Two-stage interference-resistant adaptive periodically time-varying CMA blind equalization

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Abstract—**In this paper, we consider the problem of blindly equalizing a digital communication signal distorted by a linear time-invariant channel, and contaminated by severe co-channel or adjacent-channel digital interference, under the assumption that the latter exhibits a different symbol rate from the desired signal. The proposed equalizer is composed of two stages, both periodically time-varying (PTV), in order to better match the periodical statistics of the received signal. The first stage employs linear PTV filtering to mitigate interference, allowing thus the second stage, based on the constant modulus algorithm (CMA), to reliably recover the transmitted information symbols. Computer simulations confirm the effectiveness of the new approach, and comparisons with existing blind methods show that a significant performance gain can be attained.**

Keywords— **Blind equalization, time-varying filters, cyclostationary signals, constrained optimization, constant-modulus algorithm.**

I. Introduction

IN high-speed digital communication systems, the transmitted signal is subjected to time dispersion due to non-ideal transmission media, which gives rise to *intersymbol interference* (ISI), and is affected by thermal noise introduced at the receiver input. *Channel equalization* is then aimed at counteracting the harmful effects of ISI and noise, in order to reliably recover the transmitted symbol sequence. Most equalization techniques belong to the class of *non-blind* or *trained* algorithms, since they make use of training sequences in order to solve for the equalizer parameters, usually under a zero-forcing, minimum mean-square error (MMSE), or maximum-likelihood criterion. To overcome the waste of resources associated to transmission of training symbols, the *blind* equalization approach has been receiving a great deal of attention in the last years [1], [2], [3]. Usually, blind techniques perform channel equalization by exploiting some known property of the desired signal (e.g., second-order or higher-order statistics, cyclostationarity, constant modulus, or finite alphabet). A major breakthrough in blind channel equalization was the pioneering work of Tong *et al.* [3], where it was recognized that possibly nonminimum-phase channels can be blindly identified using only the second-order statistics (SOS) of the received signal, by exploiting *temporal* diversity (induced by fractionally-spaced sampling) and/or *spatial* diversity (associated to the use of multiple sensors). In both cases, the problem can be described by a single-input/multiple-output (SIMO) or multi-channel model [1], [2], [3], whose outputs are corrupted by wide-sense stationary (WSS) thermal noise; equalization is then performed by a multiple-input/single-output (MISO) equalizer, composed by a bank of linear time-invariant (LTI) filters.

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However, in both wired and wireless systems, one might be faced with the more challenging problem of counteracting not only the ubiquitous presence of ISI, but also the deleterious effects of *co-channel interference* (CCI) and *adjacent-channel interference* (ACI). Recently, the problem of channel equalization in the presence of CCI and/or ACI has been considered in [4], [5], where it is assumed that the desired and the interfering signals exhibit the same symbol rate. Thus, since the disturbance is WSS and the received signal is sampled at the symbol rate, the equalizers proposed in [4], [5] turn out to be LTI as well. However, a more general situation arises when the desired and interference signals exhibit different symbol rates. This happens, for example, in wired networks, due to crosstalk between adjacent pairs transmitting at different rates; or in wireless overlay networks, where the same geographical area is covered by different systems sharing the same spectrum band; or finally in wireless multi-rate networks, where different services with different rates need to be accommodated efficiently in an unified bandwidth-on-demand manner. If the received signal is fractionally-sampled with respect to the symbol period of the desired signal, or multiple sensors are employed at the receiver and their outputs are sampled at the symbol rate, the problem can be conveniently described by a linear multiple-input/multiple-output system model, which, however, due to the different symbol rates of the desired and interfering signals, turns out to be linear periodically time-varying (LPTV) or linear almost periodically time-varying (LAPTV). An equivalent LTI SIMO model can be obtained regarding as input only the desired symbol sequence, but now the additive disturbances affecting the outputs of the SIMO system turn out to be the sum of WSS thermal noise plus cyclostationary or almost cyclostationary interference. In both cases, optimal equalization and interference suppression is achieved by a MISO *time-varying* equalizer, composed by a bank of LPTV or LAPTV filters [6]. Preliminary work on this topic has been carried out in [7], where a non-blind PTV MMSE equalization technique has been proposed. In particular, in [7] it has been shown that, by exploiting the time-varying properties of the additive disturbances at the outputs of the SIMO channel, it is possible to derive the equalizer structure without explicit knowledge or estimation of the interfering channel. A blind PTV approach has also been proposed in [8], where estimation of the desired-signal channel is based on the different circularity and/or cyclostationarity properties of the desired and interfering signals. Simulation results in [7], [8] confirm that the PTV equalizers achieve a significant performance gain over their LTI counterparts.

In this paper, we propose a new approach to blind PTV equalization, based on the well known *constant modulus algorithm* (CMA) [9], which can cope with the presence of high-level CCI or ACI. A major drawback of CMA, in environments containing strong interference signals, is the *capture effect*, i.e., extraction of the interference rather than the desired signal. An accurate analytical study of the capture effect has been carried out by Treichler *et al.* in [10], under the assumption that both the desired and the interfering signals are pure sinusoids. In particular, the authors in [10] determine the regions of signal-to-interference (SIR) values and filter initial conditions leading to extraction of the desired signal or capture of the interference. However, in order to avoid the capture effect, *a priori* knowledge of the input amplitudes and frequencies of the sinusoids are required; moreover, more sophisticated techniques are needed to accommodate dynamic environments, where interferers may suddenly appear or disappear. To overcome the previous drawback, under the assumption that the interference symbol period is known and different from the desired symbol period, we propose in this paper to employ a two-stage equalizer, where a linear *pre-filtering* stage performs preliminary interference mitigation, so as to alleviate the capture problem of the CMA employed in the second stage. Such a pre-filtering stage is designed blindly, by resorting to a *constrained optimization* procedure, which is based on a linear parameterization of the unknown signal channel response. The use of constrained optimization procedures in blind equalization was originally proposed by Tsatsanis *et al.* in [11], where a similar parameterization of the channel response was introduced. However, our formulation of the optimization problem is completely different from [11], leading to a computationally-efficient solution which, unlike [11], does not require burdensome eigendecompositions. Moreover, our parameterization is more general than that considered in [11], which makes our method robust to possible mismatches in channel-length knowledge. Another key aspect of the proposed solution is that, since the disturbances at the outputs of the SIMO channel present PTV statistical properties, both stages of the proposed equalizer are implemented as PTV filters. Finally, since the CMA can be efficiently implemented by resorting to the stochastic gradient algorithm, we develop a recursive implementation of the pre-filtering stage, so that the two-stage structure can operate adaptively with a reasonable computational burden.

The paper is organized as follows. In Section II, the system model is introduced and a mathematical formulation of the problem is provided. Section III presents the proposed two-stage PTV-CMA equalizer, together with an adaptive implementation of the pre-filtering stage. Results of Montecarlo computer simulations are provided in Section IV and, finally, conclusions are drawn in Section V.

II. THE MATHEMATICAL MODEL

Let us consider a digital communication system, wherein the complex envelope of the received signal, at the input of the A/D converter, can be expressed as

$$r_a(t) = u_a(t) + i_a(t) + w_a(t) \, , \tag{1}$$

where $u_a(t)$, $i_a(t)$, and $w_a(t)$ denote the desired signal, the CCI or ACI, and thermal noise, respectively. If we assume that both the desired signal and the interference employ linear modulation formats, with (possibly) different symbol periods T_U and T_I, and that both the signal and interference channels can be modeled as LTI systems, $u_a(t)$ and $i_a(t)$ can be expressed as:

$$u_a(t) = \sum_{q=-\infty}^{\infty} s(q)\, c_a(t - q\, T_U) \, , \tag{2}$$

$$i_a(t) = \left[\sum_{q=-\infty}^{\infty} s_I(q)\, c_{I,a}(t - q\, T_I) \right] e^{j 2\pi f_I t} \, , \tag{3}$$

where $s(q)$ and $s_I(q)$ are the complex information sequences, $c_a(t)$ and $c_{I,a}(t)$ denote the overall impulse responses (including transmitting filters, channels, and receiving filter) of the signal and interference channels, and, finally, f_I is the frequency offset of the interference, measured with respect to the carrier frequency of the desired signal. The following customary assumptions will be considered throughout the paper: A1) $s(q)$ and $s_I(q)$ are mutually independent zero-mean and independent identically-distributed (iid) sequences, with variance σ_s^2 and σ_I^2, respectively; A2) $w_a(t)$ is a WSS zero-mean complex process, which is independent from $s(q)$ and $s_I(q)$. Under assumption A1, the continuous-time signals $u_a(t)$ and $i_a(t)$ turn out to be *second-order cyclostationary* [12], with period T_U and T_I, respectively.

We will explicitly consider only the case of temporal diversity, even though most of the results can be applied with minor modifications to the case of multiple sensors. Consequently, assume that the received signal (1) is fractionally sampled at rate N/T_U, with $N > 1$ denoting the *oversampling* factor, obtaining hence the discrete-time signal

$$r(n) = u(n) + i(n) + w(n) \, , \tag{4}$$

where $i(n) \triangleq i_a(nT_U/N)$, $w(n) \triangleq w_a(nT_U/N)$, and, finally,

$$u(n) \triangleq u_a(nT_U/N) = \sum_{q=-\infty}^{\infty} s(q)\, c(n - q\, N) \, , \tag{5}$$

with $c(n) \triangleq c_a(nT_U/N)$. It can be seen that the discrete-time signal $u(n)$ is second-order cyclostationary, that is, its statistical autocorrelation function $R_u(n, m) \triangleq E[u(n)\, u^*(n - m)]$ is periodic in n with period N, where $E[\cdot]$ denotes statistical averaging and the superscript $*$ denotes conjugation. A customary representation for such a signal is in terms of its *polyphase decomposition* [13] with respect to N:

$$u^{(\ell)}(k) \triangleq u(kN + \ell) = \sum_{q=-\infty}^{\infty} s(q)\, c^{(\ell)}(k - q) \, , \tag{6}$$

for $\ell = 0, 1, \ldots, N-1$, where $c^{(\ell)}(k) \triangleq c(kN + \ell)$. Indeed, the polyphase components (6) turn out to be jointly WSS in this case. In the sequel, we will refer to the processes $u^{(\ell)}(k)$ as the *phases* of $u(n)$. If we assume that $c_a(t) = 0$ for $t \notin [\, 0, L_c T_U \,)$, it follows that the discrete-time channels $c^{(\ell)}(k)$, $\ell = 0, 1, \ldots, N-1$, are finite impulse-response filters of order less than or equal to L_c, i.e., $c^{(\ell)}(k) = 0$ for $k \notin \{\, 0, 1, \ldots, L_c - 1 \,\}$.

In order to conveniently exploit the temporal diversity at the receiver, consider the polyphase decomposition of $r(n)$ with respect to N:

$$r^{(\ell)}(k) \triangleq r(kN+\ell) = u^{(\ell)}(k) + i^{(\ell)}(k) + w^{(\ell)}(k) \,, \tag{7}$$

for $\ell = 0, 1, \ldots, N-1$, where $i^{(\ell)}(k) \triangleq i(kN+\ell)$ and $w^{(\ell)}(k) \triangleq w(kN+\ell)$. A more compact SIMO model can be obtained by collecting the N different phases of $r(n)$ in the N-column vector $\boldsymbol{r}(k) \triangleq [\, r^{(0)}(k), r^{(1)}(k), \ldots, r^{(N-1)}(k)\,]^T$, with T denoting transpose, obtaining thus:

$$\boldsymbol{r}(k) = \sum_{q=0}^{L_c-1} \boldsymbol{c}(q)\, s(k-q) + \boldsymbol{i}(k) + \boldsymbol{w}(k) \,, \tag{8}$$

where

$$\begin{aligned} \boldsymbol{c}(k) &\triangleq [\, c^{(0)}(k),\, c^{(1)}(k),\, \ldots,\, c^{(N-1)}(k)\,]^T \,, \\ \boldsymbol{i}(k) &\triangleq [\, i^{(0)}(k),\, i^{(1)}(k),\, \ldots,\, i^{(N-1)}(k)\,]^T \,, \\ \boldsymbol{w}(k) &\triangleq [\, w^{(0)}(k),\, w^{(1)}(k),\, \ldots,\, w^{(N-1)}(k)\,]^T \,. \end{aligned} \tag{9}$$

In the following, we will extensively exploit the time-varying properties (in k) of the statistical correlation matrix $\boldsymbol{R}_{rr}(k,m) \triangleq E[\,\boldsymbol{r}(k)\,\boldsymbol{r}^H(k-m)\,]$, where H denotes conjugate transpose. Having already observed that the phases of $u(n)$ are jointly WSS, the desired signal part of the *vectorized* process $\boldsymbol{r}(k)$ [i.e., the first term of (8)] turns out to be WSS. Moreover, assumption A2 implies that the vector process $\boldsymbol{w}(k)$ is WSS in its turn. Thus, the only possibly non-WSS part in $\boldsymbol{r}(k)$ is indeed $\boldsymbol{i}(k)$, hence the time-varying behavior with k of $\boldsymbol{R}_{rr}(k,m)$ coincide with that of $\boldsymbol{R}_{ii}(k,m) \triangleq E[\,\boldsymbol{i}(k)\,\boldsymbol{i}^H(k-m)\,]$. Observing that the elements of $\boldsymbol{i}(k)$ are the polyphase components $i^{(\ell)}(k)$ for $\ell = 0, 1, \ldots, N-1$, it is shown in [7] that the statistical autocorrelation functions of such polyphase components are possibly time-varying in k, according to the ratio T_U/T_I: when T_U/T_I is integer, the N phases $i^{(\ell)}(k)$ are jointly WSS, and $\boldsymbol{R}_{ii}(k,m)$ does not depend on k; when $T_U/T_I = p/P$ is a rational number (with p and P co-prime integers), the N phases $i^{(\ell)}(k)$ are jointly cyclostationary with period P, and $\boldsymbol{R}_{ii}(k,m)$ is periodic in k with period P; finally, when T_U/T_I is not a rational number, the N phases $i^{(\ell)}(k)$ are jointly *almost* cyclostationary [12], and $\boldsymbol{R}_{ii}(k,m)$ is *almost* periodic [14] in k.

III. THE TWO-STAGE PTV-CMA BLIND EQUALIZER

Our aim is to design a linear equalizer, which is able to extract the desired symbol $s(k-d)$, with d denoting a suitable delay, by simultaneously minimizing the effects of ISI, interference, and noise. Denoting with L_e the equalizer length (expressed in symbol intervals), the equalizer can be compactly described as follows:

$$\hat{s}(k-d) = \boldsymbol{b}^H(k)\,\boldsymbol{z}(k) \,, \tag{10}$$

where $\boldsymbol{b}(k)$ and $\boldsymbol{z}(k)$ are (NL_e)-column vectors, with $\boldsymbol{z}(k) \triangleq [\,\boldsymbol{r}^T(k), \boldsymbol{r}^T(k-1), \ldots, \boldsymbol{r}^T(k-L_e+1)\,]^T$. We are assuming explicitly that $\boldsymbol{b}(k)$ is time-varying, since, due to the presence of the interference, the received vector $\boldsymbol{r}(k)$ might exhibit (periodically or almost periodically) time-varying statistics (see Section II). To determine the vector $\boldsymbol{b}(k)$ in a blind manner, we propose here to adopt the CMA; however, since the CMA is prone to the interference capture problem, we resort here to a two-stage blind equalizer (Fig. 1), where the first stage performs preliminary interference mitigation, allowing thus the CMA in the second stage to reliably recover the desired symbol sequence. In the sequel, we will focus our attention to the case where T_U/T_I is a rational number, and therefore the input of the equalizer exhibits PTV second-order statistics with (known) period P, which, in turn, implies that the two stages turn out to be PTV of the same period. It is worthwhile to observe that the case of not rational T_U/T_I can be tackled as well by formulating the problem in the frequency domain [6].

A. Design of the first stage

The first stage performs interference mitigation by linear transformation of the input vector $\boldsymbol{z}(k)$:

$$\boldsymbol{x}(k) = \boldsymbol{T}^H(k)\,\boldsymbol{z}(k) \,, \tag{11}$$

where $\boldsymbol{T}(k)$ is a suitable (time-varying) filtering matrix. To derive a criterion for blindly determining $\boldsymbol{T}(k)$, let us rewrite $\boldsymbol{z}(k)$, accounting for (8), as

$$\boldsymbol{z}(k) = \boldsymbol{C}\,\boldsymbol{s}(k) + \boldsymbol{j}(k) + \boldsymbol{v}(k) \,, \tag{12}$$

where

$$\boldsymbol{C} \triangleq \begin{pmatrix} \boldsymbol{c}(0) & \ldots & \boldsymbol{c}(L_c-1) & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{c}(0) & \ldots & \boldsymbol{c}(L_c-1) & \boldsymbol{0} \\ \vdots & & & & \vdots \\ \vdots & & & & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{c}(0) & \ldots & \boldsymbol{c}(L_c-1) \end{pmatrix} \tag{13}$$

is the $(NL_e) \times (L_c + L_e - 1)$ signal channel block Toeplitz matrix, $\boldsymbol{j}(k) \triangleq [\,\boldsymbol{i}^T(k),\, \boldsymbol{i}^T(k-1),\, \ldots,\, \boldsymbol{i}^T(k-L_e+1)\,]^T$, $\boldsymbol{v}(k) \triangleq [\,\boldsymbol{w}^T(k),\, \boldsymbol{w}^T(k-1),\, \ldots,\, \boldsymbol{w}^T(k-L_e+1)\,]^T$, and $\boldsymbol{s}(k) \triangleq [\, s(k),\, s(k-1),\, \ldots,\, s(k-L_e-L_c+2)\,]^T$. Denoting with $\boldsymbol{s}_{\text{ISI}}(k)$ the vector including all elements in $\boldsymbol{s}(k)$ except for the $(d+1)$th element $s(k-d)$, and with $\boldsymbol{C}_{\text{ISI}}$ the matrix including all the columns in $\boldsymbol{C}$ except for the $(d+1)$th column $\boldsymbol{c}_d$, equation (12) can be rewritten as

$$\boldsymbol{z}(k) = \boldsymbol{c}_d\, s(k-d) + \boldsymbol{C}_{\text{ISI}}\, \boldsymbol{s}_{\text{ISI}}(k) + \boldsymbol{j}(k) + \boldsymbol{v}(k) \,, \tag{14}$$

for any $d \in \{0, 1, \ldots, L_e + L_c - 2\}$. Finally, by defining the overall disturbance (i.e., ISI, interference, and noise) vector $\boldsymbol{d}(k) \triangleq \boldsymbol{C}_{\text{ISI}}\, \boldsymbol{s}_{\text{ISI}}(k) + \boldsymbol{j}(k) + \boldsymbol{v}(k)$, we obtain the simple model of the equalizer input

$$\boldsymbol{z}(k) = \boldsymbol{c}_d\, s(k-d) + \boldsymbol{d}(k) \,. \tag{15}$$

At this point, in order to solve for $\boldsymbol{T}(k)$ in a blind manner, we exploit the result that the vector $\boldsymbol{c}_d$ can be linearly parameterized as

$$\boldsymbol{c}_d = \boldsymbol{Q}_d\, \widetilde{\boldsymbol{c}}_d \,, \tag{16}$$

where $\boldsymbol{Q}_d$ is a $(NL_e) \times (NL_d)$ *known* full column rank parameterization matrix satisfying $\boldsymbol{Q}_d^H \boldsymbol{Q}_d = \boldsymbol{I}$, with $\boldsymbol{I}$ denoting the identity matrix,[1] $\widetilde{\boldsymbol{c}}_d$ is an *unknown* vector collecting some of the samples of the discrete-time signal channel $c(n)$, and finally L_d depends on the particular parameterization used (see next subsection for detailed expressions of $\boldsymbol{Q}_d$ and $\widetilde{\boldsymbol{c}}_d$). Substitution of (15) and (16) in (11) yields now:

$$\boldsymbol{x}(k) = \boldsymbol{T}^H(k)\,\boldsymbol{Q}_d\,\widetilde{\boldsymbol{c}}_d\,s(k-d) + \boldsymbol{T}^H(k)\,\boldsymbol{d}(k)\,, \qquad (17)$$

from which it results that, by constraining $\boldsymbol{T}(k)$ to satisfy $\boldsymbol{T}^H(k)\,\boldsymbol{Q}_d = \boldsymbol{I}$, the contribution of the desired symbol $s(k-d)$ is passed to the output of the first stage with response $\widetilde{\boldsymbol{c}}_d$. Under the previous constraint, $\boldsymbol{T}(k)$ can be chosen so as to minimize the output power of the first stage, i.e., as the solution of the following constrained optimization problem:

$$\min_{\boldsymbol{T}(k)} E[\|\boldsymbol{x}(k)\|^2] \quad \text{subject to} \quad \boldsymbol{T}^H(k)\,\boldsymbol{Q}_d = \boldsymbol{I}\,, \qquad (18)$$

with $\|\cdot\|$ denoting the Euclidean norm, which assures that the disturbance power is minimized, without cancellation of the desired signal. The solution of (18), derived in Appendix A, is given by

$$\boldsymbol{T}(k) = \boldsymbol{R}_{zz}^{-1}(k)\,\boldsymbol{Q}_d\,(\,\boldsymbol{Q}_d^H\,\boldsymbol{R}_{zz}^{-1}(k)\,\boldsymbol{Q}_d\,)^{-1}\,, \qquad (19)$$

where $\boldsymbol{R}_{zz}(k) \triangleq E[\,\boldsymbol{z}(k)\,\boldsymbol{z}^H(k)\,]$ is the correlation matrix of $\boldsymbol{z}(k)$. The corresponding minimum value of output power, obtained by substituting (19) in (18), is given by:

$$\mathcal{P}_{\min}(k) = \text{trace}[(\,\boldsymbol{Q}_d^H\,\boldsymbol{R}_{zz}^{-1}(k)\,\boldsymbol{Q}_d\,)^{-1}]\,, \qquad (20)$$

with $\text{trace}(\cdot)$ denoting the trace operator. Observe that both the optimal filtering matrix (19) and $\mathcal{P}_{\min}(k)$ depend[2] on k since, due to the presence of the interference $\boldsymbol{j}(k)$ in $\boldsymbol{z}(k)$, $\boldsymbol{R}_{zz}(k)$ exhibits the same time-varying behavior with k as $\boldsymbol{R}_{ii}(k,m)$. Therefore, under the assumption that T_U/T_I is rational, the prefiltering stage can be implemented as in [7], [8] by the time-sequenced approach. More precisely, let us represent the cyclostationary vectorized process $\boldsymbol{z}(k)$ in terms of its polyphase decomposition with respect to P, i.e., $\boldsymbol{z}^{(h)}(i) \triangleq \boldsymbol{z}(iP+h)$ for $h = 0, 1, \ldots, P-1$. Then, the PTV transformation $\boldsymbol{T}(k)$ is implemented (see Fig. 1) by using P time-invariant matrices $\boldsymbol{T}^{(h)}$, each filtering a different phase of the input vector $\boldsymbol{z}(k)$:

$$\boldsymbol{T}^{(h)} = (\boldsymbol{R}_{zz}^{(h)})^{-1}\,\boldsymbol{Q}_d\,[\,\boldsymbol{Q}_d^H\,(\boldsymbol{R}_{zz}^{(h)})^{-1}\,\boldsymbol{Q}_d\,]^{-1}\,, \qquad (21)$$

for $h = 0, 1, \ldots, P-1$, where $\boldsymbol{R}_{zz}^{(h)} \triangleq E[\,\boldsymbol{z}^{(h)}(i)\,\boldsymbol{z}^{(h)}(i)^H\,]$ is the correlation matrix of the hth phase of $\boldsymbol{z}(k)$. As a result, if we consider the polyphase decomposition of the output $\boldsymbol{x}(k)$ with respect to P, its hth phase is given by $\boldsymbol{x}^{(h)}(i) = [\boldsymbol{T}^{(h)}]^H\,\boldsymbol{z}^{(h)}(i)$.

[1] For the sake of notation simplicity, we have chosen here and in the following not to indicate the dimensions of the zero/identity matrix, which can be easily deduced from the context.

[2] Both quantities depend on the value of d as well, hence one could in principle find the optimum delay by minimizing $\mathcal{P}_{\min}(k)$ with respect to d. Since, however, this further minimization does not admit a closed-form solution, in the following we will consider only a single, fixed value of d.

Note that (21) is amenable to a straightforward implementation, where the matrices $\boldsymbol{R}_{zz}^{(h)}$ are estimated on the basis of the available data and the estimates substituted in (21). However, this procedure has the disadvantage of requiring two matrix inversions at each step, and therefore is not suited to real-time operation. Therefore, we propose a recursive algorithm (summarized in Fig. 2 and derived in detail in Appendix B) for updating $\boldsymbol{T}^{(h)}(i)$, which denotes the estimate, at iteration i, of $\boldsymbol{T}^{(h)}$ given by (21). The algorithm, similar to the well-known RLS algorithm [15], is based on the matrix inversion lemma for updating, at iteration i, the inverse $\boldsymbol{P}^{(h)}(i)$ of the correlation matrix $\boldsymbol{R}_{zz}^{(h)}$.

B. Choice of the parameterization

The parameterization of $\boldsymbol{c}_d$ given by (16) was originally proposed in [11]. However, the approach of [11] can be applied only when $L_c \le L_e$ and does not globally characterize the columns $\boldsymbol{c}_d$ of the signal channel matrix $\boldsymbol{C}$ over the entire range of the delays $d \in \{0, 1, \ldots, L_e + L_c - 2\}$. To overcome this drawback, we propose here a more general expression for $\boldsymbol{Q}_d$ and $\widetilde{\boldsymbol{c}}_d$ to be used in (16), which, depending on the relative values of d, L_e, and L_c, is given by one of the following cases:

Case C_1 ($0 \le d < \min[L_e, L_c] - 1$): the vector $\boldsymbol{c}_d$ is given by

$$\boldsymbol{c}_d = [\,\underbrace{\boldsymbol{c}^T(d), \ldots, \boldsymbol{c}^T(0)}_{N(d+1)},\ \underbrace{0, \ldots, 0}_{N(L_e-d-1)}\,]^T\,, \qquad (22)$$

hence parameterization (16) is given by

$$\boldsymbol{Q}_d = \begin{pmatrix} \boldsymbol{I}_{N(d+1)\times N(d+1)} \\ \boldsymbol{O}_{N(L_e-d-1)\times N(d+1)} \end{pmatrix} \quad \text{and} \quad \widetilde{\boldsymbol{c}}_\mathrm{d} = \begin{pmatrix} \boldsymbol{c}(d) \\ \vdots \\ \boldsymbol{c}(0) \end{pmatrix}, \qquad (23)$$

with $\boldsymbol{O}$ denoting the zero matrix.

Case C_2 ($\min[L_e, L_c] - 1 \le d \le \max[L_e, L_c] - 1$): there is no parameterization matrix of full column rank when $L_c > L_e$, whereas when $L_c \le L_e$, as observed in [11], the vector $\boldsymbol{c}_d$ is given by

$$\boldsymbol{c}_d = [\,\underbrace{0, \ldots, 0}_{N(d-L_c+1)},\ \underbrace{\boldsymbol{c}^T(L_c-1), \ldots, \boldsymbol{c}^T(0)}_{NL_c},\ \underbrace{0, \ldots, 0}_{N(L_e-d-1)}\,]^T\,, \qquad (24)$$

hence parameterization (16) is given by

$$\boldsymbol{Q}_d = \begin{pmatrix} \boldsymbol{O}_{N(d-L_c+1)\times NL_c} \\ \boldsymbol{I}_{NL_c\times NL_c} \\ \boldsymbol{O}_{N(L_e-d-1)\times NL_c} \end{pmatrix} \quad \text{and} \quad \widetilde{\boldsymbol{c}}_\mathrm{d} = \begin{pmatrix} \boldsymbol{c}(L_c-1) \\ \vdots \\ \boldsymbol{c}(0) \end{pmatrix}. \qquad (25)$$

Case C_3 ($\max[L_e, L_c] - 1 < d \le L_e + L_c - 2$): the vector $\boldsymbol{c}_d$ is given by

$$\boldsymbol{c}_d = [\,\underbrace{0, \ldots, 0}_{N(d-L_c+1)}\ \underbrace{\boldsymbol{c}^T(L_c-1), \ldots, \boldsymbol{c}^T(d-L_e+1)}_{N(L_c+L_e-d-1)}\,]^T\,, \qquad (26)$$

hence parameterization (16) is given by

$$\boldsymbol{Q}_d = \begin{pmatrix} \boldsymbol{O}_{N(d-L_c+1)\times N(L_c+Le-d-1)} \\ \boldsymbol{I}_{N(L_c+L_e-d-1)\times N(L_c+Le-d-1)} \end{pmatrix} \qquad (27)$$

and

$$\widetilde{\boldsymbol{c}}_d = \begin{pmatrix} \boldsymbol{c}(L_c-1) \\ \vdots \\ \boldsymbol{c}(d-L_e+1) \end{pmatrix} . \qquad (28)$$

The choice of which parameterization to use deserves, at this point, some comments, mainly with regard to its sensitivity to possible inaccuracies in signal channel-length knowledge. It is worthwhile to note that, in cases C_2 and C_3, matrix $\boldsymbol{Q}_d$ depends explicitly on the channel length L_c, whereas, in case C_1, its structure is independent of L_c. This implies that, similarly to many other parametric algorithms [16] for blind equalization, exact knowledge of the channel length L_c is a crucial issue when parameterizations C_2 and C_3 are used. In fact, let $\widehat{L}_c \neq L_c$ denote the estimated channel length, $\widehat{\boldsymbol{Q}}_d$ the corresponding parameterization matrix, and $\widehat{\boldsymbol{T}}(k)$ the filtering matrix built from $\widehat{\boldsymbol{Q}}_d$; thus, channel-length underestimation (i.e., $\widehat{L}_c < L_c$) can cause partial cancellation of the desired symbol, since the filtering matrix $\widehat{\boldsymbol{T}}(k)$ satisfies the constraint $\widehat{\boldsymbol{T}}^H(k)\,\widehat{\boldsymbol{Q}}_d = \boldsymbol{I}$ instead of $\widehat{\boldsymbol{T}}^H(k)\,\boldsymbol{Q}_d = \boldsymbol{I}$. On the other hand, similarly to other SOS-based equalization techniques [2], the pre-filtering stage of the proposed method is robust with respect to channel-length overestimation (i.e., $\widehat{L}_c > L_c$): in this case, indeed, the estimated matrix $\widehat{\boldsymbol{T}}(k)$ does not cancel the desired symbol, even though its disturbance suppression capability decreases for increasing value of the channel-length mismatch. Interestingly, when parameterizations C_2 and C_3 fail to adequately preserve the desired symbol while simultaneously minimizing the contribution of the disturbance, one can advantageously use parameterization C_1, since: (i) when $L_c \leq L_e$, it requires only an approximate knowledge of the channel length in order to select a suitable delay d such that $d < L_c - 1$; (ii) when $L_c > L_e$, and in particular when the channel is very long, the delay d and the equalizer length L_e can be fixed (provided that $d < L_e - 1$) in order to reduce the complexity of the pre-filtering stage, since the dimensions of $\boldsymbol{Q}_d$ are independent of L_c.

C. Design of the second stage

After the first stage, unless very severe interference is present, the desired signal component will exceed the interfering one, i.e., a significant signal-to-interference-plus-noise ratio (SINR) improvement will be achieved. Therefore, the CMA employed in the second stage is expected to converge to the extraction of the desired signal, rather than to the extraction of the interfering one. It is worthwhile to note that, since the first stage does not operate perfect interference cancellation, there might still exist a significant cyclostationary interference residual in $\boldsymbol{x}(k)$. Thus, to better track the time-varying statistics of such a residual, we propose to use P different time-sequenced CMAs (see Fig. 1), each operating on a different (stationary) phase $\boldsymbol{x}^{(h)}(i)$ of the output of the first stage. The hth CMA utilizes a weight vector $\boldsymbol{u}^{(h)}$ minimizing the cost function

$$J_{\text{CMA}}(\boldsymbol{u}^{(h)}) \triangleq E[(\gamma - |\boldsymbol{u}^{(h)H}\,\boldsymbol{x}^{(h)}(i)|^2)^2] , \qquad (29)$$

where $\gamma \triangleq \frac{E\{|s(k)|^4\}}{\sigma_s^2}$ is the dispersion coefficient of the desired sequence $s(k)$. Solution of (29) is obtained by resorting to the stochastic gradient method, which yields:

$$\boldsymbol{u}^{(h)}(i+1) = \boldsymbol{u}^{(h)}(i) + \mu^{(h)} y^{(h)}(i)^* \,(\gamma - |y^{(h)}(i)|^2)\,\boldsymbol{x}^{(h)}(i) , \qquad (30)$$

where $\boldsymbol{u}^{(h)}(i)$ is the updated estimate of $\boldsymbol{u}^{(h)}$ at the ith iteration, $\mu^{(h)}$ is the step-size, and $y^{(h)}(i) \triangleq \boldsymbol{u}^{(h)}(i)^H\,\boldsymbol{x}^{(h)}(i)$ denotes the output of the hth CMA equalizer at iteration i. Note that the time-sequenced approach consists in selecting, at each time i, the CMA output $y^{(h)}(i)$ such that $h = (i \mod P)$.

A final remark about computational complexity and convergence speed of the overall two-stage algorithm is in order. As to computational complexity, it should be observed that, at each iteration i, only one phase of the filter must be updated and one output $y^{(h)}(i)$ evaluated, corresponding to the hth branch out of P in Fig. 1. This means that the amount of computations per output symbol involved is essentially the same as for a single time-invariant filter of the same length, whereas the amount of memory required is P-fold increased, due to the need to store P different versions of the weight vectors and matrices utilized in the algorithm. In regard to convergence speed, since each phase is adapted independently to track the time-varying statistics of the interference, the equalizer takes P times as long to converge, in comparison with a time-invariant filter of the same length, operating however in a stationary environment. It should be noted that a convergence-speed improvement can be achieved by resorting to a frequency-domain implementation [6] of the proposed equalizer.

IV. Simulation results

In this section, we present the results of Montecarlo computer simulations, carried out over 100 independent trials, aimed at assessing the performance of the proposed method and providing a comparison with other blind and non-blind equalization techniques.

In all the experiments, unless otherwise specified, the following simulation setting is assumed. The desired signal is BPSK, which corresponds to a dispersion coefficient $\gamma = 1$ to be used in the CMA cost function (29), while the interference is QPSK. The ratio T_U/T_I is 5/3, which implies that the period of the considered PTV equalizers is $P = 3$, and the oversampling factor is set to $N = 5$. The channels $c_a(t)$ and $c_{I,a}(t)$ are modeled as two-ray multipath channels, whose impulse responses

$$p(t) = a_1\, e^{j2\pi\xi_1}\, g(t-\tau_1) + a_2\, e^{j2\pi\xi_2}\, g(t-\tau_2) , \qquad (31)$$

are truncated in the interval $(0, T_0)$, where $g(t)$ is a Nyquist-shaped pulse with 35 % excess bandwidth, and $a_1 = 1\,[1]$, $a_2 = 0.8\,[0.75]$, $\xi_1 = 0.15\,[0.25]$, $\xi_2 = 0.6\,[0.9]$, $\tau_1 = 0.25\,T_U\,[0]$, $\tau_2 = T_U\,[T_I]$, and $T_0 = 4\,T_U\,[5\,T_I]$ for the desired signal [for the interference, respectively]. The resulting discrete-time channels $c^{(\ell)}(k)$, $\ell = 0, 1, \ldots, N-1$, span $L_c = 4$ symbol periods of the desired signal, and their z-transforms have no common zeros. The thermal noise is modeled as a complex circular Gaussian process, and the signal-to-noise ratio (SNR) at the equalizer input is defined as

$$\text{SNR} \triangleq \frac{\sigma_s^2 \sum_{n=0}^{L_c-1} \|\, \boldsymbol{c}(n)\, \|^2}{E[\, \|\boldsymbol{w}(k)\|^2\,]} , \qquad (32)$$

which is set to 30 dB in all the experiments, whereas the SIR at the equalizer input is defined as

$$\mathrm{SIR} \triangleq \frac{\sigma_s^2 \sum_{n=0}^{L_c-1} \| \boldsymbol{c}(n) \|^2}{\langle E[\, \|\boldsymbol{i}(k)\|^2 \,] \rangle} \,, \tag{33}$$

with $\langle \cdot \rangle$ denoting temporal averaging. As performance measure, we resorted to the SINR at the output of the equalizer, averaged over the different phases, which is defined [see (10) and (15)] as

$$\mathrm{SINR} \triangleq \frac{1}{P} \sum_{h=0}^{P-1} \left(\frac{\sigma_s^2 \, |\boldsymbol{b}^{(h)H} \, \boldsymbol{c}_d|^2}{\boldsymbol{b}^{(h)H} \, \boldsymbol{R}_{dd}^{(h)} \, \boldsymbol{b}^{(h)}} \right) , \tag{34}$$

where $\boldsymbol{b}^{(h)}$ is the hth phase of $\boldsymbol{b}(k)$, and $\boldsymbol{R}_{dd}^{(h)}$ is the correlation matrix of the hth phase of $\boldsymbol{d}(k)$.

For the proposed algorithm (see Fig. 2), we choose $\lambda = \delta = 1$ in the first stage, whereas, in the second stage, the CMA weight vectors $\boldsymbol{u}^{(h)}$ are initialized with a single one in the first component, i.e., $\boldsymbol{u}^{(h)}(0) = [1, 0, \ldots, 0]^T$. Due to the large initial estimation error of the inverse of the correlation matrices $\boldsymbol{R}_{zz}^{(h)}$, the output power of the first stage might exhibit a remarkable increase during the first iterations, converging then to the steady-state solution. It has been found experimentally that such a large dynamic variation at the output of the first stage might prevent the CMA in the second stage to converge. Therefore, as an appropriate countermeasure, we choose to "freeze" (i.e., to keep fixed) the CMA weight vectors initially, while the filtering matrices $\boldsymbol{T}^{(h)}(i)$ are still adapting (i.e., for the first 100 iterations). Finally, we choose to continuously adjust the step-size of the CMA, in order to achieve fast convergence without compromising stability; thus, we set $\mu^{(h)}(i) = 0.2 \, \mu_{\max}^{(h)}(i)$, where, according to [17], $\mu_{\max}^{(h)}(i)$ is the maximum value of the step-size that assures CMA stability at iteration i, and can be evaluated in real-time, since it depends only on the output $y^{(h)}(i)$ of the equalizer and the dispersion coefficient γ.

A. *Experiment 1: co-channel interference*

In the first experiment, we considered the CCI case, i.e., we set $f_I = 0$ in (3). First, we evaluated the steady-state performance of the proposed two-stage PTV-CMA equalizer, in comparison with those of the following receivers: (i) non-blind LPTV MMSE receiver (referred to as LPTV in the plots) proposed in [7], which has exact knowledge of the signal channel and is implemented in a batch mode; (ii) PTV version of the blind receiver proposed by Tsatsanis *et al.* in [11] (referred to as PTV-TSA), which is implemented in a time-sequenced batch mode; (iii) PTV version of the CMA receiver without pre-filtering (referred to as PTV-CMA), which is implemented in a time-sequenced adaptive mode, and utilizes the same initialization and the same step-size adaptation strategy as the two-stage PTV-CMA. For all receivers, we fixed the equalizer length at $L_e = 10$ and the equalization delay at $d = 6$. Such a choice corresponds to adopting, for the proposed two-stage PTV-CMA algorithm, parameterization C_2 given by (25), which is a reasonable choice in this experiment since the channel length L_c is assumed to be known. The maximum sample-size considered for all the equalizers under comparison is 6000 desired symbols per-phase. We reported in Fig. 3 the values of SINR obtained with the different receivers as a function of SIR ranging from 5 to 40 dB. In order to avoid transient effects, the SINR is computed, for all the considered equalizers, using only the last 1000 equalized symbols per-phase. Results show that the proposed equalizer provides good performances also for low values of SIR. Moreover, for values of SIR exceeding 20 dB, its performances are remarkably close to those of the non-blind LPTV receiver. On the contrary, both PTV-TSA and PTV-CMA suffer a severe performance degradation for all the considered values of SIR.

To assess the convergence performance of the proposed algorithm, we reported in Fig. 4 the values of SINR as a function of the number i of iterations, for different values of SIR. To allow for a fair comparison, the values of SINR are normalized to the corresponding SINR of the non-blind LPTV MMSE. It can be observed that for moderate-to-high values of SIR a satisfactory convergence (say, within 3 dB from the non-blind case) is achieved within 6000 iterations, whereas for SIR = 5 dB the SINR degradation is about 8 dB for $i = 6000$ and, hence, a larger number of iterations are needed for the algorithm to converge.

We also investigated the tracking performance of the proposed algorithm, i.e., its capability to operate in a situation where the power of the interference is dynamically varying. More specifically, we fixed the power of both the desired signal and noise, and let the interference power vary, according to the following law: for the first 8000 iterations, the SIR is equal to 5 dB; from iteration 8001 to 9000, it increases linearly from 5 to 40 dB; from iteration 9001 to 17000, it is equal to 40 dB; from iteration 17001 to 18000, it decreases linearly from 40 to 5 dB; from iteration 18001 to 26000, it is equal to 5 dB. Unlike the previous simulations, where the choice $\lambda = 1$ was justified, since we were operating in stationary settings, we chose $\lambda = 0.998$ in this case, in order to better track the SIR variations. We reported in Fig. 5 the values of SINR as a function of the number i of iterations, in comparison with the SINR of the non-blind LPTV MMSE receiver. The proposed method is able to operate satisfactorily in this dynamic environment, exhibiting a better tracking behavior when the interference power is diminishing. Finally, as expected, using values of $\lambda < 1$, and thereby reducing the memory of the pre-filtering stage, results in faster tracking but worst steady-state performance.

B. *Experiment 2: adjacent-channel interference*

In the second experiment, we compared the steady-state performances of the equalizers already considered in experiment 1, with the only difference that the interference is adjacent-channel, i.e., $f_I \neq 0$ in (3). We reported in Fig. 6 the values of SINR for a fixed SIR = 15 dB, as a function of f_I (normalized to the desired symbol rate T_U^{-1}) ranging from 0 to 1.8, where the latter value corresponds to the case where the desired signal and the interference are spectrally disjoint. The performance of the proposed method are quite robust to the value of f_I, exhibiting only a slight improvement as f_I increases. On the contrary, both PTV-CMA and PTV-TSA perform unsatisfactorily for any value of f_I and, moreover, performances of PTV-CMA are quite sensitive to the value of f_I, since the latter is one of the parameters, together with SIR and initialization choice, affecting the interference capture phenomenon (see [10] for a discussion, albeit in a simplified case).

C. Experiment 3: channel with common zeros

The third experiment is aimed at showing that the proposed method is able to work also when the sub-channels $c^{(\ell)}(k)$, $\ell = 0, 1, \ldots, N-1$, exhibit common zeros in their z-transforms, a situation that happens very often in wireless channels. To this end, we considered the same scenario of experiment 1, with the only exception that the desired channel, given by (31), was modified in order to exactly force all the sub-channels to share one zero. More precisely, let $z_{\max}^{(0)}$ denote the first zero (i.e., the one with maximum module) of the first sub-channel; thus, we modified $c^{(\ell)}(k)$ for $\ell \neq 0$ so as to force the first zero of each sub-channel to be equal to $z_{\max}^{(0)}$, without modifying all the other zeros. Simulation results, reported in Fig. 7, show that the proposed method yields satisfactory performances for all considered values of SIR, assuring a significant gain over PTV-TSA and PTV-CMA. A comparison with Fig. 3 evidence, however, a more marked performance degradation with respect to the non-blind LPTV equalizer, whose performances are practically not affected from the presence of common zeros.

D. Experiment 4: channel-length mismatch

In the last experiment, we demonstrated the robustness of the proposed method to possible errors in channel-length knowledge. As observed in Section III, the effects of such errors can be effectively counteracted by resorting to parameterization C_1, whose form [see (23)] does not depend on the true channel length. Thus, we considered again the same scenario of experiment 1, with the exception that the estimated channel length is $\widehat{L}_c = 2$, different from the true one $L_c = 4$. It should be remarked that this is a challenging situation, since SOS-based blind methods are much more sensitive to channel underestimation than to overestimation. To counteract the channel-length mismatch, the proposed algorithm utilizes parameterization C_1 with $d = 0$. A comparison with Fig. 3, where the correct order of the channel was assumed, shows that the performances of the proposed method in this situation, reported in Fig. 8, exhibit only a slight degradation in correspondence of the lowest values of SIR. On the contrary, the performance of PTV-TSA are heavily affected by channel-length underestimation and, moreover, do not improve as the SIR increases, which clearly evidences the inconsistency of such a method when the channel is undermodeled. Moreover, the performances of LPTV-MMSE and PTV-CMA are not affected by channel-length mismatch, since the former assumes perfect knowledge of the channel, while the latter does not rely on channel-length knowledge. As a final remark, it should be observed that, although not originally proposed in [11], also the PTV-TSA method could benefit from using other kinds of parameterization, such as C_1, in order to improve its robustness with respect to channel-length mismatches.

V. CONCLUSIONS

In this paper, we proposed a new blind adaptive equalization technique, which is able to compensate for channel distortion (i.e., ISI) while simultaneously mitigating the effects of high-level CCI and/or ACI. The proposed equalizer is composed of two stages: in the first stage, a linearly-constrained adaptive transformation is performed on the received data, in order to blindly preserve the desired symbol while minimizing the contribution of disturbances; in the second stage, a CMA equalizer is used to extract the desired symbol. Due to the presence of the interference, both stages turn out to be PTV, and are implemented by using the time-sequenced approach. Simulation results show that the proposed two-stage PTV-CMA equalizer exhibits satisfactory performance also for low values of SIR and for channels exhibiting common zeros, significantly outperforming other blind techniques proposed in the literature. Further research is needed to investigate how the pre-filtering matrix $\boldsymbol{T}(k)$ employed in the first stage affects the shape and minima of the CMA cost function utilized in the second stage, in order to analytically delineate the conditions under which the capture effect is avoided. Moreover, the advantages deriving from the frequency-domain implementation of the proposed PTV equalizer deserve further investigation.

APPENDIX

I. DERIVATION OF THE FILTERING MATRIX $\boldsymbol{T}(k)$

We seek the solution of the following constrained optimization problem:

$$\min_{\boldsymbol{T}(k)} E[\|\boldsymbol{x}(k)\|^2] \quad \text{subject to} \quad \boldsymbol{T}^H(k)\,\boldsymbol{Q}_d = \boldsymbol{I}\,. \tag{35}$$

Since, accounting for (11),

$$E[\|\boldsymbol{x}(k)\|^2] = E[\boldsymbol{z}^H(k)\,\boldsymbol{T}(k)\,\boldsymbol{T}^H(k)\,\boldsymbol{z}(k)] \tag{36}$$

and observing that, due to the trace identity and to linearity of the trace operator, one has

$$\begin{aligned} &E[\boldsymbol{z}^H(k)\,\boldsymbol{T}(k)\,\boldsymbol{T}^H(k)\,\boldsymbol{z}(k)] \\ &\qquad = \text{trace}\{\,\boldsymbol{T}^H(k)\,E[\boldsymbol{z}(k)\,\boldsymbol{z}^H(k)]\,\boldsymbol{T}(k)\,\}\,, \end{aligned} \tag{37}$$

the optimization problem (35) can be reformulated as follows:

$$\min_{\boldsymbol{T}(k)} \text{trace}[\,\boldsymbol{T}^H(k)\,\boldsymbol{R}_{zz}(k)\,\boldsymbol{T}(k)\,] \tag{38}$$

subject to $\boldsymbol{T}^H(k)\,\boldsymbol{Q}_d = \boldsymbol{I}$. The classical approach for solving problems like (38) is the method of Lagrange multipliers. Then, let us consider the *Lagrangian* function

$$\begin{aligned} \mathcal{L}[\,\boldsymbol{T}(k), \boldsymbol{\Lambda}(k)\,] \;&=\; \text{trace}\{\,\boldsymbol{T}^H(k)\,\boldsymbol{R}_{zz}(k)\,\boldsymbol{T}(k) + \\ &+\;\; 2\,\text{Re}[(\,\boldsymbol{I} - \boldsymbol{T}^H(k)\,\boldsymbol{Q}_d\,)\,\boldsymbol{\Lambda}(k)]\,\}\,, \end{aligned} \tag{39}$$

where $\boldsymbol{\Lambda}(k)$ is the matrix of complex Lagrange multipliers. The constrained optimization problem (38) is turned into the following unconstrained one:

$$\min_{\boldsymbol{T}(k)} \mathcal{L}[\,\boldsymbol{T}(k), \boldsymbol{\Lambda}(k)\,]\,, \tag{40}$$

whose solutions $\boldsymbol{T}(k)$ are potential solutions of the constrained optimization problem (38). After some manipulations of (39) and using again the linearity of the trace, one obtains:

$$\begin{aligned}\mathcal{L}[\boldsymbol{T}(k),\boldsymbol{\Lambda}(k)\,] &= \text{trace}[\,\boldsymbol{T}^H(k)\,\boldsymbol{R}_{zz}(k)\,\boldsymbol{T}(k)+ \\ &- \boldsymbol{T}^H(k)\,\boldsymbol{Q}_d\,\boldsymbol{\Lambda}(k)-\boldsymbol{\Lambda}^H(k)\,\boldsymbol{Q}_d^H\,\boldsymbol{T}(k)\,] \\ &+ \text{trace}[\,\boldsymbol{\Lambda}(k)+\boldsymbol{\Lambda}^H(k)\,]\,, \end{aligned} \tag{41}$$

and thus (40) can be equivalently written as

$$\min_{\boldsymbol{T}(k)} \text{trace}[\boldsymbol{T}^H(k)\,\boldsymbol{R}_{zz}(k)\,\boldsymbol{T}(k)+ \\ -\boldsymbol{T}^H(k)\,\boldsymbol{V}(k)-\boldsymbol{V}^H(k)\,\boldsymbol{T}(k)]\,, \tag{42}$$

with $\boldsymbol{V}(k) \stackrel{\triangle}{=} \boldsymbol{Q}_d\,\boldsymbol{\Lambda}(k)$. The matrix $\boldsymbol{T}(k)$ that satisfies (42) is given by (see [18])

$$\boldsymbol{T}(k)=\boldsymbol{R}_{zz}^{-1}(k)\,\boldsymbol{V}(k)=\boldsymbol{R}_{zz}^{-1}(k)\,\boldsymbol{Q}_d\,\boldsymbol{\Lambda}(k)\,. \tag{43}$$

In order to find the values of the multipliers $\boldsymbol{\Lambda}(k)$, let us impose that (43) satisfies the constraint:

$$\boldsymbol{\Lambda}(k)^H(\,\boldsymbol{Q}_d^H\,\boldsymbol{R}_{zz}^{-1}(k)\,\boldsymbol{Q}_d\,)=\boldsymbol{I}\,, \tag{44}$$

obtaining so

$$\boldsymbol{\Lambda}(k)=(\,\boldsymbol{Q}_d^H\,\boldsymbol{R}_{zz}^{-1}(k)\,\boldsymbol{Q}_d\,)^{-1} \tag{45}$$

and, by substitution of (45) in (43), solution (19). Finally, by substitution of (19) in (37), the minimum output power $\mathcal{P}_{\min}(k)$ is easily found to be given by (20).

II. Derivation of the updating equations for $\boldsymbol{T}^{(h)}(i)$

We seek the recursive equation for updating the matrix

$$\boldsymbol{T}^{(h)}(i)=\boldsymbol{P}^{(h)}(i)\,\boldsymbol{Q}_d\,[\,\boldsymbol{Q}_d^H\,\boldsymbol{P}^{(h)}(i)\,\boldsymbol{Q}_d\,]^{-1}\,, \tag{46}$$

which is the estimate, at iteration i, of $\boldsymbol{T}^{(h)}$ given by (21), where $\boldsymbol{P}^{(h)}(i)$ denotes the inverse of $\boldsymbol{R}_{zz}^{(h)}$ at iteration i. Starting from the recursive equation for updating $\boldsymbol{P}^{(h)}(i)$ (see Fig. 2), which is initialized as $\boldsymbol{P}^{(h)}(-1)=\delta^{-1}\,\boldsymbol{I}$, with δ denoting a positive scalar, we find the updating equation for the matrix $[\boldsymbol{Q}_d^H\,\boldsymbol{P}^{(h)}(i)\,\boldsymbol{Q}_d]^{-1}$, which is given by

$$\begin{aligned}[\,\boldsymbol{Q}_d^H\,\boldsymbol{P}^{(h)}(i)\,\boldsymbol{Q}_d\,]^{-1} &= [\,\lambda^{-1}\,\boldsymbol{Q}_d^H\,\boldsymbol{P}^{(h)}(i-1)\,\boldsymbol{Q}_d+ \\ &-\lambda^{-1}\,\boldsymbol{Q}_d^H\,\boldsymbol{p}^{(h)}(i)\,\boldsymbol{z}^{(h)}(i)^H\,\boldsymbol{P}^{(h)}(i-1)\,\boldsymbol{Q}_d]^{-1}\,.\end{aligned} \tag{47}$$

Applying the matrix inversion lemma to (47), one obtains:

$$[\,\boldsymbol{Q}_d^H\,\boldsymbol{P}^{(h)}(i)\,\boldsymbol{Q}_d\,]^{-1} = \lambda\left\{[\boldsymbol{Q}_d^H\,\boldsymbol{P}^{(h)}(i-1)\,\boldsymbol{Q}_d]^{-1}+ \\ \frac{[\boldsymbol{Q}_d^H\,\boldsymbol{P}^{(h)}(i-1)\,\boldsymbol{Q}_d]^{-1}\,\boldsymbol{Q}_d^H\,\boldsymbol{p}^{(h)}(i)\,\boldsymbol{z}^{(h)}(i)^H\,\boldsymbol{T}^{(h)}(i-1)}{1-\boldsymbol{z}^{(h)}(i)^H\,\boldsymbol{T}^{(h)}(i-1)\,\boldsymbol{Q}_d^H\,\boldsymbol{p}^{(h)}(i)}\right\}\,, \tag{48}$$

where

$$\boldsymbol{T}^{(h)}(i-1)=\boldsymbol{P}^{(h)}(i-1)\,\boldsymbol{Q}_d\,[\boldsymbol{Q}_d^H\,\boldsymbol{P}^{(h)}(i-1)\,\boldsymbol{Q}_d]^{-1}\,. \tag{49}$$

In order to seek the equation for updating $\boldsymbol{T}^{(h)}(i)$, we substitute (48) and the updating equation for $\boldsymbol{P}^{(h)}(i)$ in (46), obtaining thus

$$\begin{aligned}\boldsymbol{T}^{(h)}(i) &= \boldsymbol{T}^{(h)}(i-1)-\boldsymbol{p}^{(h)}(i)\,\boldsymbol{z}^{(h)}(i)^H\,\boldsymbol{T}^{(h)}(i-1)+ \\ &+ \frac{\boldsymbol{T}^{(h)}(i-1)\,\boldsymbol{Q}_d^H\,\boldsymbol{p}^{(h)}(i)\,\boldsymbol{z}^{(h)}(i)^H\,\boldsymbol{T}^{(h)}(i-1)}{1-\boldsymbol{z}^{(h)}(i)^H\,\boldsymbol{T}^{(h)}(i-1)\,\boldsymbol{Q}_d^H\,\boldsymbol{p}^{(h)}(i)}+ \\ &- \left[\frac{\boldsymbol{p}^{(h)}(i)\,\boldsymbol{z}^{(h)}(i)^H\,\boldsymbol{T}^{(h)}(i-1)\,\boldsymbol{Q}_d^H\,\boldsymbol{p}^{(h)}(i)}{1-\boldsymbol{z}^{(h)}(i)^H\,\boldsymbol{T}^{(h)}(i-1)\,\boldsymbol{Q}_d^H\,\boldsymbol{p}^{(h)}(i)}\right]\cdot \\ &\cdot\ \boldsymbol{z}^{(h)}(i)^H\,\boldsymbol{T}^{(h)}(i-1)\,.\end{aligned} \tag{50}$$

After some manipulations, (50) can be rewritten as

$$\begin{aligned}\boldsymbol{T}^{(h)}(i) &= \boldsymbol{T}^{(h)}(i-1)+ \\ &- \left[\frac{\boldsymbol{p}^{(h)}(i)-\boldsymbol{T}^{(h)}(i-1)\,\boldsymbol{Q}_d^H\,\boldsymbol{p}^{(h)}(i)}{1-\boldsymbol{z}^{(h)}(i)^H\,\boldsymbol{T}^{(h)}(i-1)\,\boldsymbol{Q}_d^H\,\boldsymbol{p}^{(h)}(i)}\right]\cdot \\ &\cdot\ \boldsymbol{z}^{(h)}(i)^H\,\boldsymbol{T}^{(h)}(i-1)\,,\end{aligned} \tag{51}$$

which, by introducing the overall gain vector $\boldsymbol{q}^{(h)}(i)$ (see Fig. 2) yields the updating equation for $\boldsymbol{T}^{(h)}(i)$, with $\boldsymbol{T}^{(h)}(-1)=\boldsymbol{Q}_d$.

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