# Successive NBI Cancellation Using Soft Decisions for OFDM Systems

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***Abstract*—Recently, we proposed a successive narrowband interference (NBI) cancellation method for orthogonal frequency-division multiplexing (OFDM) systems, which empowers the conventional one-tap frequency-domain equalizer to gain in robustness against NBI. In this letter, we generalize such an approach by using soft (instead of hard) feedback from the decoding unit, when the digitally-modulated NBI is (possibly) improper. It is shown that, with respect to our previous work, the devised equalizers allow one to achieve a significant performance gain, with a moderate increase in computational complexity.**

***Index Terms*—Narrowband interference (NBI) suppression, orthogonal frequency-division multiplexing (OFDM), soft decision feedback, widely-linear processing.**

## I. Introduction

IN SPITE of their indisputable equalization capabilities, orthogonal frequency-division multiplexing (OFDM) systems are very sensitive to narrowband interference (NBI), which might be the dominant source of performance degradation [1]–[5]. Advanced NBI cancellation techniques can be adopted to improve overall performance of OFDM systems, by relying on Hadamard linear precoding at the transmitter [2] or exploiting at the receiver some forms of redundancy, such as cyclic prefix (CP) and virtual carrier insertion [3] or constellation symmetry [4], [5]. However, all these approaches involve a significant modification of the conventional OFDM transmitter and/or receiver. An alternative viable approach to NBI mitigation was proposed in [1], which can be adapted to existing multicarrier standards more readily since it is based on the conventional inverse discrete Fourier transform (IDFT) precoding at the transmitter and involves at the receiver the conventional one-tap frequency-domain equalizer (FEQ). The basic idea underlying the approach of [1] is that, after performing (complete) CP removal, discrete Fourier transform (DFT), and one-tap FEQ to suppress interblock interference (IBI) and intercarrier interference (ICI), detection of the received block might be improved sequentially one subcarrier after the other. These improvements are obtained by using *hard* decisions of the OFDM symbols to predict the NBI contribution on the used subcarriers, modeled as a *proper* [6] random process;[1] then, the predicted NBI is successively subtracted from each subcarrier to reach a better final decision on the transmitted OFDM symbol. However, for low values of the signal-to-interference ratio (SIR), using *soft* instead of hard decisions for predicting the NBI contribution might mitigate the detrimental effects of wrong decisions more effectively [7]. Moreover, many digitally-modulated NBI signals of interest are *improper* [6] since, for the sake of bandwidth efficiency, they are generated by using improper modulation formats, such as offset quadrature phase-shift keying (OQPSK), offset quadrature amplitude modulation (OQAM), minimum shift-keying (MSK), and Gaussian MSK (GMSK).

In this letter, we generalize the successive NBI cancellation (S-NBI-C) algorithm of [1] by using soft decisions of the OFDM symbols for NBI prediction and, additionally, accounting for the (possible) improper nature of the NBI. Soft interference cancellation was originally employed in the area of multiuser detection [8], [9] and, more recently, has also proven fruitful for IBI and/or ICI cancellation in NBI-free OFDM systems [10]. Herein, we show that soft estimates of the OFDM symbols significantly improve the NBI prediction process. Furthermore, the synthesis of the proposed receiver is carried out in a general framework and the obtained receiver subsumes as a particular case not only the equalizer of [1] but also other novel receivers.

## II. Proposed S-NBI-C Algorithm

Let[2] $\mathbf{W}_{\text{DFT}} \in \mathbb{C}^{M\times M}$ be the unitary DFT matrix and $\boldsymbol{\mathcal{H}} \triangleq \text{diag}[H(0), H(1), \ldots, H(M-1)]$, with $M$ denoting the number of subcarriers of the OFDM system and $\{H(m)\}_{m=0}^{M-1}$ being the nonzero DFT of the discrete-time channel $\{h(n)\}_{n=0}^{L_h}$. After conventional OFDM processing (CP removal, DFT, and one-tap FEQ), the received IBI- and ICI-free block is (see [1])

$$\mathbf{y}(n) = \mathbf{s}(n) + \underbrace{\boldsymbol{\mathcal{H}}^{-1}\mathbf{W}_{\text{DFT}}[\mathbf{w}(n) + \boldsymbol{\jmath}(n)]}_{\mathbf{d}(n)\in\mathbb{C}^M} = \mathbf{s}(n) + \mathbf{d}(n) \tag{1}$$

[1]A random process $r(n)$ is proper (or circular) if $\text{E}[r(n)\, r^*(n-m)] \equiv 0$, $\forall n, m \in \mathbb{Z}$; otherwise, it is referred to as improper (or noncircular).

[2]Matrices [vectors] are denoted with uppercase [lowercase] boldface letters (e.g., $\mathbf{A}$ or $\boldsymbol{a}$); the field of $m \times n$ complex [real] matrices is denoted as $\mathbb{C}^{m\times n}[\mathbb{R}^{m\times n}]$, with $\mathbb{C}^m[\mathbb{R}^m]$ used as a shorthand for $\mathbb{C}^{m\times 1}[\mathbb{R}^{m\times 1}]$; the superscripts $*, T, H$, and $-1$ denote the conjugate, the transpose, the Hermitian, and the inverse of a matrix; $\mathbf{0}_m \in \mathbb{R}^m$ denote the null vector, $\mathbf{O}_{m\times n} \in \mathbb{R}^{m\times n}$ the null matrix, and $\mathbf{I}_m \in \mathbb{R}^{m\times m}$ the identity matrix; $\mathbf{A} = \text{diag}(a_1, a_2, \ldots, a_n) \in \mathbb{C}^{n\times n}$ is a diagonal matrix whose $(i,i)$th entry is $a_i$ and $\|\mathbf{A}\|$ is the Frobenius norm of $\mathbf{A}$; $\{\mathbf{A}\}_{i_1,i_2}$ is the $(i_1, i_2)$th element of $\mathbf{A}$; $\delta(n)$ is the Kronecker delta; $\text{E}[\cdot]$ denotes ensemble averaging, $j \triangleq \sqrt{-1}$ is the imaginary unit, and $\Re\{\cdot\}$ and $\Im\{\cdot\}$ denote real and imaginary parts; for any stationary random vector $\mathbf{x}(n) \in \mathbb{C}^m$, we denote with $\mathbf{R}_{\mathbf{xx}} \triangleq \text{E}[\mathbf{x}(n)\, \mathbf{x}^H(n)] \in \mathbb{C}^{m\times m}$ its autocorrelation matrix and with $\mathbf{R}_{\mathbf{xx}^*} \triangleq \text{E}[\mathbf{x}(n)\, \mathbf{x}^T(n)] \in \mathbb{C}^{m\times m}$ its conjugate correlation matrix.

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where $\mathbf{s}(n) \triangleq [s_0(n), s_1(n), \ldots, s_{M-1}(n)]^T \in \mathbb{C}^M$ represents the information symbol block, whose entries are modeled as a sequence of zero-mean unit-variance independent and identically distributed (i.i.d.) complex proper random variables,[3] whereas $\boldsymbol{\jmath}(n) \in \mathbb{C}^M$ and $\mathbf{w}(n) \in \mathbb{C}^M$ account for NBI and thermal noise after CP removal. Specifically, $\mathbf{w}(n)$ is modeled as a zero-mean complex proper Gaussian random vector with autocorrelation matrix $\mathbf{R_{ww}} = \sigma_{\rm w}^2 \mathbf{I}_M$, statistically independent of $\mathbf{s}(n)$, whereas $\boldsymbol{\jmath}(n)$ is modeled as a zero-mean complex improper random vector, statistically independent of both $\mathbf{s}(n)$ and $\mathbf{w}(n)$, with autocorrelation matrix $\mathbf{R}_{\boldsymbol{\jmath\jmath}} \in \mathbb{C}^{M\times M}$ and conjugate correlation matrix[4] $\mathbf{R}_{\boldsymbol{\jmath\jmath}^*}(n) \in \mathbb{C}^{M\times M}$.

The general description of the generalized S-NBI-C algorithm parallels that of [1]. The key differences stem from the fact that the proposed S-NBI-C method uses soft estimates of the OFDM symbols for producing a "clean" version of $\mathbf{y}(n)$ and, to account for the improper nature of the processed data (a more formal motivation is given later on), involves a *widely-linear* (WL) [11] prediction of the NBI contribution. Let $y_m(n)$ and $d_m(n)$ denote the $(m+1)$th entry of the received vector $\mathbf{y}(n)$ and the *disturbance* (NBI-plus-noise) vector $\mathbf{d}(n)$, for $m \in \mathcal{J}_c \triangleq \{0, 1, \ldots, M-1\}$, respectively. Dropping out hereafter the dependence on the block index $n$ and assuming that $\hat{d}_{i_1} = 0$,[5] the detection algorithm on the $(i_k)$th subcarrier, for $k \in \{1, 2, \ldots, M\}$, can be summarized as follows (see also Fig. 1): 1) form the clean version of $y_{i_k}$ as $z_{i_k} \triangleq y_{i_k} - \hat{d}_{i_k} = s_{i_k} + d_{i_k} - \hat{d}_{i_k}$; 2) produce a soft estimation of the transmitted symbol $s_{i_k}$ as $\zeta_{i_k} = \mathcal{Q}(z_{i_k})$, where $\mathcal{Q}(\cdot)$ is a nonlinear decision function whose choice will be discussed in detail soon after; 3) estimate the NBI contribution contaminating the $(i_k)$th subcarrier as $\tilde{d}_{i_k} = y_{i_k} - \zeta_{i_k}$; 4) define the weight vectors $\boldsymbol{\alpha}_k \triangleq [\alpha_{k,i_1}, \alpha_{k,i_2}, \ldots, \alpha_{k,i_k}]^H \in \mathbb{C}^k$ and $\boldsymbol{\beta}_k \triangleq [\beta_{k,i_1}, \beta_{k,i_2}, \ldots, \beta_{k,i_k}]^H \in \mathbb{C}^k$ and, if $k < M$, predict the NBI contribution on the next subcarrier as

$$\hat{d}_{i_{k+1}} = \boldsymbol{\alpha}_k^H \, \tilde{\mathbf{d}}_k + \boldsymbol{\beta}_k^H \tilde{\mathbf{d}}_k^* = \underbrace{[\boldsymbol{\alpha}_k^H, \boldsymbol{\beta}_k^H]}_{\boldsymbol{\gamma}_k^H \in \mathbb{C}^{1\times 2k}} \underbrace{\begin{bmatrix} \tilde{\mathbf{d}}_k \\ \tilde{\mathbf{d}}_k^* \end{bmatrix}}_{\tilde{\boldsymbol{d}}_k^{\rm aug} \in \mathbb{C}^{2k}} = \boldsymbol{\gamma}_k^H \, \tilde{\boldsymbol{d}}_k^{\rm aug} \quad (2)$$

where $\tilde{\mathbf{d}}_k \triangleq [\tilde{d}_{i_1}, \tilde{d}_{i_2}, \ldots, \tilde{d}_{i_k}]^T \in \mathbb{C}^k$ gathers the disturbance estimates available at the $k$th step. Observe that the estimator in (2) is WL since both $\tilde{\mathbf{d}}_k$ and its conjugate version $\tilde{\mathbf{d}}_k^*$ are processed jointly. The hard *final* decision $\hat{s}_m$ on the $m$th data symbol $s_m$ is obtained by quantizing $z_m$ to the nearest (in terms of Euclidean distance) symbol, for $m \in \mathcal{J}_c$.

The S-NBI-C receiver of [1] was synthesized under the assumptions that $\boldsymbol{\beta}_k = \mathbf{0}_k$, i.e., *linear* prediction of the NBI is performed, and $\zeta_{i_k} = s_{i_k}$, i.e., *correct* symbol decisions are assumed. Herein, besides considering a WL prediction filter, contrary to [1], we do not assume error-free past decisions; in this case, the design of the prediction filter is more cumbersome and depends on the choice of the decision function $\mathcal{Q}(\cdot)$. On this subject, different options can be pursued. As a first option, hard decisions provided by the decoding unit can be used by setting $\zeta_{i_k} = \hat{s}_{i_k}$, for each step of the aforementioned cancellation procedure. However, as confirmed by our simulations, this choice is not the best one for low values of the SIR: in these regimes, the decisions produced in the early steps of the sequential cancellation procedure (i.e., those provided on the first decoded subcarriers) are less reliable than those in later steps (i.e., decisions produced on the last decoded subcarriers), leading to undesirable error propagation from subcarrier to subcarrier. To mitigate the detrimental effects of wrong decisions, a better option [7] consists of choosing $\mathcal{Q}(\cdot)$ as to minimize the mean-square error MSE $\triangleq \mathrm{E}[|s_{i_k} - \mathcal{Q}(z_{i_k})|^2]$, whose solution is the conditional mean of $s_{i_k}$ given $z_{i_k}$, i.e., $\zeta_{i_k} = \mathcal{Q}(z_{i_k}) = \mathrm{E}[s_{i_k} | z_{i_k}|$, which, for a given sample $\overline{z}_{i_k}$ of $z_{i_k}$, produces the soft estimate $\overline{\zeta}_{i_k} = \mathcal{Q}(\overline{z}_{i_k})$ of $s_{i_k}$.

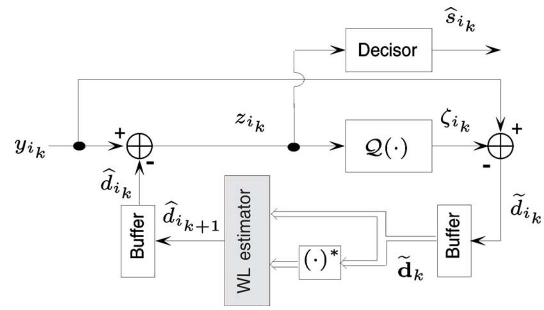

Fig. 1. Block diagram of the proposed S-NBI-C receiver.

### A. WL Prediction Filter With Soft Symbol Decisions

Accounting for (1), one has $\tilde{\mathbf{d}}_k = \mathbf{s}_k - \boldsymbol{\zeta}_k + \mathbf{d}_k$, where $\mathbf{s}_k \triangleq [s_{i_1}, s_{i_2}, \ldots, s_{i_k}]^T \in \mathbb{C}^k, \boldsymbol{\zeta}_k \triangleq [\zeta_{i_1}, \zeta_{i_2}, \ldots, \zeta_{i_k}]^T \in \mathbb{C}^k$, and $\mathbf{d}_k \triangleq [d_{i_1}, d_{i_2}, \ldots, d_{i_k}]^T \in \mathbb{C}^k$. When predicting the disturbance contribution $d_{i_{k+1}}$ at the $(k+1)$th step, the previously produced soft estimates $\overline{\zeta}_{i_\ell} = \mathcal{Q}(\overline{z}_{i_\ell})$, for $\ell \in \{1, 2, \ldots, k\}$, constitute the available prior information about $\boldsymbol{\zeta}_k$. Since, if $z_{i_\ell}$ is observed, then $\zeta_{i_\ell}$ is determined uniquely, the observed value of $\mathbf{z}_k \triangleq [z_{i_1}, z_{i_2}, \ldots, z_{i_k}]^T \in \mathbb{C}^k$ represents the *a priori* information to be used for synthesizing the WL filter. From a Bayesian perspective, let $\overline{\mathbf{z}}_k$ denote a sample of $\mathbf{z}_k$, the vector $\boldsymbol{\gamma}_k$ minimizes the *a posteriori* prediction error power $\mathcal{E}_{i_{k+1}} \triangleq \mathrm{E}[|d_{i_{k+1}} - \hat{d}_{i_{k+1}}|^2 \,|\, \mathbf{z}_k = \overline{\mathbf{z}}_k]$, and is given [11] by

$$\boldsymbol{\gamma}_k^{\rm opt} = [(\boldsymbol{\alpha}_k^{\rm opt})^T, (\boldsymbol{\beta}_k^{\rm opt})^T]^T = \boldsymbol{\Phi}_k^{-1} \mathbf{b}_k \quad (3)$$

where we defined $\boldsymbol{\Phi}_k \triangleq [[\boldsymbol{\Psi}_k, \boldsymbol{\Omega}_k]^T, [\boldsymbol{\Omega}_k, \boldsymbol{\Psi}_k]^H]^T \in \mathbb{C}^{2k\times 2k}$, with $\boldsymbol{\Psi}_k \triangleq \mathrm{E}[\tilde{\mathbf{d}}_k \, \tilde{\mathbf{d}}_k^H \,|\, \mathbf{z}_k = \overline{\mathbf{z}}_k]$ and $\boldsymbol{\Omega}_k \triangleq \mathrm{E}[\tilde{\mathbf{d}}_k \, \tilde{\mathbf{d}}_k^T \,|\, \mathbf{z}_k = \overline{\mathbf{z}}_k]$, and $\mathbf{b}_k \triangleq [\mathbf{p}_k^T, \mathbf{q}_k^H]^T \in \mathbb{C}^{2k}$, with $\mathbf{p}_k \triangleq \mathrm{E}[\tilde{\mathbf{d}}_k \, d_{i_{k+1}}^* \,|\, \mathbf{z}_k = \overline{\mathbf{z}}_k]$ and $\mathbf{q}_k \triangleq \mathrm{E}[\tilde{\mathbf{d}}_k \, d_{i_{k+1}} \,|\, \mathbf{z}_k = \overline{\mathbf{z}}_k]$. Although in principle the conditional expectations $\boldsymbol{\Psi}_k, \boldsymbol{\Omega}_k, \mathbf{p}_k$ and $\mathbf{q}_k$ can be evaluated analytically without any further assumption, the resulting receiver might exhibit a prohibitively large computational complexity. To obtain a closed-form expression for $\boldsymbol{\gamma}_k^{\rm opt}$ with an acceptable tradeoff between performance and complexity, similarly to [8] and [9], we formulate the assumptions reported at the bottom of the next page.[6] Strictly speaking, such assumptions state that the OFDM symbols are mutually and *conditionally* uncorrelated, with real and imaginary parts condition-

[3]Although one could also consider improper OFDM modulations as in [4], [5], we focus only on proper complex-valued OFDM modulation format, such as PSK or QAM, which are recommended in many multicarrier standards.

[4]The conjugate correlation function of some improper modulation formats might exhibit periodically time-varying features (see, e.g., [4]), which however can be deterministically compensated for.

[5]The decoding order influences the overall performance of any successive interference cancellation algorithm. Herein, as in [1], we assume that the ordering set $\{i_1, i_2, \ldots, i_M\}$ is chosen such that the most reliable subcarriers, i.e., those experiencing the largest signal-to-interference-plus-noise ratio (SINR) upon conventional OFDM processing, are decoded first.

[6]Given the random variables $X, Y$, and $Z$, the moments $\mathrm{E}[X\,Y \,|\, Z = z]$ and $\mathrm{E}[X\,Y^* \,|\, Z = z]$ are denoted together as $\mathrm{E}[X\,Y^{[*]} \,|\, Z = z]$, for $z \in \mathbb{C}$.

ally uncorrelated, too, whereas (7) maintains that the conditioning $\{\mathbf{z}_k = \overline{\mathbf{z}}_k\}$ does not influence the second-order statistical properties of the disturbance and the uncorrelatedness between OFDM symbols and disturbance. Even though our design can be generalized to other complex proper modulation formats, for simplicity, we assume that the OFDM symbols assume *equiprobable* values in $\mathscr{S} = \{\pm(1/\sqrt{2}) \pm (j/\sqrt{2})\}$. In such a case, since $|s_{i_\ell}|^2 = 1$, it follows that $\mathrm{E}[|s_{i_\ell}|^2 \,|\, z_{i_\ell} = \overline{z}_{i_\ell}] = 1$, for any $\ell \in \{1, 2, \ldots, k\}$. On the other hand, since $\Re^2\{s_{i_\ell}\} = \Im^2\{s_{i_\ell}\} = 1/2$, one has $\mathrm{E}[\Re^2\{s_{i_\ell}\} \,|\, z_{i_\ell} = \overline{z}_{i_\ell}] = \mathrm{E}[\Im^2\{s_{i_\ell}\} \,|\, z_{i_\ell} = \overline{z}_{i_\ell}] = 1/2$ and, thus, using also (6), one obtains $\mathrm{E}[s_{i_\ell}^2 \,|\, z_{i_\ell} = \overline{z}_{i_\ell}] = 2j\, \mathrm{E}[\Re\{s_{i_\ell}\} \Im\{s_{i_\ell}\} \,|\, z_{i_\ell} = \overline{z}_{i_\ell}] = 2j\, \Re\{\overline{\zeta}_{i_\ell}\} \Im\{\overline{\zeta}_{i_\ell}\}$. By virtue of (7) and observing that, when conditioned on $\{\mathbf{z}_k = \overline{\mathbf{z}}_k\}$, the vector $\boldsymbol{\zeta}_k$ is nonrandom, one has

$$\boldsymbol{\Psi}_k = \boldsymbol{\Lambda}_k + \mathbf{R}_{\mathbf{d}_k \mathbf{d}_k} \quad \text{and} \quad \boldsymbol{\Omega}_k = \boldsymbol{\Sigma}_k + \mathbf{R}_{\mathbf{d}_k \mathbf{d}_k^*} \tag{8}$$

$$\mathbf{p}_k = \mathrm{E}[\mathbf{d}_k\, d_{i_{k+1}}^*] \quad \text{and} \quad \mathbf{q}_k = \mathrm{E}[\mathbf{d}_k\, d_{i_{k+1}}], \tag{9}$$

where, by virtue of (4)–(6) and $\mathrm{E}[s_{i_\ell} \,|\, z_{i_\ell} = \overline{z}_{i_\ell}] = \overline{\zeta}_{i_\ell}$, the entries of $\boldsymbol{\Lambda}_k \triangleq \mathrm{E}[(\mathbf{s}_k - \boldsymbol{\zeta}_k)(\mathbf{s}_k - \boldsymbol{\zeta}_k)^H \,|\, \mathbf{z}_k = \overline{\mathbf{z}}_k]$ and $\boldsymbol{\Sigma}_k \triangleq \mathrm{E}[(\mathbf{s}_k - \boldsymbol{\zeta}_k)(\mathbf{s}_k - \boldsymbol{\zeta}_k)^T \,|\, \mathbf{z}_k = \overline{\mathbf{z}}_k]$ are given by

$$\{\boldsymbol{\Lambda}_k\}_{\ell,m} = (1 - |\overline{\zeta}_{i_\ell}|^2)\, \delta(\ell - m) \tag{10}$$

$$\{\boldsymbol{\Sigma}_k\}_{\ell,m} = (\Im^2\{\overline{\zeta}_{i_\ell}\} - \Re^2\{\overline{\zeta}_{i_\ell}\})\, \delta(\ell - m) \tag{11}$$

for $\ell, m \in \{1, 2, \ldots, k\}$, that is, under our assumptions, both $\boldsymbol{\Lambda}_k$ and $\boldsymbol{\Sigma}_k$ turn out to be diagonal matrices. Observe that, since soft decisions are used, in general $|\overline{\zeta}_{i_\ell}|^2 \neq 1$ and $\Im^2\{\overline{\zeta}_{i_\ell}\} \neq \Re^2\{\overline{\zeta}_{i_\ell}\}$ on those subcarriers hit by the NBI, especially for low-to-moderate values of the SIR.

To compute the soft estimate $\overline{\zeta}_{i_k} = \mathrm{E}[s_{i_k} | z_{i_k} = \overline{z}_{i_k}]$, we can resort to the Bayes' rule, that is

$$\overline{\zeta}_{i_k} = \frac{\sum_{\overline{s}_{i_k} \in \mathfrak{S}} \overline{s}_{i_k}\, f_{z_{i_k} \,|\, s_{i_k}, \mathbf{z}_{k-1}}(\overline{z}_{i_k} \,|\, \overline{s}_{i_k}, \overline{\mathbf{z}}_{k-1})}{\sum_{\overline{s}_{i_k} \in \mathfrak{S}} f_{z_{i_k} \,|\, s_{i_k}, \mathbf{z}_{k-1}}(\overline{z}_{i_k} \,|\, \overline{s}_{i_k}, \overline{\mathbf{z}}_{k-1})} \tag{12}$$

where $f_{z_{i_k} \,|\, s_{i_k}, \mathbf{z}_{k-1}}(\overline{z}_{i_k} \,|\, \overline{s}_{i_k}, \overline{\mathbf{z}}_{k-1})$ is the conditional pdf of $z_{i_k}$ given $\{s_{i_k} = \overline{s}_{i_k}, \mathbf{z}_{k-1} = \overline{\mathbf{z}}_{k-1}\}$,[7] which is well modeled [8], [9] as complex Gaussian [12]. To calculate the second-order conditional moments of $z_{i_k}$, accounting for (1) and (2), we observe that $z_{i_k} = s_{i_k} + d_{i_k} - (\boldsymbol{\gamma}_{k-1}^{\mathrm{opt}})^H\, \tilde{\boldsymbol{d}}_{k-1}^{\mathrm{aug}}$. By invoking (4)–(7), one has $\mathrm{E}[z_{i_k} \,|\, s_{i_k} = \overline{s}_{i_k}, \mathbf{z}_{k-1} = \overline{\mathbf{z}}_{k-1}] = \overline{s}_{i_k}$, $\eta_k \triangleq \mathrm{E}[|z_{i_k} - \overline{s}_{i_k}|^2 \,|\, \mathbf{z}_{k-1} = \overline{\mathbf{z}}_{k-1}] = \mathrm{E}[|d_{i_k}|^2] - \mathbf{b}_{k-1}^H \boldsymbol{\Phi}_{k-1}^{-1}\, \mathbf{b}_{k-1}$, and $\tau_k \triangleq \mathrm{E}[(z_{i_k} - \overline{s}_{i_k})^2 \,|\, \mathbf{z}_{k-1} = \overline{\mathbf{z}}_{k-1}] = \mathrm{E}[d_{i_k}^2] - (\mathbf{c}_{k-1}^H \boldsymbol{\Phi}_{k-1}^{-1}\, \mathbf{b}_{k-1})^*$, where $\mathbf{c}_{k-1} \triangleq [\mathbf{q}_{k-1}^T, \mathbf{p}_{k-1}^H]^T \in \mathbb{C}^{2(k-1)}$. Let $\overline{\mathbf{v}}_{i_k} \triangleq [\overline{z}_{i_k} - \overline{s}_{i_k}, \overline{z}_{i_k}^* - \overline{s}_{i_k}^*]^T \in \mathbb{C}^2$, $\{\mathbf{V}\}_{1,1} = \{\mathbf{V}\}_{2,2} = \eta_k$ and $\{\mathbf{V}\}_{1,2} = \{\mathbf{V}\}_{2,1}^* = \tau_k$, one obtains

$$f_{z_{i_k} \,|\, s_{i_k}, \mathbf{z}_{k-1}}(\overline{z}_{i_k} \,|\, \overline{s}_{i_k}, \overline{\mathbf{z}}_{k-1}) = \frac{e^{-\frac{1}{2}(\overline{\mathbf{v}}_{i_k}^H \mathbf{V}^{-1} \overline{\mathbf{v}}_{i_k})}}{\pi \sqrt{\eta_k^2 - |\tau_k|^2}}. \tag{13}$$

By substituting (13) in (12), $\overline{\zeta}_{i_k}$ can be evaluated explicitly.

### B. Discussion and Concluding Remarks

The proposed receiver has been developed under the general assumptions of soft decisions and improper NBI and, thus, it is referred to as "S-NBI-C (S-I)".[8] Under these assumptions, since $\boldsymbol{\beta}_k^{\mathrm{opt}} \neq \mathbf{0}_k$, the resulting NBI predictor is WL, i.e., according to (2), it jointly processes $\tilde{\mathbf{d}}_k$ and $\tilde{\mathbf{d}}_k^*$. Interestingly enough, the NBI predictor turns out to be WL even when the NBI is proper, i.e., $\mathbf{R}_{\mathcal{J}\mathcal{J}^*} = \mathbf{O}_{M \times M}$, which implies that $\mathbf{R}_{\mathbf{d}_k \mathbf{d}_k^*} = \mathbf{O}_{k \times k}$ in (8) and $\mathbf{q}_k = \mathbf{0}_k$ in (9). Indeed, in such a case, it is readily verified that, due to the presence of the diagonal matrix $\boldsymbol{\Sigma}_k$ in (8), the optimal weight vector $\boldsymbol{\beta}_k^{\mathrm{opt}}$ is still nonzero. This receiver is referred to as "S-NBI-C (S-P)". In other words, the feedback of soft decisions induces a certain degree of impropriety in the data processed by the NBI prediction filter, which can be exploited for improving the NBI suppression capabilities of the receiver. A similar result was obtained by [9] in the context of multiuser detection; this observation indicates that, independently of whether the received signal is proper or improper, soft feedback from the decoding unit motivates WL processing not only for multiple-access interference suppression in code-division multiple-access systems but also for NBI prediction in OFDM networks. It is also interesting to observe that, if hard estimates of the OFDM symbols are used in (10) and (11) instead of the soft ones (12), i.e., $\overline{\zeta}_{i_k} = \hat{s}_{i_k}$, it results that $|\overline{\zeta}_{i_k}|^2 = 1$ and $\Im^2\{\overline{\zeta}_{i_k}\} = \Re^2\{\overline{\zeta}_{i_k}\}$; consequently, both the matrices $\boldsymbol{\Lambda}_k$ and $\boldsymbol{\Sigma}_k$ are zero: the resulting WL receiver, referred to as "S-NBI-C (H-I)", is a straightforward extension of the S-NBI-C method of [1], which additionally exploits the improper nature of the NBI. Finally, if $\overline{\zeta}_{i_k} = \hat{s}_{i_k}$ and the NBI is proper, the proposed NBI prediction filter boils down to the linear receiver proposed in [1], i.e., $\boldsymbol{\alpha}_k^{\mathrm{opt}} = \mathbf{R}_{\mathbf{d}_k \mathbf{d}_k}^{-1}\, \mathbf{p}_k$ and $\boldsymbol{\beta}_k^{\mathrm{opt}} = \mathbf{0}_k$, which is referred to as "S-NBI-C (H-P)".

In Figs. 2 and 3, we report Monte Carlo simulation results aimed at comparing the average bit-error-rate (ABER) performances of the aforementioned four receivers. We considered a system employing either $M = 32$ or $M = 64$ subcarriers, with a CP of length $L_{\mathrm{cp}} = 8$ and sampling period $T_c$, which operates over a fourth-order nonminimum-phase finite impulse response

[7] In our framework, the distribution of $z_{i_k}$ is interpreted as the conditional distribution of $z_{i_k}$ assuming $\{\mathbf{z}_{k-1} = \overline{\mathbf{z}}_{k-1}\}$.

[8] S and H point out that soft and hard OFDM symbol decisions are used, respectively, whereas I and P indicate that the predictor synthesis is carried out by presuming the NBI to be improper and proper, respectively.

$$\mathrm{E}[s_{i_\ell} \,|\, \mathbf{z}_k = \overline{\mathbf{z}}_k] = \mathrm{E}[s_{i_\ell} \,|\, z_{i_\ell} = \overline{z}_{i_\ell}], \quad \text{for } \ell \in \{1, 2, \ldots, k\} \tag{4}$$

$$\mathrm{E}[s_{i_\ell}\, s_{i_m}^{[*]} \,|\, \mathbf{z}_k = \overline{\mathbf{z}}_k] = \begin{cases} \mathrm{E}[s_{i_\ell}\, s_{i_\ell}^{[*]} \,|\, z_{i_\ell} = \overline{z}_{i_\ell}], & \text{if } \ell = m; \\ \mathrm{E}[s_{i_\ell} \,|\, z_{i_\ell} = \overline{z}_{i_\ell}]\mathrm{E}[s_{i_m}^{[*]} \,|\, z_{i_m} = \overline{z}_{i_m}] & \text{if } \ell \neq m. \end{cases}, \quad \text{for } \ell, m \in \{1, 2, \ldots, k\} \tag{5}$$

$$\mathrm{E}[\Re\{s_{i_\ell}\} \Im\{s_{i_\ell}\} \,|\, \mathbf{z}_k = \overline{\mathbf{z}}_k] = \mathrm{E}[\Re\{s_{i_\ell}\} \,|\, z_{i_\ell} = \overline{z}_{i_\ell}]\mathrm{E}[\Im\{s_{i_\ell}\} \,|\, z_{i_\ell} = \overline{z}_{i_\ell}], \quad \text{for } \ell \in \{1, 2, \ldots, k\} \tag{6}$$

$$\mathrm{E}[\mathbf{d} \,|\, \mathbf{z}_k = \overline{\mathbf{z}}_k] = \mathbf{0}_M, \quad \mathrm{E}[\mathbf{d}(\mathbf{s}^{[*]})^T \,|\, \mathbf{z}_k = \overline{\mathbf{z}}_k] = \mathbf{O}_{M \times M}, \mathrm{E}[\mathbf{d}\,(\mathbf{d}^{[*]})^T \,|\, \mathbf{z}_k = \overline{\mathbf{z}}_k] = \mathrm{E}[\mathbf{d}\,(\mathbf{d}^{[*]})^T] \tag{7}$$

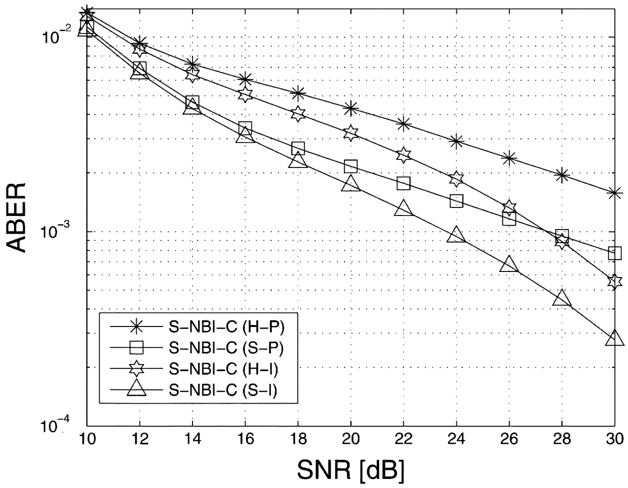


Fig. 2. ABER versus SNR ($M = 32$ and SIR $= 10$ dB).

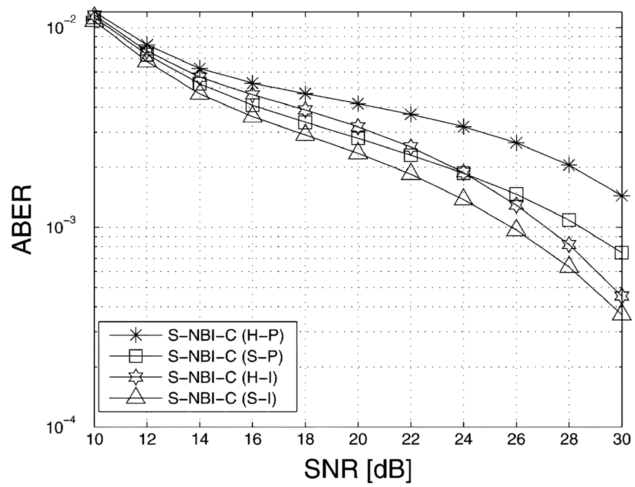


Fig. 3. ABER versus SNR ($M = 64$ and SIR $= 10$ dB).

channel modeled as in [1]. The baseband continuous-time improper NBI is modeled as a digitally modulated OQPSK signal, with symbol period $T_I = T/4$, where $T = (M + L_{\rm cp})\,T_c$ denotes the OFDM symbol period, and carrier frequency-offset $f_{\rm nbi} = 4.5/(M\,T_c)$ (measured with respect to the carrier frequency of the OFDM signal), employing a raised cosine pulse, truncated in the interval $(-5\,T_I, 5\,T_I]$. Observe that, when $M = 32$, the NBI bandwidth $W_{\rm nbi} \approx 0.1\,W_{\rm ofdm}$, where $W_{\rm ofdm}$ denotes the bandwidth of OFDM signal, whereas, for $M = 64$, the NBI bandwidth is halved, i.e., $W_{\rm nbi} \approx 0.05\,W_{\rm ofdm}$. According to (1), the signal-to-noise ratio (SNR) is defined as $\mathrm{E}[||\mathbf{s}(n)||^2]/\mathrm{E}[||\boldsymbol{\mathcal{H}}^{-1}\mathbf{W}_{\rm DFT}\,\mathbf{w}(n)||^2]$, and SIR $\triangleq \mathrm{E}[||\mathbf{s}(n)||^2]/\mathrm{E}[||\boldsymbol{\mathcal{H}}^{-1}\mathbf{W}_{\rm DFT}\,\boldsymbol{\jmath}(n)||^2]$ and set to 10 dB. Results of Fig. 2 ($M = 32$) show that the novel receivers "S-NBI-C (S-I)", "S-NBI-C (S-P)", and "S-NBI-C (H-I)" significantly outperform the "S-NBI-C (H-P)" and enlighten that using soft estimates improves the NBI prediction process, thereby allowing a better NBI suppression. In particular, the "S-NBI-C (S-P)" receiver performs worse than the "S-NBI-C (S-I)" one, because, even though it is WL, it does not take into account the impropriety of the NBI. In its turn, the "S-NBI-C (H-I)" WL predictor, which exploits the improper nature of the NBI but uses hard estimates, pays a penalty with respect to the "S-NBI-C (S-P)" one, for low-to-moderate SNR values. This fact further confirms the importance of employing soft decisions in NBI-contaminated OFDM systems. As regards Fig. 3 ($M = 64$), compared with results of Fig. 2, the performances of both "S-NBI-C (H-P)" and "S-NBI-C (S-P)" receivers do not change significantly, whereas, with respect to the "S-NBI-C (H-I)" one, the performance gain of the "S-NBI-C (S-I)" receiver is slightly reduced.

As final remarks, note that the computational complexity of the proposed S-NBI-C methods is greater than that of [1], which is $\mathcal{O}(k^3)$ flops for the $k$th step, with $k \in \{1, 2, \ldots, M\}$. Indeed, the major complexity burden in implementing $\boldsymbol{\gamma}_k^{\rm opt}$ arises from the matrix inversion $\boldsymbol{\Phi}_k^{-1}$, which requires a computational load $\mathcal{O}[(2k)^3]$, for step $k$, with $k \in \{1, 2, \ldots, M\}$, and each symbol time $n$. However, there exist (see, e.g., [8]) efficient recursive procedures that allow one to significantly reduce such an implementation complexity. The syntheses of all the considered receivers require knowledge of $\mathbf{R}_{\mathbf{dd}}$ and $\mathbf{R}_{\mathbf{dd}^*}$, which, accounting for (1), depend on the channel impulse response and the second-order statistics (SOS) of the noise vector $\mathbf{w}(n)$ and the NBI vector $\boldsymbol{\jmath}(n)$. The channel impulse response can be estimated through conventional channel estimation techniques [13]. The SOS of $\mathbf{w}(n)$ and $\boldsymbol{\jmath}(n)$ can be directly estimated from the received data [1], [14], [15] or can approximately built starting from their synthetic SOS parameters [3], such as $\sigma_w^2$, the NBI power, $W_{\rm nbi}$, and $f_{\rm nbi}$.

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