y_n[\ell]\, c_1[\ell]^*\, e^{i2\pi\nu[\ell]} \right| \,, \tag{8}$$

which can be implemented by means of a simple 1D-FFT. The estimate range is $|\tau_1| \leq (N\, T_c)/2$, which again is more than adequate for tracking (it is approximately equal to the symbol interval). In this case, the contribution of the MAI to the estimator (8) can be reduced by choosing the spreading codes such that their zero-delay frequency-shifted cross-correlation be negligible.

Numerical results: The performance of the proposed synchronization algorithm in the uplink channel has been investigated by Monte Carlo computer simulations. We considered the following parameters: BPSK signaling for all users, $N = 256$ subcarriers, $L = 32$ (CP length), $f_c = 16$ MHz, signal-to-noise ratio $\mathrm{SNR} = 20$ dB. Moreover, length-31 concatenated Gold sequences were employed both as training sequences and spreading codes. We considered $J = 4$ users with delays (normalized to T_c) $\boldsymbol{\tau} = [-54, -23, 13, 35]$ and frequency offsets (normalized to $1/T$) $\boldsymbol{\theta} = [-0.47, -0.27, 0.20, 0.48]$. Figures 1 and 2 show the cost functions for frequency offset estimation (7) and time-delay estimation (8) evaluated for the first user (similar results were obtained for the other users). It can be seen that the MAI contribution is effectively canceled out by the good frequency-shifted cross-correlation properties of the Gold sequences adopted, both for training and spreading. Results, not reported here, show moreover that the root mean-square error of the offset and delay estimates is practically insensitive to SNR.

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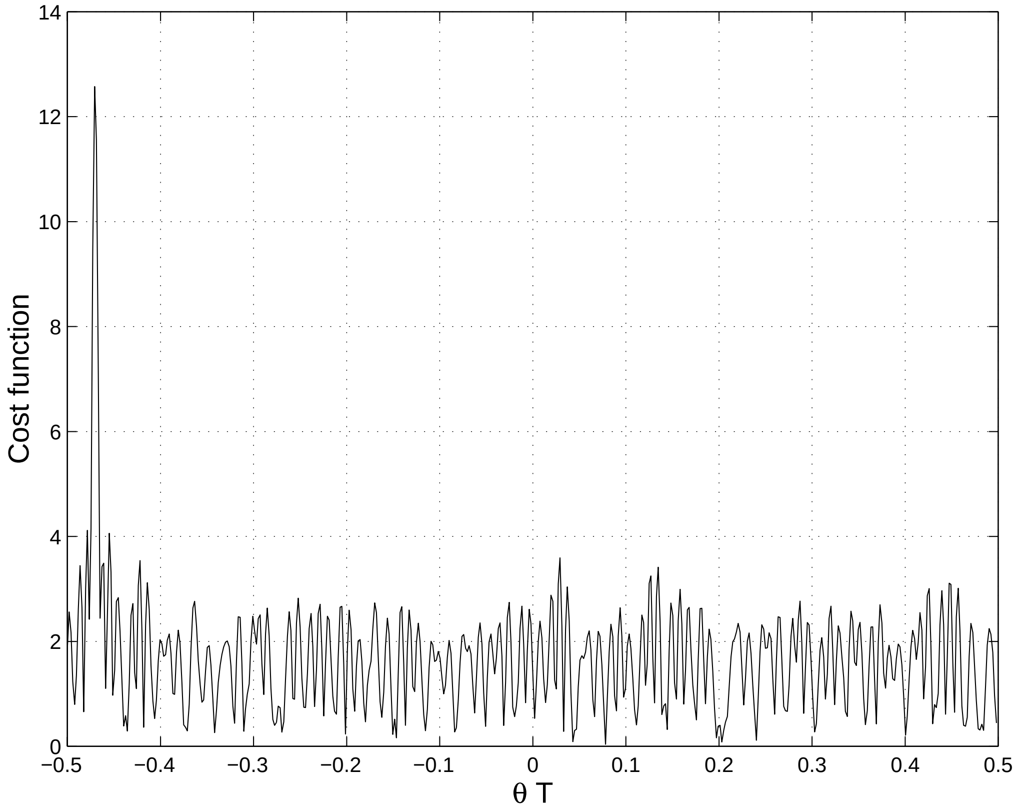

Figure 1: Cost function for frequency offset estimation of the first user (true value $\theta_1 T = -0.47$).

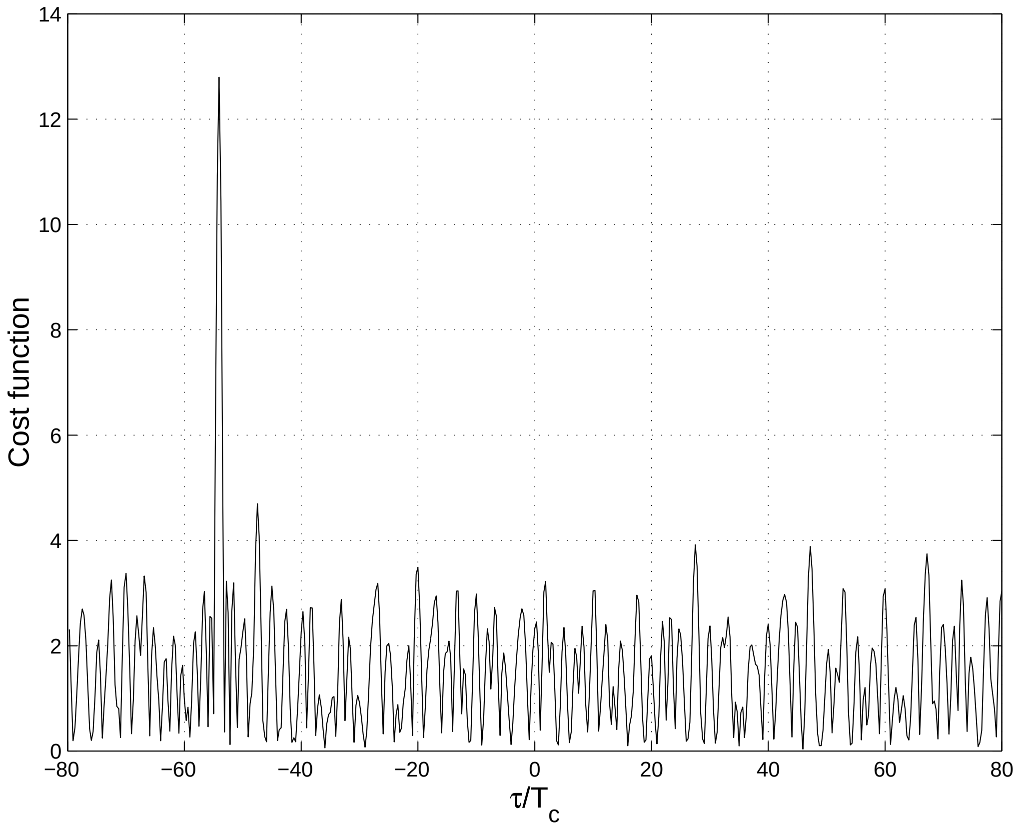

Figure 2: Cost function for time delay estimation of the first user (true value $\tau_1/T_c = -54$).