

Perfect symbol recovery and NBI suppression in MIMO-OFDM systems

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It is shown that, by exploiting the degrees of freedom arising from the use of multiple antennas and/or insertion of virtual carriers in multiple-input multiple-output (MIMO) orthogonal frequency-division multiplexing (OFDM) systems, perfect symbol recovery and complete narrowband interference (NBI) cancellation can be achieved in the absence of noise by means of constrained minimum mean-output-energy linear filtering. Moreover, a nonlinear iterative scheme is proposed to improve NBI rejection in realistic operative conditions.

Introduction: Multiple-input multiple-output (MIMO) systems have been adopted in many standards, such as, e.g., IEEE802.11n, LTE, and WiMAX, as a means to enhance data rate through spatial multiplexing and/or improve link reliability and range through spatial diversity. At the same time, the growing request for high-speed wireless services demands the MIMO signaling bandwidth to be larger than the channel coherence bandwidth, causing frequency-selective fading and, hence, intersymbol interference (ISI). A key transmission technique to combat ISI is *orthogonal frequency-division multiplexing (OFDM)*.

In future wireless communication systems, broadband MIMO-OFDM systems, which may span a bandwidth up to several tens of MHz, might share the spectrum with other systems having narrower bandwidths, e.g., 3G devices such as EDGE, DECT, CDMA-2000, and even W-CDMA [1]. Interference originated by such narrowband systems can severely degrade the performance of MIMO-OFDM systems [2]. Several algorithms have been proposed to suppress *narrowband interference (NBI)* in single-input single-output OFDM systems [1, 3, 4].

By exploiting the spatial and/or spectral redundancy contained in the MIMO-OFDM signal, we show that constrained *minimum mean-output-energy (MMOE)* linear filtering jointly ensures perfect symbol recovery and complete NBI cancellation in the absence of noise, provided that certain mild conditions are fulfilled. When the NBI cancellation of the constrained MMOE (CMMOE) filter is not perfect, we also propose to use a *nonlinear iterative cancellation scheme* to improve NBI suppression.

System model: We consider a MIMO-OFDM system with M subcarriers, M_{vc} virtual carriers (VCs), N_T transmitting and N_R receiving antennas, employing channel-independent spatial multiplexing (the discussion can be extended with minor modifications to the case of space-time block coding). Let $\mathbf{s}^\alpha(n) \triangleq [s_0^\alpha(n), s_1^\alpha(n), \dots, s_{Q-1}^\alpha(n)]^T \in \mathbb{C}^Q$ denote the block of information symbols to be transmitted on the α th antenna within the n th sampling interval, with $Q \triangleq M - M_{vc} \geq 0$ and $\alpha \in \{1, 2, \dots, N_T\}$. Such a block undergoes conventional OFDM precoding, encompassing VC insertion, inverse discrete Fourier transform (IDFT), and cyclic prefix (CP) insertion of length L_{cp} . At the β th receiving antenna, with $\beta \in \{1, 2, \dots, N_R\}$, after performing CP removal and discrete Fourier transform (DFT), if the CP length exceeds the maximum delay spread of the MIMO channel, the received signal $\mathbf{r}^\beta(n) \in \mathbb{C}^M$ is completely free of interblock interference. In this case, by vertically stacking all such data into $\mathbf{r}(n) \in \mathbb{C}^{N_R M}$, one obtains the following compact model

$$\mathbf{r}(n) = \mathbf{H} \mathbf{s}(n) + \mathbf{j}(n) + \mathbf{w}(n) \quad (1)$$

where $\mathbf{H} \triangleq \mathcal{H}(\mathbf{I}_{N_T} \otimes \Theta) \in \mathbb{C}^{(N_R M) \times (N_T Q)}$, with the matrix $\Theta \in \mathbb{R}^{M \times Q}$ performing VC insertion [4] and \otimes denoting the Kronecker product, whereas the (β, α) block of the matrix $\mathcal{H} \in \mathbb{C}^{(N_R M) \times (N_T M)}$ is the diagonal matrix collecting the values on the DFT grid of the \mathcal{Z} -transform of the L_h -order impulse channel response $\{h^{\beta\alpha}(\ell)\}_{\ell=0}^{L_h}$ between the α th transmitting antenna and the β th receiving antenna, for $\alpha \in \{1, 2, \dots, N_T\}$ and $\beta \in \{1, 2, \dots, N_R\}$, the vector $\mathbf{s}(n) \in \mathbb{C}^{N_T Q}$ is the vertical stacking of $\{\mathbf{s}^\alpha(n)\}_{\alpha=1}^{N_T}$, $\mathbf{j}(n) \in \mathbb{C}^{N_R M}$ and $\mathbf{w}(n) \in \mathbb{C}^{N_R M}$ account for NBI and thermal noise, respectively.

Hereinafter, we assume that: **(a1)** $\mathbf{s}(n)$ is a zero-mean circularly symmetric complex (ZMCSC) random vector, having correlation matrix $\mathbb{E}[\mathbf{s}(n) \mathbf{s}^H(n)] = \sigma_s^2 \mathbf{I}_{N_T Q}$; **(a2)** $\mathbf{j}(n)$ is modeled as a ZMCSC wide-sense stationary random vector, statistically independent of $\mathbf{s}(n)$, with correlation matrix $\mathbf{R}_{jj} \triangleq \mathbb{E}[\mathbf{j}(n) \mathbf{j}^H(n)] \in \mathbb{C}^{(N_R M) \times (N_R M)}$; **(a3)** $\mathbf{w}(n)$ is a ZMCSC Gaussian random vector, statistically independent of $\mathbf{s}(n)$ and $\mathbf{j}(n)$, with correlation matrix $\mathbb{E}[\mathbf{w}(n) \mathbf{w}^H(n)] = \sigma_w^2 \mathbf{I}_{N_R M}$.

Perfect symbol recovery and MMOE-based NBI rejection: To recover the information block $\mathbf{s}(n)$ and jointly mitigate NBI, we employ a linear filter $\mathbf{G} \in \mathbb{C}^{(N_T Q) \times (N_R M)}$, whose input-output relationship is given by

$$\mathbf{y}(n) = \mathbf{G} \mathbf{r}(n) = \mathbf{G} \mathbf{H} \mathbf{s}(n) + \mathbf{G} [\mathbf{j}(n) + \mathbf{w}(n)] \quad (2)$$

where $\mathbf{y}(n) \in \mathbb{C}^{N_T Q}$ contains soft estimates of the elements of $\mathbf{s}(n)$, which are then quantized to the nearest (in terms of Euclidean distance) symbol to form hard estimates of the symbols transmitted on each antenna.

Perfect recovery of the symbol block $\mathbf{s}(n)$ is ensured by enforcing the zero-forcing (ZF) constraint $\mathbf{G} \mathbf{H} = \mathbf{I}_{N_T Q}$. If the channel matrix \mathbf{H} is tall and full-column rank, i.e., $N_R M > N_T Q$ and $\text{rank}(\mathbf{H}) = N_T Q$, the solution of $\mathbf{G} \mathbf{H} = \mathbf{I}_{N_T Q}$ is *not* unique and, thus, the remaining degrees of freedom in \mathbf{G} can be exploited to mitigate the effects of the NBI. Such a result can be obtained by choosing \mathbf{G} so as to minimize the mean-output-energy (MOE) at the output of the linear filter in the *frequency-domain*, which is given by $\text{MOE}(\mathbf{G}) \triangleq \mathbb{E}[\|\mathbf{y}(n)\|^2] = \text{trace}(\mathbf{G} \mathbf{R}_{rr} \mathbf{G}^H)$, where $\mathbf{R}_{rr} \triangleq \mathbb{E}[\mathbf{r}(n) \mathbf{r}^H(n)] \in \mathbb{C}^{(N_R M) \times (N_R M)}$ is the correlation matrix of $\mathbf{r}(n)$. Therefore, we choose \mathbf{G} as the solution of the CMMOE problem

$$\min_{\mathbf{G}} \text{trace}(\mathbf{G} \mathbf{R}_{rr} \mathbf{G}^H) \quad \text{subject to} \quad \mathbf{G} \mathbf{H} = \mathbf{I}_{N_T Q} \quad (3)$$

whose solution is given by

$$\mathbf{G}_{\text{cmmoe}} = \mathbf{H}^\dagger \left[\mathbf{I}_{N_R M} - \mathbf{R}_{rr} \mathbf{\Pi}^H (\mathbf{\Pi} \mathbf{R}_{rr} \mathbf{\Pi}^H)^{-1} \mathbf{\Pi} \right] \quad (4)$$

where $\mathbf{R}_{rr} = \sigma_s^2 \mathbf{H} \mathbf{H}^H + \mathbf{R}_{jj} + \sigma_w^2 \mathbf{I}_{N_R M}$, $\mathbf{\Pi} \in \mathbb{C}^{(N_R M - N_T Q) \times (N_R M)}$ obeys $\mathbf{\Pi} \mathbf{H} = \mathbf{O}_{(N_R M - N_T Q) \times (N_T Q)}$ and $\mathbf{\Pi} \mathbf{\Pi}^H = \mathbf{I}_{(N_R M - N_T Q)}$, and \mathbf{H}^\dagger is the pseudo-inverse of \mathbf{H} . It can be readily verified that, when \mathbf{H} is square (that is, $N_R M = N_T Q$), the CMMOE solution boils down to the conventional ZF receiver $\mathbf{G}_{zf} \triangleq \mathbf{H}^{-1}$.

To highlight the NBI rejection capability of the CMMOE filter, we first derive the asymptotic expression of $\mathbf{G}_{\text{cmmoe}}$ for $\sigma_w^2 \rightarrow 0$. To this aim, we observe that, due to the narrowband nature of the interference, the first R_{nbi} eigenvalues of \mathbf{R}_{jj} are significantly different from zero, whereas the remaining ones are vanishingly small; hence, the eigenvalue decomposition of \mathbf{R}_{jj} is well-modeled as $\mathbf{R}_{jj} = \mathbf{J} \mathbf{J}^H$, with $\mathbf{J} \triangleq \mathbf{U} \mathbf{\Sigma}^{1/2} \in \mathbb{C}^{(N_R M) \times R_{\text{nbi}}}$, where the diagonal matrix $\mathbf{\Sigma} \in \mathbb{R}^{R_{\text{nbi}} \times R_{\text{nbi}}}$ collects the R_{nbi} dominant eigenvalues of \mathbf{R}_{jj} , whose corresponding eigenvectors are the columns of $\mathbf{U} \in \mathbb{C}^{(N_R M) \times R_{\text{nbi}}}$. Therefore, by using the limit formula for the pseudo-inverse, one obtains

$$\overline{\mathbf{G}}_{\text{cmmoe}} \triangleq \lim_{\sigma_w^2 \rightarrow 0} \mathbf{G}_{\text{cmmoe}} = \mathbf{H}^\dagger \left[\mathbf{I}_{N_R M} - \mathbf{J} (\mathbf{\Pi} \mathbf{J})^\dagger \mathbf{\Pi} \right]. \quad (5)$$

Accounting for (5), the noiseless mean-output-energy of the CMMOE filter is given by $\text{trace}[\overline{\mathbf{G}}_{\text{cmmoe}} (\sigma_s^2 \mathbf{H} \mathbf{H}^H + \mathbf{R}_{jj}) \overline{\mathbf{G}}_{\text{cmmoe}}^H] = \sigma_s^2 N_T Q + \text{trace}(\overline{\mathbf{G}}_{\text{cmmoe}} \mathbf{J} \mathbf{J}^H \overline{\mathbf{G}}_{\text{cmmoe}}^H)$, which allows us to infer that

$$\overline{\mathbf{G}}_{\text{cmmoe}} \mathbf{J} = \mathbf{H}^\dagger \mathbf{J} \left[\mathbf{I}_{R_{\text{nbi}}} - (\mathbf{\Pi} \mathbf{J})^\dagger (\mathbf{\Pi} \mathbf{J}) \right] = \mathbf{O}_{(N_T Q) \times R_{\text{nbi}}} \quad (6)$$

is a *sufficient* condition to achieve *complete* cancellation of the NBI in the high signal-to-noise ratio (SNR) regime. It is noteworthy that condition (6) is trivially satisfied when $\mathbf{\Pi} \mathbf{J} \in \mathbb{C}^{(N_R M - N_T Q) \times R_{\text{nbi}}}$ is full-column rank, i.e., $N_R M - N_T Q \geq R_{\text{nbi}}$ and $\text{rank}(\mathbf{\Pi} \mathbf{J}) = R_{\text{nbi}}$, since in this case $(\mathbf{\Pi} \mathbf{J})^\dagger (\mathbf{\Pi} \mathbf{J}) = \mathbf{I}_{R_{\text{nbi}}}$. We have thus proved the following basic result.

Theorem 1: In the absence of noise, the CMMOE filter completely cancels NBI if **(c1)** $N_R M \geq N_T Q + R_{\text{nbi}}$ and **(c2)** $\text{rank}(\mathbf{\Pi} \mathbf{J}) = R_{\text{nbi}}$.

Condition (c1) imposes an upper bound on the number of dominant eigenvalues of the NBI correlation matrix, i.e., $R_{\text{nbi}} \leq N_R M - N_T Q$. Since R_{nbi} depends on the (nominal) bandwidth of the interference, such an upper bound poses a limit on the number of subcarriers of the MIMO-OFDM system that are hit by the NBI signal.

For low-to-moderate SNR values and/or when the filtering matrix $\mathbf{G}_{\text{cmmoe}}$ is estimated from data, one can resort to a nonlinear iterative scheme [3] to significantly improve NBI suppression, by operating on the output $\mathbf{y}_{\text{cmmoe}}(n) \triangleq \mathbf{G}_{\text{cmmoe}} \mathbf{r}(n)$ of the linear CMMOE filter.

Nonlinear NBI cancellation scheme: Let $\mathbf{d}(n) \triangleq \mathbf{G}_{\text{cmmoe}} [\mathbf{j}(n) + \mathbf{w}(n)]$ be the overall disturbance at the output of the CMMOE filter, it results that

$$\mathbf{y}_{\text{cmmoe}}(n) = \mathbf{s}(n) + \mathbf{d}(n). \quad (7)$$

The rationale of the proposed algorithm is to detect the strongest entry of $\mathbf{s}(n)$, canceling out its contribution from the received vector (7) and, then, using this modified received signal to have a better estimation of

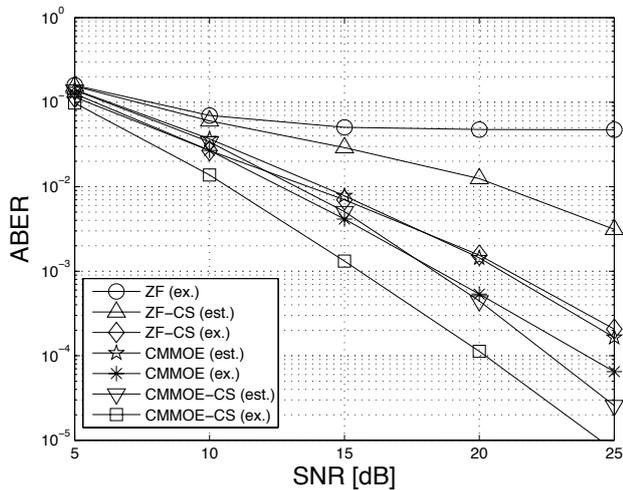


Fig. 1: Average BER versus SNR.

the NBI samples affecting the other subcarriers. The strongest entry of $s(n)$ is not known beforehand; it is detected from the strength of the output signal-to-interference-plus-noise ratio (SINR), which can be blindly estimated from (7). In the following, we drop the time dependence on the index n and assume that the entries of $\mathbf{y}_{\text{cmmoe}}$ have been already arranged in descending SINR order. The entry of $\mathbf{y}_{\text{cmmoe}}$ demodulated at the first step is $y_{\text{cmmoe},0} = s_0 + d_0$. The hard-decoded information symbol $\hat{s}_0 = \mathcal{Q}(y_{\text{cmmoe},0})$, with $\mathcal{Q}(\cdot)$ denoting the decision function, is removed from $y_{\text{cmmoe},0}$, thus obtaining $\hat{d}_0 \triangleq y_{\text{cmmoe},0} - \hat{s}_0$, which represents an estimate of the disturbance contribution corrupting the first entry of \mathbf{s} . The key idea of the proposed algorithm consists of using \hat{d}_0 to perform a reliable linear prediction \hat{d}_1 of the disturbance on the second entry $y_{\text{cmmoe},1}$ of $\mathbf{y}_{\text{cmmoe}}$. At the second step, the disturbance prediction \hat{d}_1 is removed from $y_{\text{cmmoe},1}$, hence increasing the reliability on the decision $\hat{s}_1 = \mathcal{Q}(y_{\text{cmmoe},1} - \hat{d}_1)$ of the information symbol s_1 . Such an estimate \hat{s}_1 is then used to produce an estimate $\hat{d}_1 \triangleq y_{\text{cmmoe},1} - \hat{s}_1$ of the disturbance contribution affecting $y_{\text{cmmoe},1}$, which, together with \hat{d}_0 , is employed to obtain a linear prediction \hat{d}_2 of the disturbance on the successive entry $y_{\text{cmmoe},2}$ of $\mathbf{y}_{\text{cmmoe}}$, and so on. The iterative procedure is carried out until all the entries of the block \mathbf{s} have been recovered, i.e., after $N_T Q$ steps.

At the k -th step, for $k \in \{0, 1, \dots, N_T Q - 1\}$, under the customary assumption of correct past decisions, i.e., $\hat{s}_\ell = s_\ell$, with $\ell < k$, which implies $\hat{d}_\ell = d_\ell$, for $\ell < k$, the disturbance predictor can be equivalently rewritten as $\hat{\mathbf{d}}_k = \boldsymbol{\gamma}_k^H \mathbf{d}_k$, where $\boldsymbol{\gamma}_k \in \mathbb{C}^{k \times 1}$ collects the predictor weights and $\mathbf{d}_k \triangleq [d_0, d_1, \dots, d_{k-1}]^T \in \mathbb{C}^{k \times 1}$ represents the prediction data vector. The weight vector $\boldsymbol{\gamma}_k$ can be chosen so as to minimize the prediction error power $\epsilon_k \triangleq \mathbb{E}[|d_k - \hat{d}_k|^2]$. By virtue of the orthogonality principle and exploiting the statistical independence between \mathbf{s} and \mathbf{d} , the optimal predictor is given by $\boldsymbol{\gamma}_k = \mathbf{Q}_k^{-1} \mathbf{q}_k$, where $\mathbf{Q}_k \triangleq \mathbb{E}[\hat{\mathbf{d}}_k \hat{\mathbf{d}}_k^H] \in \mathbb{C}^{k \times k}$ is the correlation matrix of $\hat{\mathbf{d}}_k \triangleq [\hat{d}_0, \hat{d}_1, \dots, \hat{d}_{k-1}]^T \in \mathbb{C}^{k \times 1}$ and $\mathbf{q}_k \triangleq \mathbb{E}[\hat{\mathbf{d}}_k y_{\text{cmmoe},k}^*] \in \mathbb{C}^{k \times 1}$ represents the correlation between the disturbance estimated vector $\hat{\mathbf{d}}_k$ and the k -th pre-filtered signal $y_{\text{cmmoe},k}$.

Simulations: In this section, we present the average BER (ABER) at the output of the proposed receiver, obtained via Monte Carlo simulations. The number of subcarriers is $M = 32$, with $Q = 28$ and $M_{\text{vc}} = 4$. Gray labeled QPSK signaling is used, the CP length is $L_{\text{cp}} = 8$, and $N_T = 2$ transmitting and $N_R = 4$ receiving antennas are employed. The continuous-time NBI signal is modeled as $\tilde{y}^\beta(t) = h_I^\beta [\sum_{k=-\infty}^{\infty} s_I(k) \psi_I(t - kT_I)] e^{j2\pi f_I t}$, where h_I^β is the complex channel gain, modeled as a zero-mean Gaussian random variable, having unitary variance, T_I and f_I are the symbol interval and carrier frequency offset, respectively, $\{s_I(k)\}$ are independent and identically distributed symbols, with variance σ_I^2 , and $\psi_I(t)$ models the convolution between the NBI shaping pulse and the analog-to-digital converter. We set $T_I = T$ and $f_I = 4P/T_c$, with $P \triangleq M + L_{\text{cp}}$, resulting in $R_{\text{nbi}} = 4$. For each (α, β) , the samples of the channel $h^{\beta\alpha}(\ell)$, whose maximum order is set to $L_h = 4$, are ZMCSC Gaussian random variables, with $\mathbb{E}[\sum_{\ell=0}^{L_h} |h^{\beta\alpha}(\ell)|^2] = 1$. The SNR and the signal-to-interference ratio, which is set equal to 5 dB, are defined starting from (1).

In Fig. 1 we reported the ABER performance of the CMMOE filter without the nonlinear iterative NBI cancellation stage (labeled as

“CMMOE”), compared to that of the same filter equipped with the nonlinear iterative NBI cancellation stage (labeled as “CMMOE-CS”). We also reported the performances of the conventional ZF filter (labeled as “ZF”) and of the same filter equipped with the nonlinear iterative NBI cancellation stage (labeled as “ZF-CS”). In addition to the exact (i.e., with perfect knowledge of the MIMO channel and the correlation matrices \mathbf{R}_{rr} and \mathbf{Q}_k) versions of the considered receivers [referred to as “(ex.)”], we also implemented their data-estimated counterparts [referred to as “(est.)”], with diagonal-loading (see [3] for details).

Results show that, as the SNR increases, the “CMMOE-CS (ex.)” receiver exhibits a significant performance gain with respect to the conventional “ZF (ex.)” one, outperforming both the “CMMOE (ex.)” and the “ZF-CS (ex.)” alternatives. Turning to the data-estimated versions, we observe that both the “CMMOE” and the “CMMOE-CS” receivers, differently from the “ZF-CS” equalizer, pay only a moderate performance penalty with respect to their exact counterparts.

Conclusion: Even when joint perfect symbol recovery and complete NBI rejection are guaranteed for linear MMOE-based filtering in the absence of noise, a noticeable performance gain is achieved in MIMO-OFDM systems by adding a nonlinear NBI cancellation stage.

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