Joint Equalization and Interference Suppression in OFDM Systems

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The paper deals with the joint equalization and narrowband interference (NBI) suppression in orthogonal frequency-division multiplexing (OFDM) systems. Since the conventional zero-forcing (ZF) receiver does not operate satisfactorily in interference-limited environments, we investigate the potential of the MMSE receiver, evaluating the advantage gained by exploiting the redundancy contained in the cyclic prefix (CP).

Introduction: When employed in high data-rate applications, OFDM systems suffer from the presence of NBI originated by low-rate systems operating in the same frequency band. In these cases, the ZF receiver, which discards the CP information, can perform very poor. Better reception strategies, referred to as *windowing receivers* [1, 2], use samples from the CP to perform a windowing before the DFT, aimed to reduce the noise and interference contributions [both NBI, interblock interference (IBI) and/or interchannel interference (ICI)] without modifying the desired signal component. In particular, *data-dependent window* (DDW) design procedures have been proposed, based on the MMSE criterion [1, 2]: in order to account for complexity limitations or special channel conditions, such procedures introduce some constraints on the window shape.

The present letter provides a unifying framework to the problem of the windowing-based receiver synthesis. Unlike the previous methods, the proposed MMSE procedure does not introduce any constraint, allowing so one to reach the optimum performance. In our framework, the windowing receivers can be easily recognized as *constrained* versions of the proposed MMSE procedure and, hence, an interesting theoretical and numerical comparison among the different techniques can be easily carried out.

The OFDM system model: Let us consider the baseband-equivalent of an OFDM system with N subcarriers (see [3] for details). The information stream $a(m)$ (modeled as a zero-mean iid sequence with variance σ_a^2) undergoes the following transformations at the transmitter: (i) serial-to-parallel conversion (S/P), which splits the stream $a(m)$ into N substreams $a_k(m) \triangleq a(mN + k)$, $k = 0, 1, \ldots, N-1$; (ii) N-point IDFT; (iii) insertion of a CP of length L, obtaining the P-column vector $\boldsymbol{u}(m)$, with $P = N + L$. By defining $\boldsymbol{a}(m) \triangleq [a_0(m), a_1(m), \ldots, a_{N-1}(m)]^T$, with T denoting transpose, one has

$$\boldsymbol{u}(m) = \boldsymbol{T}_{\rm cp}\, \boldsymbol{W}_{\rm IDFT}\, \boldsymbol{a}(m) = \boldsymbol{T}_0\, \boldsymbol{a}(m)\,, \tag{1}$$

where $\boldsymbol{W}_{\rm IDFT}$ is the $N \times N$ IDFT matrix, $\boldsymbol{T}_{\rm cp} \triangleq [\boldsymbol{I}_{\rm cp}^T, \boldsymbol{I}_N]^T$ is the $P \times N$ CP insertion matrix, with $\boldsymbol{I}_{\rm cp}$ denoting the $L \times N$ matrix obtained from $\boldsymbol{I}_N$ by picking its last L rows, and $\boldsymbol{T}_0 \triangleq \boldsymbol{T}_{\rm cp}\, \boldsymbol{W}_{\rm IDFT}$ is the $P \times N$ *precoding matrix*. Vector $\boldsymbol{u}(m)$ undergoes parallel-to-serial (P/S) conversion and the resulting serial stream is transformed by the D/A converter, operating at rate $1/T_c$, into the signal $u(t) = \sum_{m=-\infty}^{\infty} \sum_{p=-L}^{N-1} u_p(m)\, \phi(t - p\, T_c - m\, T)$, where $\phi(t)$ is the D/A impulse response, $T \triangleq P\, T_c$ represents the symbol period, and the elements of vector $\boldsymbol{u}(m)$ have been indicized as $u_p(m)$.

The signal $u(t)$ propagates through a linear time-invariant (LTI) channel $g(t)$, and then is filtered at the receiver by $\psi(t)$ and contaminated by wide-sense stationary (WSS) additive disturbance $d(t)$, modeled as the sum of thermal noise plus NBI. By collecting the samples of the received signal $r(t) = u(t) * g(t) * \psi(t) + d(t)$ at time epochs $t_{n,\ell} = n\, T + \ell\, T_c$, for $\ell = -L, -L+1, \ldots, N-1$, we obtain the vector model

$$\boldsymbol{r}(n) = \boldsymbol{H}_0\,\boldsymbol{T}_0\,\boldsymbol{a}(n) + \boldsymbol{H}_1\,\boldsymbol{T}_0\,\boldsymbol{a}(n-1) + \boldsymbol{d}(n)\;, \tag{2}$$

where perfect time and frequency synchronization at the receiver side is assumed, $\boldsymbol{d}(n)$ is defined similarly to $\boldsymbol{r}(n)$, the $P \times P$ Toeplitz matrices $\boldsymbol{H}_0$ and $\boldsymbol{H}_1$ are given as in [3], and the assumption that the CP length L exceeds the channel memory L_1 assures that IBI can be due to only symbol block $\boldsymbol{a}(n-1)$ and, moreover, it can be canceled by removing the CP portion.

Linear receivers for OFDM: To detect $\boldsymbol{a}(n)$, we resort to a LTI filter, namely, $\boldsymbol{y}(n) = \boldsymbol{F}^H\,\boldsymbol{r}(n)$, where $\boldsymbol{F} \triangleq [\boldsymbol{f}_1, \boldsymbol{f}_2, \ldots, \boldsymbol{f}_N]$ is a $P \times N$ matrix, whose choice can be based on several optimality criteria. Accounting for (2), one has (the superscript H denotes conjugate transpose):

$$\boldsymbol{y}(n) = \boldsymbol{F}^H \boldsymbol{H}_0\,\boldsymbol{T}_0\,\boldsymbol{a}(n) + \boldsymbol{F}^H \boldsymbol{H}_1\,\boldsymbol{T}_0\,\boldsymbol{a}(n-1) + \boldsymbol{F}^H\,\boldsymbol{d}(n)\;. \tag{3}$$

ZF receivers assure perfect reconstruction in the absence of disturbance and are obtained by imposing in (3) the conditions C1) $\boldsymbol{F}^H \boldsymbol{H}_0\,\boldsymbol{T}_0 = \boldsymbol{I}_N$ and C2) $\boldsymbol{F}^H \boldsymbol{H}_1\,\boldsymbol{T}_0 = \boldsymbol{0}_{N\times N}$, which result in ICI and IBI suppression, respectively. The simplest ZF receiver is the conventional ZF-OFDM receiver given by

$$\boldsymbol{F}^H_{\text{ZF}-\text{OFDM}} = \boldsymbol{\mathcal{H}}^{-1}\,\boldsymbol{W}_{\text{DFT}}\,\boldsymbol{R}_{\text{cp}}\;, \tag{4}$$

where $\boldsymbol{\mathcal{H}}$ is the $N \times N$ diagonal matrix containing the samples of the N-point DFT of the (zero-padded) L_1-column channel vector $\boldsymbol{h} \triangleq [h(0), h(1), \ldots, h(L_1-1)]^T$, $\boldsymbol{W}_{\text{DFT}}$ is the $N \times N$ DFT matrix and $\boldsymbol{R}_{\text{cp}} \triangleq [\boldsymbol{0}_{N\times L}, \boldsymbol{I}_N]$ is the $N \times P$ CP removal matrix. Such a receiver does not attempt to counteract the presence of the NBI and, moreover, it discards the CP, although it contains useful information.

MMSE receivers: When a strong NBI is present, one can trade-off a small amount of ICI and IBI for a better NBI suppression. Therefore, we propose to estimate the vector $\boldsymbol{a}(n)$ by elaborating the CP portion and resorting to the MMSE criterion, i.e., by minimizing the MSE $\triangleq E\left[\|\boldsymbol{F}^H \boldsymbol{r}(n) - \boldsymbol{a}(n)\|^2\right]$ with respect to $\boldsymbol{F}$, which leads to $\boldsymbol{F}_{\mathrm{MMSE}} = \sigma_a^2 \, \boldsymbol{R}_{\boldsymbol{rr}}^{-1} \, \boldsymbol{H}_{\boldsymbol{0}} \boldsymbol{T}_{\boldsymbol{0}}$, where $\boldsymbol{R}_{\boldsymbol{rr}} \triangleq E[\boldsymbol{r}(n) \, \boldsymbol{r}^H(n)]$. The minimum value of MSE is given by

$$\mathrm{MSE}_{\min} = \sigma_a^2 \left[N - \sigma_a^2 \, \mathrm{trace}(\boldsymbol{R}_{\boldsymbol{rr}}^{-1} \, \boldsymbol{H}_0 \boldsymbol{T}_0 \boldsymbol{T}_0^H \boldsymbol{H}_0^H)\right] \, . \tag{5}$$

The MMSE receiver cannot be implemented by means of the FFT and, hence, its computational complexity can be high for OFDM systems with large number N of subcarriers (e.g., ADSL). However, for moderate values of N, it can offer a noticeable performance advantage over the ZF receivers, with a tolerable increase in complexity. Moreover, observe that MMSE receiver can work also when the condition $L_1 \leq L$ is not satisfied, i.e., in the case of a CIR longer than the CP or even when the CP is absent altogether.

ZF-MMSE and windowing-based receivers: The MSE minimization can be also carried out by imposing the conditions C1) and C2): the resulting receiver will be referred to as a *constrained ZF-MMSE* equalizer. Belonging to this class are the windowing-based receivers [1, 2], which resort to DDW to improve performance, both in terms of disturbance rejection [1] and of robustness against ICI induced by frequency offset errors [2]. We focus, in particular, on the approach of Redfern [1], which can be interpreted in our framework as a modification of the ZF-OFDM receiver, obtained by substituting the CP removal matrix $\boldsymbol{R}_{\mathrm{cp}}$ in (4) with a $P \times N$ *windowing matrix* $\boldsymbol{W}$; hence the overall receiver is given by $\boldsymbol{F}^H \triangleq \boldsymbol{\mathcal{H}}^{-1} \, \boldsymbol{W}_{\mathrm{DFT}} \, \boldsymbol{W}$, and the matrix $\boldsymbol{W}$ is determined by minimizing the MSE, while satisfying the ZF conditions C1) and C2). To assure an affordable complexity, in [1] the matrix $\boldsymbol{W}$

is assumed to have a parametric structure depending only on a W-dimensional vector, where W is chosen to be much smaller than N.[1] Shortly, the receiver described by [1] can be interpreted as a constrained ZF-MMSE receiver, with an additional structural constraint about $\boldsymbol{W}$. Hence, its MSE cannot be smaller than the $\text{MSE}_{\min}$ given by (5); moreover, since the number of available degrees of freedom is given by W, it can be too small to achieve a satisfactory disturbance rejection.

Simulation results: We compare the performance of the conventional ZF-OFDM receiver, the MMSE receiver without constraints and the constrained ZF-MMSE or windowing receiver of Redfern [1], with $W = 4$. The OFDM system parameters are $N = 32$ and $L = 8$, whereas $h(n) = 0.5^n$, for $n = 0, 1, \ldots, L_1 - 1$, with $L_1 = 4$ (discrete-time channel). The NBI is modeled as a WSS Gaussian process, with autocorrelation function $r_{\text{NBI}}(m) = \sigma^2_{\text{NBI}}\, a^{|m|}$, where a can be related to the 3-dB bandwidth ν_3, and is set to 0.8, unless otherwise stated; the thermal noise is modeled as a Gaussian iid sequence with power σ_n^2. Unless otherwise specified, the $\text{SNR} \triangleq \sigma_a^2/\sigma_n^2$ and the $\text{SIR} \triangleq \sigma_a^2/\sigma^2_{\text{NBI}}$ are set to 30 and 10 dB, respectively.

In Fig. 1 we reported the maximal mutual information [4] between the recovered block $\boldsymbol{y}(n)$ and the input block $\boldsymbol{a}(n)$. The MMSE receiver outperforms both the ZF-OFDM and the ZF-MMSE one, the increase in capacity being between 15% and 20% over the ZF-OFDM receiver and between 8% and 10% over the ZF-MMSE one, where the advantage decreases with increasing values of SNR. In Fig. 2 the mutual information is evaluated as a function of the 3-dB NBI bandwidth ν_3, normalized to $1/N$, which is the discrete-time intercarrier spacing of the OFDM system. The mutual information decreases with increasing values of the NBI bandwidth; this can be explained by observing that a larger number of subcarriers is corrupted by the NBI, although the overall NBI power is kept

[1] It should be noted that the windowing receiver [1] requires $L_1 \leq L - W$, which is a more restrictive condition than $L_1 \leq L$ needed by the ZF-OFDM receiver.

constant. The capacity improvement of the MMSE receiver is about 18% over ZF-OFDM and about 8% over ZF-MMSE, and does not vary significantly with ν_3.

In conclusion, we observe that the MMSE receiver, which elaborates the CP, assures an increase of capacity up to 20% over the ZF-OFDM and up to 10% over the ZF-MMSE receiver. However, one should take into account that the ZF-OFDM and ZF-MMSE receivers can be implemented by FFT and, hence, exhibit a lower computational complexity with respect to the *unconstrained* MMSE solution.

References

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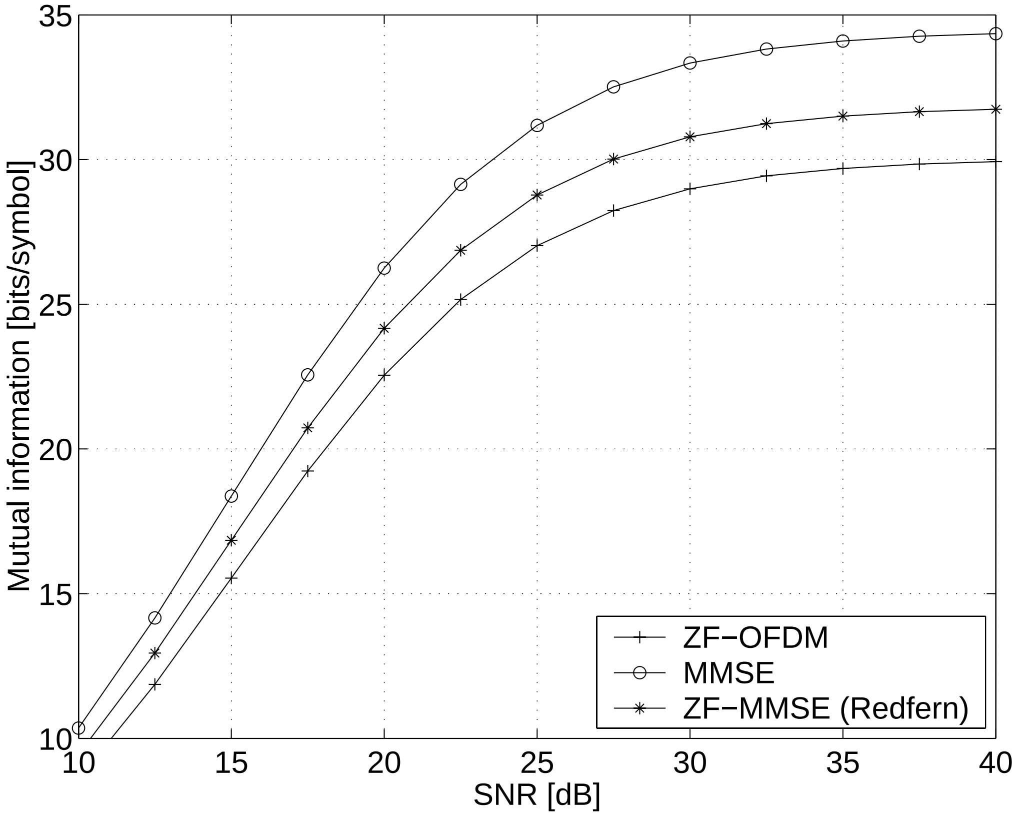

Figure 1: Mutual information versus SNR (in dB).

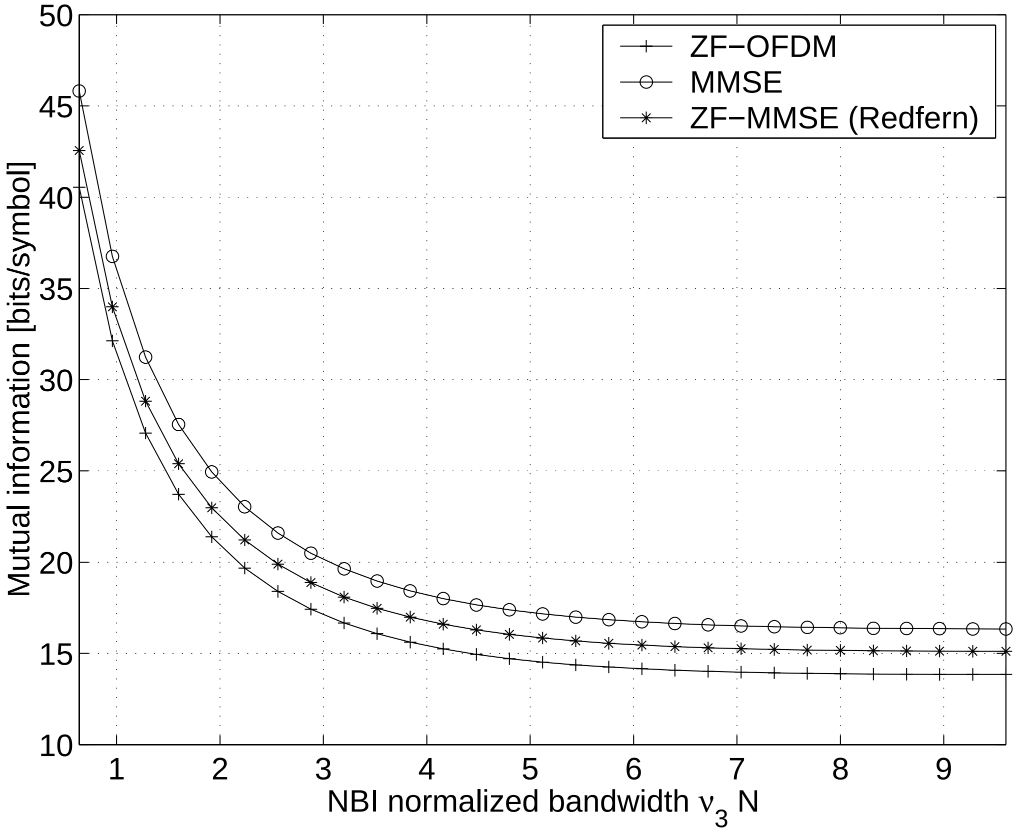

Figure 2: Mutual information versus NBI normalized bandwidth.