Widely-linear versus linear blind multiuser detection with subspace-based channel estimation: finite sample-size effects

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***Abstract*—In a recent paper [1], we presented the finite-sample theoretical performance comparison between linear (L) and widely-linear (WL) minimum output-energy (MOE) receivers for direct-sequence code-division multiple-access (DS-CDMA) systems, worked out under the assumption that the channel impulse response of the desired user is exactly known. The main scope of the present work is to extend such an analysis, taking into account not only autocorrelation matrix (ACM) estimation effects, but also the accuracy of subspace-based blind channel estimation (CE). We aim to answer the two following questions: *Which of the two estimation processes (ACM or CE) is the main source of degradation when implementing the receivers on the basis of a finite sample-size? Compared with the L-MOE one, is the finite-sample WL-MOE receiver with blind CE capable of achieving the performance gains predicted by the theory?* To this goal, simple and easily interpretable formulas are developed for the signal-to-interference-plus-noise ratio (SINR) at the output of the L- and WL-MOE receivers with blind CE, when they are implemented using either the sample ACM or its eigendecomposition. In addition, the derived formulas, which are validated by simulations, allow one to recognize and discuss interesting tradeoffs between the main parameters of the DS-CDMA system.**

***Index Terms*—Blind multiuser detection, channel estimation, direct-sequence code-division multiple-access (DS-CDMA) systems, linear and widely-linear filtering, minimum output-energy (MOE) criterion, perturbation analysis, proper and improper random processes, subspace methods.**

I. Introduction

In multiuser communication systems, such as non-orthogonal direct-sequence code-division multiple-access (DS-CDMA) systems, the multiple-access interference (MAI) often represents the main source of performance degradation. During the last decades, in order to counteract such a degradation, a great bulk of research activities has been devoted to multiuser detection (MUD) [2]. Among most recent MUD developments, *widely-linear* (WL) techniques [3]–[7] perform MAI mitigation by exploiting the *noncircular* or *improper* [8] features of many digital modulation schemes, such as ASK, differential BPSK (DBPSK), offset QPSK (OQPSK), offset QAM (OQAM), MSK and its variant Gaussian MSK (GMSK). Unlike conventional L-MUD receivers [9]–[11], which process only the complex envelope $r(t)$ of the received signal, to extract the information contained in its statistical autocorrelation function $R_{rr}(t,\tau) \triangleq \mathrm{E}[r(t)\, r^*(t-\tau)]$, WL-MUD receivers jointly elaborate $r(t)$ and its complex-conjugate version, to take advantage also of the information contained in their statistical *cross-correlation* function $R_{rr^*}(t,\tau) \triangleq \mathrm{E}[r(t)\, r(t-\tau)]$, assuring thus potentially larger performance gains.

In a recent paper [1], the theoretical performance analysis of the WL receiver designed according to the minimum output-energy (MOE) criterion [11], is considered in terms of signal-to-interference-plus-noise ratio (SINR). Implementation of the WL-MOE receiver requires knowledge of the (augmented) *autocorrelation matrix* (ACM) of the received signal, and of the *received signature* (possibly distorted by the channel) of each user to be demodulated. In particular, by applying a first-order perturbative analysis, the performance degradation due to finite-sample ACM estimation is evaluated in [1], with reference to two common implementations of the WL-MOE receiver: the WL-SMI one, which is based on sample matrix inversion (SMI), and the WL-SUB one, which resorts to ACM subspace (SUB) decomposition to improve robustness against finite-sample errors.[1] The results of the analysis show that the WL-MOE receiver is more sensitive than its linear counterpart to finite sample-size effects associated to ACM estimation, and its subspace-based implementation is a viable strategy to achieve in practice the performance gains predicted by theory.

The analysis of [1] is exhaustive only when the received signature is perfectly known at the receiver. When operating over a multipath channel, however, due to the unknown channel response, the received signature is a distorted version of the transmitted one, making channel estimation (CE) a necessary step to implement both the L- and WL-MOE receivers. In such a scenario, the performance of the receivers is affected not only by imperfect ACM estimation, but also by inaccurate CE. Conventional CE methods are *training-based*, which might lead to significant waste of resources in mobile communications scenarios, especially when training must be frequently

Manuscript received May 28, 2008; revised October 2, 2008. This work has been partially supported by the Italian National Project: Wireless multiplatfOrm mimo active access netwoRks for QoS-demanding muLtimedia Delivery (WORLD), under grant number 2007R989S. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Subhrakanti Dey.

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Publisher Item Identifier S 0000-0000(00)00000-3.

[1] It should be mentioned that other robust techniques, suitably generalized to the WL case, could be employed, such as those developed in [12]–[14], which are in their turn based on the robust beamforming technique [15].

repeated due to rapidly-changing channel conditions. Therefore, the past few decades witnessed many contributions in the area of *blind* CE for DS-CDMA systems, which assume the only knowledge of the transmitted signature of the desired user. Blind CE approaches relying on second-order statistics (SOS) of the received data are particularly attractive, since they require far fewer samples than methods based on higher-order statistics. SOS-based approaches for blind CE exploit the channel information contained in $R_{rr}(t, \tau)$ by processing the received signal $r(t)$. Among existing SOS-based approaches, the subspace-based CE method, originally proposed in [16], is one of the most popular blind algorithms for DS-CDMA systems, due to its closed-form expression and noise robustness [17], and its amenability to low-complexity and fast recursive implementation [18]. Moreover, channel identifiability conditions for subspace-based CE are well assessed [19] and, finally, subspace-based blind methods can be easily combined with existing training-based approaches (so-called *semi-blind* methods [20]). Among the main drawbacks of subspace-based CE are its performance degradation when the number of active users is comparable to the processing gain, and the need for accurate rank estimation of the noise-free ACM. Under the assumption that the transmitted symbols are improper and the noise is proper, the first drawback can be overcome by resorting to a generalized subspace-based CE method, which allows one to enlarge the dimension of the observation space, by jointly processing both $r(t)$ and its conjugate version $r^*(t)$ to exploit the channel information contained in both $R_{rr}(t, \tau)$ and $R_{rr^*}(t, \tau)$. Such an approach was originally proposed in [21]–[24] to improve channel identification in many application fields, including multicarrier CDMA [23] and single-carrier DS-CDMA systems [24]. To face up to the second drawback, one can use conventional rank estimation techniques, such as the Akaike information criterion (AIC) [25] or the minimum description length (MDL) method [26], or, alternatively, a subspace tracking procedure with successive cancellation techniques [27].

In this paper, the first-order perturbation analysis carried out in [1] is extended to incorporate the effects of errors due to subspace-based blind CE on the synthesis of the L- and WL-MOE receivers. It is worthwhile to note that, when the desired channel vector has been estimated through the subspace method and, hence, the subspace decomposition of the ACM is already available, it is preferable from a computational viewpoint to implement the L- and WL-SUB receivers rather than their SMI counterparts, since they do not require direct ACM inversion. Notwithstanding this, we have chosen to carry out also the performance analysis of the SMI versions of the receivers with CE since, in this way, an interesting comparison with the SUB versions of the receivers, as well as with our previous results [1], can be established. A distinct advantage of our analysis is that it leads to easily interpretable formulas, clearly showing the fundamental relationships among the main parameters (sample-size, processing gain, channel length, and number of users) of the DS-CDMA system. Although a similar perturbative performance analysis was addressed in [28]–[30] for the blind L-MMSE (minimum mean-square error) receiver, the analysis carried out in this paper for the L-MOE receiver with blind CE allows a more direct and fruitful comparison with the WL-MOE one and, moreover, leads to more easily interpretable results (albeit slightly less accurate) than those obtained in [28]–[30].

Finally, it is noteworthy that the problem considered herein exhibits interesting analogies with a well-studied topic in array processing, since the L-MOE-based multiuser detector is mathematically equivalent to the linear minimum variance (L-MV) beamformer [31], where in the latter the role of the received signature is played by the array steering vector (SV). Finite-sample performance analysis of the L-MV beamformer was carried out in [31]–[33] for the SMI version, and in [34] for the subspace-based implementation (so called projection method). Specifically, in [32] only the effects of ACM estimation were considered, whereas in [31], [34] the effects of ACM estimation and SV perturbation were *separately* studied, and a complete analysis of the *joint* effects of ACM estimation and SV perturbation was carried out only in [33]. However, the latter analysis does not explicitly account for the situation wherein the SV is blindly estimated from the received data and, consequently, the SV perturbation depends in its turn on the accuracy in ACM estimation, which is exactly the case of the subspace-based CE algorithms considered herein.

The rest of the paper is organized as follows. In Section II, the mathematical model of the DS-CDMA system is introduced, and the ideal WL-MOE and L-MOE receivers are briefly reviewed. In Section III, two data-estimated versions of the WL-MOE receiver (SMI and SUB) with CE are presented, and their finite-sample performance analysis is developed. The same analysis is carried out for the SMI and SUB implementations of the L-MOE receiver with CE in Section IV, where, moreover, a comparison between WL-MOE and L-MOE receivers is assessed. The theoretical results of Sections III and IV are validated by computer simulations in Section V, while their proofs are gathered in the Appendix. Finally, concluding remarks are given in Section VI.

A. Notations

The fields of complex, real, and integer numbers are denoted with $\mathbb{C}$, $\mathbb{R}$, and $\mathbb{Z}$, respectively; matrices [vectors] are denoted with upper case [lower case] boldface letters (e.g., $\mathbf{A}$ or $\mathbf{a}$); the field of $m \times n$ complex [real] matrices is denoted as $\mathbb{C}^{m \times n}$ [$\mathbb{R}^{m \times n}$], with $\mathbb{C}^m$ [$\mathbb{R}^m$] used as a shorthand for $\mathbb{C}^{m \times 1}$ [$\mathbb{R}^{m \times 1}$]; the superscripts $*$, T, H, -1 and $\dagger$ denote the conjugate, the transpose, the Hermitian (conjugate transpose), the inverse, and the Moore-Penrose generalized inverse [38] (pseudo-inverse) of a matrix, respectively; $\mathbf{0}_m \in \mathbb{R}^m$, $\mathbf{O}_{m \times n} \in \mathbb{R}^{m \times n}$ and $\mathbf{I}_m \in \mathbb{R}^{m \times m}$ denote the null vector, the null matrix, and the identity matrix, respectively; trace$(\cdot)$ and rank$(\cdot)$ represent the trace and the rank; $\mathcal{N}(\mathbf{A})$, $\mathcal{R}(\mathbf{A})$, and $\mathcal{R}^{\perp}(\mathbf{A})$ denote the null space, the range (column space), and the orthogonal complement of the column space of $\mathbf{A} \in \mathbb{C}^{m \times n}[\mathbb{R}^{m \times n}]$ in $\mathbb{C}^m[\mathbb{R}^m]$; for any $\mathbf{a} \in \mathbb{C}^m$, $\|\mathbf{a}\| \triangleq (\mathbf{a}^H \mathbf{a})^{1/2}$ denotes the Euclidean norm; $\mathbf{A} = \text{diag}(A_{11}, A_{22}, \ldots, A_{nn})$ is a diagonal matrix with elements A_{ii} on the main diagonal; E$[\cdot]$ and Var$[\cdot]$ denote ensemble averaging and variance, respectively, and $i \triangleq \sqrt{-1}$ is the imaginary unit; for any stationary discrete-time random vector process $\mathbf{x}(k) \in \mathbb{C}^m$, we denote with $\mathbf{R_{xx}} \triangleq \mathrm{E}[\mathbf{x}(k)\,\mathbf{x}^H(k)] \in \mathbb{C}^{m\times m}$ and with $\mathbf{R_{xx^*}} \triangleq \mathrm{E}[\mathbf{x}(k)\,\mathbf{x}^T(k)] \in \mathbb{C}^{m\times m}$ the autocorrelation matrix and the conjugate correlation matrix, respectively ($\mathbf{R_{xx}} \equiv \mathbf{R_{xx^*}} \in \mathbb{R}^{m\times m}$ when $\mathbf{x}(k) \in \mathbb{R}^m$); throughout the paper, we occasionally use the simplified notations $\mathbf{a}_\mathrm{R} \triangleq \mathrm{Re}[\mathbf{a}]$, $\mathbf{a}_\mathrm{I} \triangleq \mathrm{Im}[\mathbf{a}]$, $\mathbf{A}_\mathrm{R} \triangleq \mathrm{Re}[\mathbf{A}]$, and $\mathbf{A}_\mathrm{I} \triangleq \mathrm{Im}[\mathbf{A}]$.

II. Problem formulation and ideal WL-MOE and L-MOE receivers

In this paper, we consider a synchronous DS-CDMA system with J users, employing short spreading codes with $1/T_c = N/T$ chips/symbol and transmitting over channels that introduce interchip interference and negligible intersymbol interference (ISI) [2]. After chip-matched filtering, perfect time synchronization and sampling with rate $1/T_c$, the received vector $\mathbf{r}(k) \in \mathbb{C}^N$ collecting the N samples of the incoming signal in the time interval $[k\,T, (k+1)\,T)$, with $k \in \mathbb{Z}$, can be written [1], [2] as follows

$$\mathbf{r}(k) = \sum_{j=1}^{J} \alpha_j\, \mathbf{C}_j\, \mathbf{g}_j\, b_j(k) + \mathbf{v}(k) = \mathbf{\Phi}\, \mathbf{b}(k) + \mathbf{v}(k)\,, \quad (1)$$

where, with reference to the jth user, the real positive number α_j is the received amplitude (which is the product of the transmitted amplitude and the channel gain), the matrix $\mathbf{C}_j \in \mathbb{C}^{N\times L_j}$ is Toeplitz having $[c_j(0), 0, \ldots, 0]^T$ as first row and $[c_j(0), c_j(1), \ldots, c_j(N-1)]^T$ as first column, with the vector $\mathbf{c}_j \triangleq [c_j(0), c_j(1), \ldots, c_j(N-1)]^T \in \mathbb{C}^N$ denoting the *unit-norm* transmitted signature,[2] the sequence $g_j(n)$ is the channel impulse response of length $L_j \ll N$ ($L_j > 1$), with $\mathbf{g}_j \triangleq [g_j(0), g_j(1), \ldots, g_j(L_j-1)]^T \in \mathbb{C}^{L_j}$ being the corresponding *unit-norm* channel vector, and $b_j(k)$ is the transmitted symbol. Moreover, the matrix $\mathbf{\Phi} \triangleq \mathbf{\Psi}\,\mathbf{A} \in \mathbb{C}^{N\times J}$ embodies the effects of channels and signatures, with $\mathbf{\Psi} \triangleq [\mathbf{C}_1\,\mathbf{g}_1, \ldots, \mathbf{C}_J\,\mathbf{g}_J] \in \mathbb{C}^{N\times J}$ and $\mathbf{A} \triangleq \mathrm{diag}(\alpha_1, \alpha_2, \ldots, \alpha_J) \in \mathbb{R}^{J\times J}$, whereas $\mathbf{b}(k) \triangleq [b_1(k), b_2(k), \ldots, b_J(k)]^T \in \mathbb{R}^J$ is the symbol vector, and $\mathbf{v}(k) \in \mathbb{C}^N$ accounts for additive thermal noise.

Throughout the paper, we will rely on the same assumptions formulated in [1]: **(a1)** $\mathbf{b}(k)$ is a binary *real* zero-mean random vector, whose entries are independent and identically distributed (i.i.d.) random variables assuming equiprobable values in $\mathcal{B} = \{-1, 1\}$, with $\mathbf{b}(k_1)$ and $\mathbf{b}(k_2)$ statistically independent for $k_1 \neq k_2$; **(a2)** $\mathbf{v}(k)$ is a *complex proper* [8] zero-mean Gaussian random vector, independent of $\mathbf{b}(k)$, having $\mathbf{R_{vv}} = \sigma_v^2\,\mathbf{I}_N$ and $\mathbf{R_{vv^*}} = \mathbf{O}_{N\times N}$, with $\mathbf{v}(k_1)$ and $\mathbf{v}(k_2)$ statistically independent of each other for $k_1 \neq k_2$. As regards the considered signal model, it is noteworthy that the assumptions of synchronous transmissions and negligible ISI are taken only for the sake of simplicity, since our analysis can be readily generalized to other scenarios (e.g., asynchronous users and/or channels with ISI). For instance, an asynchronous system with J_a users can be described by the synchronous model (1) with $J \leq 2J_a$ equivalent users (see [10] for details).

[2]The signature $\mathbf{c}_j$ accounts also for possible precoding phases, whose role in downlink is discussed in [1, Theorem 1].

The output of a WL receiver for user $j \in \{1, 2, \ldots, J\}$ can be expressed [1], [35] as

$$y_j(k) = \mathbf{f}_{j,1}^H\,\mathbf{r}(k) + \mathbf{f}_{j,2}^H\,\mathbf{r}^*(k) = \mathbf{f}_j^H\,\mathbf{z}(k)\,, \quad (2)$$

where $\mathbf{f}_j \triangleq [\mathbf{f}_{j,1}^T, \mathbf{f}_{j,2}^T]^T \in \mathbb{C}^{2N}$ and the *augmented* vector $\mathbf{z}(k) \triangleq [\mathbf{r}^T(k), \mathbf{r}^H(k)]^T \in \mathbb{C}^{2N}$ is the received block. According to (1), vector $\mathbf{z}(k)$ can be expressed as

$$\mathbf{z}(k) = \mathbf{H}\,\mathbf{b}(k) + \mathbf{d}(k)\,, \quad (3)$$

with the augmented structure $\mathbf{H} \triangleq [\mathbf{\Phi}^T, \mathbf{\Phi}^H]^T \in \mathbb{C}^{2N\times J}$ and $\mathbf{d}(k) \triangleq [\mathbf{v}^T(k), \mathbf{v}^H(k)]^T \in \mathbb{C}^{2N}$. Moreover, the following condition is assumed to hold: **(c1)** when $J \leq N$ (*underloaded systems*), the matrix $\mathbf{\Phi}$ is full-column rank, i.e., $\mathrm{rank}(\mathbf{\Phi}) = J$. It can be readily shown (see, e.g., [1]) that, in the downlink case, wherein all the user signals propagate through a common multipath channel, the linear independence of the signatures $\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_J$ is a necessary and sufficient condition to ensure **(c1)**. It is noteworthy that, if $\mathbf{\Phi}$ is full-column rank, the augmented matrix $\mathbf{H}$ is full-column rank, too, i.e., $\mathrm{rank}(\mathbf{H}) = J$. In other words, in underloaded environments, condition **(c1)** additionally assures the full-column rank property of $\mathbf{H}$. However, the matrix $\mathbf{H}$ can be full-column rank even when $N < J \leq 2\,N$ (*overloaded systems*), wherein $\mathbf{\Phi}$ is inherently rank-deficient. Thus, in addition to condition **(c1)**, we assume hereinafter that: **(c2)** when $N < J \leq 2\,N$, the matrix $\mathbf{H}$ is full-column rank, i.e., $\mathrm{rank}(\mathbf{H}) = J$. With reference to the downlink scenario, fulfillment of condition **(c2)** is thoroughly discussed in [1, Theorem 1].

Accounting for (3), (2) can be equivalently written as

$$\begin{aligned} y_j(k) &= \mathbf{f}_j^H \mathbf{h}_j\, b_j(k) + \mathbf{f}_j^H [\overline{\mathbf{H}}_j\, \overline{\mathbf{b}}_j(k) + \mathbf{d}(k)] \\ &= \mathbf{f}_j^H \mathbf{h}_j\, b_j(k) + \mathbf{f}_j^H\, \mathbf{q}_j(k)\,, \end{aligned} \quad (4)$$

where $\mathbf{h}_j \triangleq [\boldsymbol{\phi}_j^T, \boldsymbol{\phi}_j^H]^T \in \mathbb{C}^{2N}$, with $\boldsymbol{\phi}_j \in \mathbb{C}^N$ being the jth column of $\mathbf{\Phi}$, whereas $\overline{\mathbf{H}}_j \triangleq [\overline{\mathbf{\Phi}}_j^T, \overline{\mathbf{\Phi}}_j^H]^T \in \mathbb{C}^{2N\times(J-1)}$, with $\overline{\mathbf{\Phi}}_j \in \mathbb{C}^{N\times(J-1)}$ denoting the matrix that includes all the columns of $\mathbf{\Phi}$ except for the jth column $\boldsymbol{\phi}_j$, $\overline{\mathbf{b}}_j(k) \in \mathbb{R}^{J-1}$ denotes the vector that includes all the elements of $\mathbf{b}(k)$ except for the jth entry $b_j(k)$, and $\mathbf{q}_j(k) \triangleq \overline{\mathbf{H}}_j\,\overline{\mathbf{b}}_j(k) + \mathbf{d}(k) \in \mathbb{C}^{2N}$ is the augmented disturbance (interference-plus-noise) vector. Since, by virtue of assumption **(a1)**, $b_j(k)$ is real-valued, an appropriate performance measure for the jth user is the output SINR [29] defined as

$$\mathrm{SINR}(\mathbf{f}_j) \triangleq \frac{\mathrm{E}^2\{\mathrm{Re}[y_j(k)] \,|\, b_j(k)\}}{\mathrm{Var}\{\mathrm{Re}[y_j(k)] \,|\, b_j(k)\}}\,. \quad (5)$$

Indeed, if the disturbance contribution $\mathbf{f}_j^H\,\mathbf{q}_j(k)$ in (4) can be approximated as a Gaussian random variable, the error probability is well approximated as $P_{e,j} \triangleq \Pr\{\hat{b}_j(k) \neq b_j(k)\} \approx Q(\sqrt{\mathrm{SINR}(\mathbf{f}_j)})$, where $Q(x) \triangleq (1/\sqrt{2\pi})\int_x^{+\infty} e^{-u^2/2}\,du$ denotes the Q function. Definition (5) of the SINR is quite general and allows for relatively simple calculations also when $\mathbf{h}_j$ and/or $\mathbf{f}_j$ are estimated from data. As discussed in [1, Lemma 1], any receiver maximizing (5) can be found, without loss of generality, within the class of receivers possessing the *conjugate symmetry* (CS) property, that is, $\mathbf{f}_{j,1} = \mathbf{f}_{j,2}^*$ in (2), which assures that the output given by (2) or (4) is *real-valued*. Belonging to this class is the WL-MOE receiver [1], i.e.,

$$\mathbf{f}_{j,\text{WL-MOE}} = \arg \min_{\substack{\mathbf{f}_j \in \mathbb{C}^{2N} \\ \mathbf{f}_j^H \mathbf{h}_j = 1}} \mathrm{E}\{\mathrm{Re}^2[y_j(k)]\}$$
$$= (\mathbf{h}_j^H \mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}^{-1} \mathbf{h}_j)^{-1} \mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}^{-1} \mathbf{h}_j = (\mathbf{h}_j^H \mathbf{R}_{\mathbf{zz}}^{-1} \mathbf{h}_j)^{-1} \mathbf{R}_{\mathbf{zz}}^{-1} \mathbf{h}_j \ , \tag{6}$$

whose corresponding *maximum* SINR can be expressed as

$$\mathrm{SINR}_{j,\text{WL-MOE}} \triangleq \mathrm{SINR}(\mathbf{f}_{j,\text{WL-MOE}}) = \mathbf{h}_j^H \mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}^{-1} \mathbf{h}_j$$
$$= (\mathbf{f}_{j,\text{WL-MOE}}^H \mathbf{R}_{\mathbf{q}_j\mathbf{q}_j} \mathbf{f}_{j,\text{WL-MOE}})^{-1} \ . \tag{7}$$

Under assumptions **(a1)** and **(a2)**, the matrix $\mathbf{R}_{\mathbf{zz}}$ assumes the form $\mathbf{R}_{\mathbf{zz}} = \mathbf{H}\,\mathbf{H}^H + \sigma_v^2\,\mathbf{I}_{2N}$. Moreover, by virtue of conditions **(c1)** and **(c2)**, the matrix $\mathbf{H}\,\mathbf{H}^H$ has only J nonzero eigenvalues $\lambda_1 \geq \lambda_2 \cdots \geq \lambda_J > 0$. An equivalent form of the WL-MOE receiver can be obtained thus by exploiting the eigenvalue decomposition (EVD) of $\mathbf{R}_{\mathbf{zz}}$, which is given by $\mathbf{R}_{\mathbf{zz}} = \mathbf{U}_s \mathbf{\Lambda}_s \mathbf{U}_s^H + \mathbf{U}_n \mathbf{\Lambda}_n \mathbf{U}_n^H$, where $\mathbf{U}_s \in \mathbb{C}^{2N\times J}$ collects the eigenvectors associated with the J largest eigenvalues of the autocorrelation matrix $\mathbf{R}_{\mathbf{zz}}$, which span the *signal subspace*, i.e., the subspace $\mathcal{R}(\mathbf{H})$, $\mathbf{\Lambda}_s \triangleq \mathrm{diag}(\lambda_1 + \sigma_v^2, \lambda_2 + \sigma_v^2, \ldots, \lambda_J + \sigma_v^2) \in \mathbb{R}^{J\times J}$, $\mathbf{\Lambda}_n = \sigma_v^2\,\mathbf{I}_{2N-J} \in \mathbb{R}^{(2N-J)\times(2N-J)}$, and, finally, $\mathbf{U}_n \in \mathbb{C}^{2N\times(2N-J)}$ collects the eigenvectors associated with the eigenvalue σ_v^2, which span the *noise subspace*, i.e., the subspace $\mathcal{R}^\perp(\mathbf{H})$ in $\mathbb{C}^{2N}$. By substituting the EVD of $\mathbf{R}_{\mathbf{zz}}$ in (6) and exploiting the orthogonality between signal and noise subspaces, one obtains the *subspace-based form* of the WL-MOE receiver as follows

$$\mathbf{f}_{j,\text{WL-MOE}} = (\mathbf{h}_j^H \mathbf{U}_s \mathbf{\Lambda}_s^{-1} \mathbf{U}_s^H \mathbf{h}_j)^{-1} \mathbf{U}_s \mathbf{\Lambda}_s^{-1} \mathbf{U}_s^H \,\mathbf{h}_j \ . \tag{8}$$

Turning to L-MUD receivers, it should be observed that (2) encompasses, as a particular case, the linear input-output relationship $y_j(k) = \mathbf{w}_j^H\,\mathbf{r}(k)$, with $\mathbf{w}_j \in \mathbb{C}^N$, which can be obtained by setting $\mathbf{f}_{j,1} = \mathbf{w}_j$ and $\mathbf{f}_{j,2} = \mathbf{0}_N$. It is worth noticing that, since a linear receiver does not satisfy the CS property, i.e., $\mathbf{f}_{j,1} \neq \mathbf{f}_{j,2}^*$, its output is not necessarily real-valued and, most important, a linear receiver does *not* generally belong to the family of receivers maximizing (5). A popular linear MUD receiver is the L-MOE one [11], i.e.,

$$\mathbf{w}_{j,\text{L-MOE}} = \arg \min_{\substack{\mathbf{w}_j \in \mathbb{C}^{N} \\ \mathbf{w}_j^H \boldsymbol{\phi}_j = 1}} \mathrm{E}[|y_j(k)|^2]$$
$$= (\boldsymbol{\phi}_j^H \mathbf{R}_{\mathbf{rr}}^{-1} \boldsymbol{\phi}_j)^{-1} \mathbf{R}_{\mathbf{rr}}^{-1} \boldsymbol{\phi}_j = (\boldsymbol{\phi}_j^H \mathbf{R}_{\mathbf{p}_j\mathbf{p}_j}^{-1} \boldsymbol{\phi}_j)^{-1} \mathbf{R}_{\mathbf{p}_j\mathbf{p}_j}^{-1} \boldsymbol{\phi}_j \ , \tag{9}$$

with $\mathbf{p}_j(k) \triangleq \overline{\mathbf{\Phi}}_j\,\overline{\mathbf{b}}_j(k) + \mathbf{v}(k) \in \mathbb{C}^N$ being the disturbance vector, and the resulting SINR (5) is given by (see [1])

$$\mathrm{SINR}_{j,\text{L-MOE}} \triangleq \mathrm{SINR}(\mathbf{f}_{j,\text{L-MOE}})$$
$$= \frac{2}{\mathbf{w}_{j,\text{L-MOE}}^H \mathbf{R}_{\mathbf{p}_j\mathbf{p}_j} \mathbf{w}_{j,\text{L-MOE}} + \mathrm{Re}[\mathbf{w}_{j,\text{L-MOE}}^H \mathbf{R}_{\mathbf{p}_j\mathbf{p}_j^*} \mathbf{w}_{j,\text{L-MOE}}^*]} \tag{10}$$

where $\mathbf{f}_{j,\text{L-MOE}} \triangleq [\mathbf{w}_{j,\text{L-MOE}}^T, \mathbf{0}_N^T]^T \in \mathbb{C}^{2N}$. Clearly, since the WL-MOE (6) is a maximum-SINR receiver, it results that $\mathrm{SINR}_{j,\text{L-MOE}} \leq \mathrm{SINR}_{j,\text{WL-MOE}}$. Similarly to the WL-MOE receiver, under condition **(c1)**, the L-MOE one (9) can be equivalently represented in subspace-based form[3]

$$\mathbf{w}_{j,\text{L-MOE}} = (\boldsymbol{\phi}_j^H\,\mathbf{V}_s\,\mathbf{\Upsilon}_s^{-1}\,\mathbf{V}_s^H\,\boldsymbol{\phi}_j)^{-1}\,\mathbf{V}_s\,\mathbf{\Upsilon}_s^{-1}\,\mathbf{V}_s^H\,\boldsymbol{\phi}_j \ , \tag{11}$$

where the columns of $\mathbf{V}_s \in \mathbb{C}^{N\times J}$ coincide with the eigenvectors corresponding to the J largest eigenvalues of $\mathbf{R}_{\mathbf{rr}} = \mathbf{\Phi}\,\mathbf{\Phi}^H + \sigma_v^2\,\mathbf{I}_N$, and $\mathbf{\Upsilon}_s \triangleq \mathrm{diag}(\mu_1 + \sigma_v^2, \mu_2 + \sigma_v^2, \ldots, \mu_J + \sigma_v^2) \in \mathbb{R}^{J\times J}$, with $\mu_1 \geq \mu_2 \cdots \geq \mu_J > 0$ being the J nonzero eigenvalues of $\mathbf{\Phi}\,\mathbf{\Phi}^H$.

III. PERFORMANCE ANALYSIS OF WL-MOE RECEIVERS WITH CHANNEL ESTIMATION

Implementation of the WL-MOE receiver defined by (6) or (8) requires estimation from the received data of the autocorrelation matrix $\mathbf{R}_{\mathbf{zz}}$ in (6) or its EVD in (8), and of $\mathbf{h}_j$ (in both). Under mild conditions, a consistent estimate $\widehat{\mathbf{R}}_{\mathbf{zz}}$ of $\mathbf{R}_{\mathbf{zz}}$ is the sample ACM obtained as

$$\widehat{\mathbf{R}}_{\mathbf{zz}} = \frac{1}{K}\sum_{k=1}^{K} \mathbf{z}(k)\,\mathbf{z}^H(k) \ , \tag{12}$$

where K denotes the estimation sample size. Applying the EVD to $\widehat{\mathbf{R}}_{\mathbf{zz}}$, one obtains $\widehat{\mathbf{R}}_{\mathbf{zz}} = \widehat{\mathbf{U}}_s \widehat{\mathbf{\Lambda}}_s \widehat{\mathbf{U}}_s^H + \widehat{\mathbf{U}}_n \widehat{\mathbf{\Lambda}}_n \widehat{\mathbf{U}}_n^H$, where the matrices $\widehat{\mathbf{U}}_s \in \mathbb{C}^{2N\times J}$, $\widehat{\mathbf{U}}_n \in \mathbb{C}^{2N\times(2N-J)}$, $\widehat{\mathbf{\Lambda}}_s \in \mathbb{R}^{J\times J}$, and $\widehat{\mathbf{\Lambda}}_n \in \mathbb{R}^{(2N-J)\times(2N-J)}$ are estimates of $\mathbf{U}_s$, $\mathbf{U}_n$, $\mathbf{\Lambda}_s$, and $\mathbf{\Lambda}_n = \sigma_v^2\,\mathbf{I}_{2N-J}$, respectively. As regards $\mathbf{h}_j$, we preliminarily observe that, according to (1), the jth column $\boldsymbol{\phi}_j$ of the matrix $\mathbf{\Phi}$ assumes the form

$$\boldsymbol{\phi}_j = \alpha_j\,\mathbf{C}_j\,\mathbf{g}_j \tag{13}$$

and, consequently, one has

$$\mathbf{h}_j = \begin{bmatrix}\boldsymbol{\phi}_j \\ \boldsymbol{\phi}_j^*\end{bmatrix} = \alpha_j \underbrace{\begin{bmatrix}\mathbf{C}_j & \mathbf{O}_{N\times L_j} \\ \mathbf{O}_{N\times L_j} & \mathbf{C}_j^*\end{bmatrix}}_{\boldsymbol{\mathcal{C}}_j \in \mathbb{C}^{2N\times 2L_j}} \begin{bmatrix}\mathbf{g}_j \\ \mathbf{g}_j^*\end{bmatrix}$$
$$= \underbrace{\alpha_j\sqrt{2}}_{\widetilde{\alpha}_j}\,\boldsymbol{\mathcal{C}}_j\,\underbrace{\frac{1}{\sqrt{2}}\begin{bmatrix}\mathbf{I}_{L_j} & i\,\mathbf{I}_{L_j} \\ \mathbf{I}_{L_j} & -i\,\mathbf{I}_{L_j}\end{bmatrix}}_{\mathbf{T}_j \in \mathbb{C}^{2L_j\times 2L_j}}\,\underbrace{\begin{bmatrix}\mathbf{g}_{j,\mathrm{R}} \\ \mathbf{g}_{j,\mathrm{I}}\end{bmatrix}}_{\boldsymbol{\varrho}_j \in \mathbb{R}^{2L_j}} = \widetilde{\alpha}_j\,\boldsymbol{\mathcal{C}}_j\,\mathbf{T}_j\,\boldsymbol{\varrho}_j \ , \tag{14}$$

where $\mathbf{T}_j$ is a *unitary* matrix, i.e., $\mathbf{T}_j\,\mathbf{T}_j^H = \mathbf{T}_j^H\mathbf{T}_j = \mathbf{I}_{2L_j}$. Assuming that the receiver has the only knowledge of the transmitted signature $\mathbf{c}_j$, the matrix $\boldsymbol{\mathcal{C}}_j$ in (14) is known, whereas, under conditions **(c1)** and **(c2)** and accounting for (14), blind estimation of $\boldsymbol{\varrho}_j$ can be accomplished [16] by exploiting the orthogonality between the signal subspace $\mathcal{R}(\mathbf{H})$ and the noise subspace $\mathcal{R}^\perp(\mathbf{H}) \equiv \mathcal{R}(\mathbf{U}_n)$, that is,

$$\mathbf{U}_n^H\,\mathbf{h}_j = \mathbf{U}_n^H\,\boldsymbol{\mathcal{C}}_j\,\mathbf{T}_j\,\boldsymbol{\varrho}_j = \mathbf{0}_{2N-J} \ , \quad \forall j \in \{1, \ldots., J\} \ . \tag{15}$$

The unknown vector $\boldsymbol{\varrho}_j$ can be obtained as the solution of the linear system (15), provided that this system *uniquely*

[3]The subspace receivers (8) and (11) are mathematically equivalent to the projection method, proposed in [31] to improve the robustness of the L-MV beamformer against ACM estimation and SV perturbation errors.

characterizes the channel coefficients for each user, i.e., an arbitrary unit-norm vector $\mathbf{g}'_j \in \mathbb{C}^{L_j}$ (with corresponding $\boldsymbol{\varrho}'_j \in \mathbb{R}^{2L_j}$), satisfies (15) if and only if (iff) $\mathbf{g}'_j = e^{i\,\psi_j}\,\mathbf{g}_j$, with $\psi_j \in [0, 2\pi)$ and $\forall j \in \{1, 2, \ldots, J\}$. It is clear that (15) has a unique solution (up to a scaling factor) iff the following condition is satisfied: **(c3)** the null space of $\mathbf{U}_n^H\,\boldsymbol{\mathcal{C}}_j\,\mathbf{T}_j$ has dimension one or, equivalently,[4] $\text{rank}(\mathbf{U}_n^H\,\boldsymbol{\mathcal{C}}_j) = 2\,L_j - 1$. A reformulation of condition **(c3)** is given in [24]. It can be readily proven that, under **(c3)**, the following two statements are equivalent: (i) the unit-norm vector $\mathbf{g}'_j \in \mathbb{C}^{L_j}$ is a solution of (15); (ii) $\mathbf{g}'_j = \pm\mathbf{g}_j$, i.e., $\psi_j = n\,\pi$, with $n \in \mathbb{Z}$. In other words, differently from conventional subspace-based multiuser CE [17], where the estimated channel might differ from the true one by an unknown *rotation* $e^{i\,\psi_j}$, in generalized subspace-based CE based on (15) the residual channel ambiguity is limited to a possible *sign inversion*. It is important to observe that condition **(c3)** necessarily imposes that the number of rows of the matrix $\mathbf{U}_n^H\,\boldsymbol{\mathcal{C}}_j\,\mathbf{T}_j \in \mathbb{C}^{(2N-J)\times 2L_j}$ be greater than or equal to its number of columns, i.e., $2N - J \geq 2L_j \iff J \leq 2(N - L_j)$. Thereby, it follows that, from the point of view[5] of the jth user, the maximum number $J_{\text{max,WL}} = 2(N - L_j)$ of users supported by the system is smaller than the maximum number $2\,N$ of users in the known-channel case [1]. We assume hereinafter that condition **(c3)** is satisfied.

In practice, however, (15) cannot be satisfied exactly when $\mathbf{R}_{\mathbf{zz}}$ (and hence $\mathbf{U}_n$) is estimated from a finite sample size. In this case, a channel estimate $\widehat{\boldsymbol{\varrho}}_j \stackrel{\triangle}{=} [\widehat{\mathbf{g}}_{j,\text{R}}^T, (\widehat{\mathbf{g}}_{j,\text{I}}^T]^T$ can still be obtained by solving (15) in the least-squares sense

$$\begin{aligned}\widehat{\boldsymbol{\varrho}}_j &= \underset{\mathbf{x}\in\mathbb{R}^{2L_j}}{\text{argmin}}\,\|\widehat{\mathbf{U}}_n^H\,\boldsymbol{\mathcal{C}}_j\,\mathbf{T}_j\,\mathbf{x}\|^2 = \\ &= \underset{\mathbf{x}\in\mathbb{R}^{2L_j}}{\text{argmin}}\left(\mathbf{x}^H\,\mathbf{T}_j^H\,\boldsymbol{\mathcal{C}}_j^H\,\widehat{\mathbf{U}}_n\,\widehat{\mathbf{U}}_n^H\,\boldsymbol{\mathcal{C}}_j\,\mathbf{T}_j\,\mathbf{x}\right), \\ &\text{subject to } \|\mathbf{x}\|^2 = 1\,, \end{aligned} \tag{16}$$

whose solution [36] is given by the eigenvector associated with the smallest eigenvalue of the matrix $\mathbf{T}_j^H\,\widehat{\boldsymbol{\mathcal{Q}}}_{j,\text{WL}}\,\mathbf{T}_j \in \mathbb{C}^{2L_j\times 2L_j}$, with $\widehat{\boldsymbol{\mathcal{Q}}}_{j,\text{WL}} \stackrel{\triangle}{=} \boldsymbol{\mathcal{C}}_j^H\,\widehat{\mathbf{U}}_n\,\widehat{\mathbf{U}}_n^H\,\boldsymbol{\mathcal{C}}_j \in \mathbb{C}^{2L_j\times 2L_j}$. By substituting in (6) and (8) the sample ACM (12) and its EVD, respectively, as well as the resulting estimate $\widehat{\mathbf{h}}_j = \widetilde{\alpha}_j\,\boldsymbol{\mathcal{C}}_j\,\mathbf{T}_j\,\widehat{\boldsymbol{\varrho}}_j$ of the received signature $\mathbf{h}_j$ [see (14)], the WL-SMI[6] and WL-SUB receivers with CE are given by

$$\mathbf{f}_{j,\text{WL-SMI-CE}} \stackrel{\triangle}{=} (\widehat{\mathbf{h}}_j^H\,\widehat{\mathbf{R}}_{\mathbf{zz}}^{-1}\,\widehat{\mathbf{h}}_j)^{-1}\,\widehat{\mathbf{R}}_{\mathbf{zz}}^{-1}\,\widehat{\mathbf{h}}_j\,, \tag{17}$$

$$\mathbf{f}_{j,\text{WL-SUB-CE}} \stackrel{\triangle}{=} (\widehat{\mathbf{h}}_j^H\,\widehat{\mathbf{U}}_s\,\widehat{\boldsymbol{\Lambda}}_s^{-1}\,\widehat{\mathbf{U}}_s^H\,\widehat{\mathbf{h}}_j)^{-1}\,\widehat{\mathbf{U}}_s\,\widehat{\boldsymbol{\Lambda}}_s^{-1}\,\widehat{\mathbf{U}}_s^H\,\widehat{\mathbf{h}}_j\,. \tag{18}$$

[4]The dimension of the null space of $\mathbf{U}_n^H\,\boldsymbol{\mathcal{C}}_j\,\mathbf{T}_j \in \mathbb{C}^{(2N-J)\times 2L_j}$ is equal to $2\,L_j - \text{rank}(\mathbf{U}_n^H\,\boldsymbol{\mathcal{C}}_j\,\mathbf{T}_j)$. Moreover, since $\mathbf{T}_j$ is unitary and, hence, nonsingular, it results that $\text{rank}(\mathbf{U}_n^H\,\boldsymbol{\mathcal{C}}_j\,\mathbf{T}_j) = \text{rank}(\mathbf{U}_n^H\,\boldsymbol{\mathcal{C}}_j)$.

[5] In order to meaningfully define the maximum number of users that can be supported by the system, we could consider the worst case, i.e., set $L_{\max} \stackrel{\triangle}{=} \max_{1\leq j\leq J} L_j$ as the maximum channel length, obtaining $J \leq 2(N - L_{\max})$.

[6]The subsequent performance analysis could be also extended to WL generalizations of the robust multiuser detectors proposed in [12]–[14], capitalizing on the fact that, as recognized by the same authors, such receivers can be regarded as *diagonally-loaded* versions of the SMI detector.

Note that, while (6) and (8) are equivalent, their estimated counterparts (17) and (18) are different, even when $\widehat{\mathbf{h}}_j = \mathbf{h}_j$. A final remark is in order about knowledge of the real scalar $\widetilde{\alpha}_j$ and of the sign inversion inherent to channel estimate $\widehat{\boldsymbol{\varrho}}_j$, which are needed to correctly build the estimated signature $\widehat{\mathbf{h}}_j$. These parameters cannot be estimated by means of SOS-based blind techniques; in practice, they can be recovered by resorting to automatic gain control and differential modulation or, more robustly, by using a few training symbols. It should be noted, however, that their possible inaccurate knowledge merely introduce a *real* multiplicative factor in the expressions of the WL-SMI-CE and WL-SUB-CE receivers, which does not affect SINR calculation based on (5). Therefore, to simplify matters, we assume that they are both known exactly.

A. Finite-sample SINR analysis

In this section, we provide the finite-sample performance analysis of the WL receivers with CE (17) and (18). To this aim, we generalize the analysis carried out in [1] in the known-channel case, by taking into account also the estimation errors in the received signature $\mathbf{h}_j$. We adopt as in [1] a first-order perturbative approach [30], [37] to model all estimation errors, and derive closed-form expressions of the SINR defined by (5) for the WL-SMI-CE and WL-SUB-CE receivers. In the following, in order to carry out the analysis in an unified framework, we denote with $\widehat{\mathbf{f}}_j$ *any* data-estimated WL-MOE receiver, i.e, $\widehat{\mathbf{f}}_j = \mathbf{f}_{j,\text{WL-SMI-CE}}$ or $\widehat{\mathbf{f}}_j = \mathbf{f}_{j,\text{WL-SUB-CE}}$, and set $\mathbf{f}_j = \mathbf{f}_{j,\text{WL-MOE}}$, where $\mathbf{f}_{j,\text{WL-MOE}}$ is the ideal WL-MOE receiver given by (6) or (8). When $\widehat{\mathbf{f}}_j$ is employed, accounting for (4), it can be shown that (5) yields

$$\text{SINR}(\widehat{\mathbf{f}}_j) = \frac{\text{E}^2\{\text{Re}[\widehat{\mathbf{f}}_j^H\,\mathbf{h}_j]\}}{\text{E}\{\text{Re}^2[\widehat{\mathbf{f}}_j^H\,\mathbf{q}_j(k)]\} + \text{Var}\{\text{Re}[\widehat{\mathbf{f}}_j^H\,\mathbf{h}_j]\}}\,. \tag{19}$$

Since $\widehat{\mathbf{f}}_j$, $\mathbf{h}_j$ and $\mathbf{q}_j(k)$ exhibit the CS property, the real parts in (19) can be omitted, thus yielding

$$\text{SINR}(\widehat{\mathbf{f}}_j) = \frac{\text{E}^2[\widehat{\mathbf{f}}_j^H\,\mathbf{h}_j]}{\text{E}\{|\widehat{\mathbf{f}}_j^H\,\mathbf{q}_j(k)|^2\} + \text{Var}[\widehat{\mathbf{f}}_j^H\,\mathbf{h}_j]}\,. \tag{20}$$

According to the perturbative approach, the vectors $\widehat{\mathbf{f}}_j$ and $\widehat{\mathbf{h}}_j$ are expressed as $\widehat{\mathbf{f}}_j = \mathbf{f}_j + \boldsymbol{\delta}\mathbf{f}_j$ and $\widehat{\mathbf{h}}_j = \mathbf{h}_j + \boldsymbol{\delta}\mathbf{h}_j$, respectively, where $\boldsymbol{\delta}\mathbf{f}_j$ and $\boldsymbol{\delta}\mathbf{h}_j$ are *small* (i.e., $\|\boldsymbol{\delta}\mathbf{f}_j\| \ll 1$ and $\|\boldsymbol{\delta}\mathbf{h}_j\| \ll 1$) and *zero-mean* CS perturbation terms. Thus, we have $\widehat{\mathbf{f}}_j^H\,\mathbf{h}_j = \mathbf{f}_j^H\mathbf{h}_j + \boldsymbol{\delta}\mathbf{f}_j^H\mathbf{h}_j = 1 + \boldsymbol{\delta}\mathbf{f}_j^H\mathbf{h}_j$, since, from (6), $\mathbf{f}_j^H\mathbf{h}_j = 1$. Moreover, denoting with $\text{E}_{\boldsymbol{\delta}\mathbf{f}_j}[\cdot]$ the average with respect to (w.r.t.) $\boldsymbol{\delta}\mathbf{f}_j$, since $\boldsymbol{\delta}\mathbf{f}_j$ is zero-mean and $\boldsymbol{\delta}\mathbf{f}_j^H\mathbf{h}_j$ is a real-valued scalar, it turns out that $\text{E}_{\boldsymbol{\delta}\mathbf{f}_j}[\widehat{\mathbf{f}}_j^H\,\mathbf{h}_j] = 1$ and

$$\begin{aligned}\text{E}_{\boldsymbol{\delta}\mathbf{f}_j}[(\widehat{\mathbf{f}}_j^H\,\mathbf{h}_j)^2] &= \text{E}_{\boldsymbol{\delta}\mathbf{f}_j}[1 + 2\boldsymbol{\delta}\mathbf{f}_j^H\mathbf{h}_j + (\boldsymbol{\delta}\mathbf{f}_j^H\mathbf{h}_j)^2] \\ &= 1 + \text{E}_{\boldsymbol{\delta}\mathbf{f}_j}[|\boldsymbol{\delta}\mathbf{f}_j^H\mathbf{h}_j|^2] = 1 + \text{E}_{\boldsymbol{\delta}\mathbf{f}_j}[\boldsymbol{\delta}\mathbf{f}_j^H\mathbf{h}_j\mathbf{h}_j^H\boldsymbol{\delta}\mathbf{f}_j]\,,\end{aligned} \tag{21}$$

and, therefore, $\mathrm{Var}[\widehat{\mathbf{f}}_j^H \mathbf{h}_j] = \mathrm{E}_{\boldsymbol{\delta}\mathbf{f}_j}[(\widehat{\mathbf{f}}_j^H \mathbf{h}_j)^2] - \mathrm{E}^2_{\boldsymbol{\delta}\mathbf{f}_j}[\widehat{\mathbf{f}}_j^H \mathbf{h}_j] = \mathrm{E}_{\boldsymbol{\delta}\mathbf{f}_j}[\boldsymbol{\delta}\mathbf{f}_j^H \mathbf{h}_j \mathbf{h}_j^H \boldsymbol{\delta}\mathbf{f}_j]$, which substituted in (20) leads to

$$\mathrm{SINR}(\widehat{\mathbf{f}}_j) = \frac{1}{\mathrm{E}_{\widehat{\mathbf{f}}_j \mathbf{q}_j}[\widehat{\mathbf{f}}_j^H \mathbf{q}_j(k)\mathbf{q}_j^H(k)\widehat{\mathbf{f}}_j] + \mathrm{E}_{\boldsymbol{\delta}\mathbf{f}_j}[\boldsymbol{\delta}\mathbf{f}_j^H \mathbf{h}_j \mathbf{h}_j^H \boldsymbol{\delta}\mathbf{f}_j]}, \tag{22}$$

where $\mathrm{E}_{\widehat{\mathbf{f}}_j \mathbf{q}_j}[\cdot]$ denotes joint average w.r.t. $\widehat{\mathbf{f}}_j$ and $\mathbf{q}_j(k)$. Under the simplifying and reasonable assumption [32] that $\widehat{\mathbf{f}}_j$ is independent of $\mathbf{q}_j(k)$,

$$\begin{aligned}\mathrm{E}_{\widehat{\mathbf{f}}_j \mathbf{q}_j}[\widehat{\mathbf{f}}_j^H \mathbf{q}_j(k)\mathbf{q}_j^H(k)\widehat{\mathbf{f}}_j] &= \mathrm{E}_{\widehat{\mathbf{f}}_j}\{\widehat{\mathbf{f}}_j^H \mathrm{E}_{\mathbf{q}_j}[\mathbf{q}_j(k)\mathbf{q}_j^H(k)]\widehat{\mathbf{f}}_j\} \\ &= \mathrm{E}_{\widehat{\mathbf{f}}_j}[\widehat{\mathbf{f}}_j^H \mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}\widehat{\mathbf{f}}_j], \end{aligned} \tag{23}$$

which, accounting for $\widehat{\mathbf{f}}_j = \mathbf{f}_j + \boldsymbol{\delta}\mathbf{f}_j$ and $\mathrm{E}[\boldsymbol{\delta}\mathbf{f}_j] = 0$ leads to $\mathrm{E}_{\widehat{\mathbf{f}}_j}[\widehat{\mathbf{f}}_j^H \mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}\widehat{\mathbf{f}}_j] = \mathbf{f}_j^H \mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}\mathbf{f}_j + \mathrm{E}_{\boldsymbol{\delta}\mathbf{f}_j}[\boldsymbol{\delta}\mathbf{f}_j^H \mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}\boldsymbol{\delta}\mathbf{f}_j]$. By substituting such a result into (22), one obtains (24) shown at the top of the next page. Since $\mathrm{E}_{\boldsymbol{\delta}\mathbf{f}_j}[\boldsymbol{\delta}\mathbf{f}_j^H \mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}\boldsymbol{\delta}\mathbf{f}_j] + \mathrm{E}_{\boldsymbol{\delta}\mathbf{f}_j}[\boldsymbol{\delta}\mathbf{f}_j^H \mathbf{h}_j\mathbf{h}_j^H \boldsymbol{\delta}\mathbf{f}_j] = \mathrm{E}_{\boldsymbol{\delta}\mathbf{f}_j}[\boldsymbol{\delta}\mathbf{f}_j^H \mathbf{R}_{\mathbf{zz}}\boldsymbol{\delta}\mathbf{f}_j]$, noting also that, according to (7), $\mathbf{f}_j^H \mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}\mathbf{f}_j = (\mathrm{SINR}_{j,\text{WL-MOE}})^{-1}$ we obtain the compact expression

$$\mathrm{SINR}(\widehat{\mathbf{f}}_j) = \frac{\mathrm{SINR}_{j,\text{WL-MOE}}}{1 + \mathrm{SINR}_{j,\text{WL-MOE}}\,\mathrm{E}_{\boldsymbol{\delta}\mathbf{f}_j}[\boldsymbol{\delta}\mathbf{f}_j^H\, \mathbf{R}_{\mathbf{zz}}\, \boldsymbol{\delta}\mathbf{f}_j]}, \tag{25}$$

where only the average w.r.t. $\boldsymbol{\delta}\mathbf{f}_j$ is left to be evaluated. To proceed further, explicit expressions for the perturbation $\boldsymbol{\delta}\mathbf{f}_j$ of the WL-SMI-CE and WL-SUB-CE receivers are needed.

Lemma 1: Let $\approx$ denote *first-order equality*,[7] the first-order perturbation term of the WL-SMI-CE and WL-SUB-CE receivers can be expressed as

$$\boldsymbol{\delta}\mathbf{f}_j = \boldsymbol{\delta}\mathbf{f}_j^{(1)} + \boldsymbol{\delta}\mathbf{f}_j^{(2)}, \tag{26}$$

with

$$\boldsymbol{\delta}\mathbf{f}_j^{(1)} \approx -\boldsymbol{\Gamma}_{j,\mathrm{WL}}\,\widehat{\mathbf{r}}_{\mathbf{q}_j b_j}, \tag{27}$$

$$\boldsymbol{\delta}\mathbf{f}_j^{(2)} \approx \boldsymbol{\Delta}_{j,\mathrm{WL}}\boldsymbol{\delta}\mathbf{h}_j, \tag{28}$$

where $\widehat{\mathbf{r}}_{\mathbf{q}_j b_j} \triangleq \frac{1}{K}\sum_{k=0}^{K-1}\mathbf{q}_j(k)\,b_j(k) \in \mathbb{C}^{2N}$ is the sample estimate of the cross-correlation between the disturbance vector $\mathbf{q}_j(k)$ and the desired symbol $b_j(k)$, $\boldsymbol{\Gamma}_{j,\mathrm{WL}}$ and $\boldsymbol{\Delta}_{j,\mathrm{WL}}$ are given by (29) and (30) shown at the top of the next page, with $\mathbf{P}_{j,\mathrm{WL}} \triangleq \mathbf{I}_{2N} - (\mathbf{h}_j^H\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}^{-1}\,\mathbf{h}_j)^{-1}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}^{-1}\,\mathbf{h}_j\,\mathbf{h}_j^H = \mathbf{I}_{2N} - \mathbf{f}_j\,\mathbf{h}_j^H \in \mathbb{C}^{2N\times 2N}$ denoting an oblique projection matrix [32] and $\gamma_{j,\mathrm{WL}} \triangleq \sigma_v^{-2} + (\mathbf{h}_j^H\,\mathbf{R}_{\mathbf{zz}}^{-1}\,\mathbf{h}_j)^{-1}\mathbf{h}_j^H\mathbf{U}_s\,\boldsymbol{\Omega}_{\mathrm{WL}}^{-1}\,\mathbf{U}_s^H\mathbf{R}_{\mathbf{zz}}^{-1}\,\mathbf{h}_j > 0$, while the diagonal matrix $\boldsymbol{\Omega}_{\mathrm{WL}} \triangleq \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_J) \in \mathbb{R}^{J\times J}$ collects the nonzero eigenvalues of $\mathbf{H}\,\mathbf{H}^H$.

Proof: See Appendix A. ■

It should be noted that $\boldsymbol{\delta}\mathbf{f}_j^{(1)}$ and $\boldsymbol{\delta}\mathbf{f}_j^{(2)}$ represent the perturbations due to estimation of $\mathbf{R}_{\mathbf{zz}}$ and $\mathbf{h}_j$, respectively; indeed, a comparison shows that the expression of $\boldsymbol{\delta}\mathbf{f}_j^{(1)}$ is the same as that reported in [1, Lemma 2]. In order to characterize the perturbation term $\boldsymbol{\delta}\mathbf{f}_j^{(2)}$, it is necessary to evaluate the perturbation $\boldsymbol{\delta}\mathbf{h}_j$ associated with the subspace-based CE procedure given by (16).

Lemma 2: Given the estimate $\widehat{\mathbf{h}}_j = \widetilde{\alpha}_j\,\boldsymbol{\mathcal{C}}_j\,\mathbf{T}_j\,\widehat{\boldsymbol{\varrho}}_j$ of the signature $\mathbf{h}_j$, where the channel estimate $\widehat{\boldsymbol{\varrho}}_j$ is the solution of (16), the perturbation $\boldsymbol{\delta}\mathbf{h}_j$ can be expressed as

$$\boldsymbol{\delta}\mathbf{h}_j \approx \boldsymbol{\Pi}_{j,\mathrm{WL}}\,\widehat{\mathbf{r}}_{\mathbf{q}_j b_j}, \tag{31}$$

where $\boldsymbol{\Pi}_{j,\mathrm{WL}} \triangleq (\mathbf{h}_j^H\,\mathbf{U}_s\,\boldsymbol{\Omega}_{\mathrm{WL}}^{-1}\,\mathbf{U}_s^H\,\mathbf{h}_j)\,\boldsymbol{\mathcal{C}}_j\,\boldsymbol{\mathcal{Q}}_{j,\mathrm{WL}}^{\dagger}\,\boldsymbol{\mathcal{C}}_j^H\,\mathbf{U}_n\,\mathbf{U}_n^H \in \mathbb{C}^{2N\times 2N}$, with $\boldsymbol{\Omega}_{\mathrm{WL}}$ and $\widehat{\mathbf{r}}_{\mathbf{q}_j b_j}$ defined in Lemma 1, and $\boldsymbol{\mathcal{Q}}_{j,\mathrm{WL}} \triangleq \boldsymbol{\mathcal{C}}_j^H\,\mathbf{U}_n\,\mathbf{U}_n^H\,\boldsymbol{\mathcal{C}}_j \in \mathbb{C}^{2L_j\times 2L_j}$.

Proof: See Appendix B. ■

Accounting for (31) and Lemma 1, the overall perturbation of the WL-SMI-CE and WL-SUB-CE weight vectors can be expressed as a *linear* function of $\widehat{\mathbf{r}}_{\mathbf{q}_j b_j}$, as summarized by the following Lemma:

Lemma 3: The first-order overall perturbation term $\boldsymbol{\delta}\mathbf{f}_j = \boldsymbol{\delta}\mathbf{f}_j^{(1)} + \boldsymbol{\delta}\mathbf{f}_j^{(2)}$ of the WL-SMI-CE and WL-SUB-CE receivers can be expressed in a unified manner as

$$\boldsymbol{\delta}\mathbf{f}_j \approx \boldsymbol{\Sigma}_{j,\mathrm{WL}}\,\widehat{\mathbf{r}}_{\mathbf{q}_j b_j}, \tag{32}$$

where $\boldsymbol{\Sigma}_{j,\mathrm{WL}} \triangleq -\boldsymbol{\Gamma}_{j,\mathrm{WL}} + \boldsymbol{\Delta}_{j,\mathrm{WL}}\,\boldsymbol{\Pi}_{j,\mathrm{WL}} \in \mathbb{C}^{2N\times 2N}$, with $\boldsymbol{\Gamma}_{j,\mathrm{WL}} \in \mathbb{C}^{2N\times 2N}$, $\boldsymbol{\Delta}_{j,\mathrm{WL}} \in \mathbb{C}^{2N\times 2N}$ and $\widehat{\mathbf{r}}_{\mathbf{q}_j b_j}$ given by Lemma 1, whereas $\boldsymbol{\Pi}_{j,\mathrm{WL}} \in \mathbb{C}^{2N\times 2N}$ is defined in Lemma 2.

It should be observed that Lemma 3 provides a compact characterization of the overall perturbation $\boldsymbol{\delta}\mathbf{f}_j$, which is obtained under the simplifying assumption [32] that the error in estimating $\mathbf{R}_{\mathbf{zz}}$ is mainly due to the term $\widehat{\mathbf{r}}_{\mathbf{q}_j b_j}$. Equipped with such a nice result, we are now in the position to evaluate the average $\mathrm{E}_{\boldsymbol{\delta}\mathbf{f}_j}[\boldsymbol{\delta}\mathbf{f}_j^H \mathbf{R}_{\mathbf{zz}}\,\boldsymbol{\delta}\mathbf{f}_j]$ at the denominator of (25). Dropping the subscript $\boldsymbol{\delta}\mathbf{f}_j$ in $\mathrm{E}_{\boldsymbol{\delta}\mathbf{f}_j}[\cdot]$ for notational simplicity, by accounting for (32) and using the trace identity, we have

$$\begin{aligned}\mathrm{E}[\boldsymbol{\delta}\mathbf{f}_j^H \mathbf{R}_{\mathbf{zz}}\,\boldsymbol{\delta}\mathbf{f}_j] &= \mathrm{E}[\widehat{\mathbf{r}}_{\mathbf{q}_j b_j}^H\,\boldsymbol{\Sigma}_{j,\mathrm{WL}}^H\,\mathbf{R}_{\mathbf{zz}}\,\boldsymbol{\Sigma}_{j,\mathrm{WL}}\,\widehat{\mathbf{r}}_{\mathbf{q}_j b_j}] \\ &= \mathrm{trace}\left\{\boldsymbol{\Sigma}_{j,\mathrm{WL}}^H\,\mathbf{R}_{\mathbf{zz}}\,\boldsymbol{\Sigma}_{j,\mathrm{WL}}\,\mathrm{E}[\widehat{\mathbf{r}}_{\mathbf{q}_j b_j}\,\widehat{\mathbf{r}}_{\mathbf{q}_j b_j}^H]\right\}, \end{aligned} \tag{33}$$

where, moreover, by virtue of assumptions **(a1)** and **(a2)**, it can be shown (see [1] for details) that $\mathrm{E}[\widehat{\mathbf{r}}_{\mathbf{q}_j b_j}\,\widehat{\mathbf{r}}_{\mathbf{q}_j b_j}^H] = \frac{1}{K}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}$. Therefore, by substituting such relation in (33), and the result back in (25), we get

$$\mathrm{SINR}(\widehat{\mathbf{f}}_j) = \frac{\mathrm{SINR}_{j,\text{WL-MOE}}}{1 + \frac{\mathrm{trace}(\boldsymbol{\Sigma}_{j,\mathrm{WL}}^H\,\mathbf{R}_{\mathbf{zz}}\,\boldsymbol{\Sigma}_{j,\mathrm{WL}}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j})}{K}\,\mathrm{SINR}_{j,\text{WL-MOE}}}. \tag{34}$$

The final result is obtained by evaluating the trace term in (34), on the basis of the different expressions for $\boldsymbol{\Sigma}_{j,\mathrm{WL}}$ given by Lemmas 1–3. In order to do this, it is convenient to consider the SMI and SUB cases separately. With reference to the WL-SMI-CE receiver, it is shown in Appendix C that

$$\begin{aligned}\mathrm{trace}(\boldsymbol{\Sigma}_{j,\mathrm{WL}}^H\,\mathbf{R}_{\mathbf{zz}}\,\boldsymbol{\Sigma}_{j,\mathrm{WL}}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}) &= (2\,N-1) - \\ 2\,\zeta_{j,\mathrm{WL}}(2\,L_j - 1) &+ \zeta_{j,\mathrm{WL}}^2\,\sigma_v^2\,\mathrm{trace}(\mathbf{R}_{\mathbf{zz}}^{-1}\,\boldsymbol{\mathcal{C}}_j\,\boldsymbol{\mathcal{Q}}_{j,\mathrm{WL}}^{\dagger}\,\boldsymbol{\mathcal{C}}_j^H), \end{aligned} \tag{35}$$

[7] First-order equality means that, as the sample size K approaches infinity, we neglect all the summands that tend to zero faster than the norm of the corresponding perturbation term.

$$\mathrm{SINR}(\widehat{\mathbf{f}}_j) = \frac{1}{\mathbf{f}_j^H \mathbf{R}_{\mathbf{q}_j\mathbf{q}_j} \mathbf{f}_j + \mathrm{E}_{\boldsymbol{\delta}\mathbf{f}_j}[\boldsymbol{\delta}\mathbf{f}_j^H \mathbf{R}_{\mathbf{q}_j\mathbf{q}_j} \boldsymbol{\delta}\mathbf{f}_j] + \mathrm{E}_{\boldsymbol{\delta}\mathbf{f}_j}[\boldsymbol{\delta}\mathbf{f}_j^H \mathbf{h}_j \mathbf{h}_j^H \boldsymbol{\delta}\mathbf{f}_j]} \tag{24}$$

$$\boldsymbol{\Gamma}_{j,\mathrm{WL}} \triangleq \begin{cases} \mathbf{P}_{j,\mathrm{WL}}\, \mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}^{-1}\,, & \text{(WL-SMI-CE)} \\ \mathbf{P}_{j,\mathrm{WL}}\, \mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}^{-1} - \gamma_{j,\mathrm{WL}}\, \mathbf{U}_n \mathbf{U}_n^H\,, & \text{(WL-SUB-CE)} \end{cases} \tag{29}$$

$$\boldsymbol{\Delta}_{j,\mathrm{WL}} \triangleq \begin{cases} (\mathbf{h}_j^H \mathbf{R}_{\mathbf{zz}}^{-1} \mathbf{h}_j)^{-1} \mathbf{R}_{\mathbf{zz}}^{-1} - 2\, \mathbf{f}_j\, \mathbf{f}_j^H\,, & \text{(WL-SMI-CE)} \\ (\mathbf{h}_j^H \mathbf{R}_{\mathbf{zz}}^{-1} \mathbf{h}_j)^{-1} \mathbf{U}_s \boldsymbol{\Lambda}_s^{-1} \mathbf{U}_s^H - 2\, \mathbf{f}_j\, \mathbf{f}_j^H\,, & \text{(WL-SUB-CE)} \end{cases} \tag{30}$$

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where $\zeta_{j,\mathrm{WL}} \triangleq (\mathbf{h}_j^H \mathbf{R}_{\mathbf{zz}}^{-1} \mathbf{h}_j)^{-1}\, \mathbf{h}_j^H\, \mathbf{U}_s\, \boldsymbol{\Omega}_{\mathrm{WL}}^{-1}\, \mathbf{U}_s^H\, \mathbf{h}_j > 0$. Instead, as regards the WL-SUB-CE receiver, it is shown in Appendix C that

$$\begin{aligned} \mathrm{trace}(\boldsymbol{\Sigma}_{j,\mathrm{WL}}^H\, \mathbf{R}_{\mathbf{zz}}\, \boldsymbol{\Sigma}_{j,\mathrm{WL}}\, \mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}) &= (J-1) \\ &+ (2\,N - J)|1 - \gamma_{j,\mathrm{WL}}\, \sigma_v^2|^2 - \zeta_{j,\mathrm{WL}}^2 (2\,L_j - 1) \\ &+ \zeta_{j,\mathrm{WL}}^2\, \sigma_v^2\, \mathrm{trace}(\mathbf{R}_{\mathbf{zz}}^{-1}\, \boldsymbol{\mathcal{C}}_j\, \boldsymbol{\mathcal{Q}}_{j,\mathrm{WL}}^\dagger\, \boldsymbol{\mathcal{C}}_j^H)\,. \end{aligned} \tag{36}$$

The trace expressions (35) and (36) are still too complicated to allow for a simple discussion, but they can be considerably simplified in the high-SNR region, i.e., by studying their behavior as $\sigma_v^2 \to 0$. Let us first examine the trace term, which is present in both (35) and (36). One has

$$\begin{aligned} &\sigma_v^2\, \mathrm{trace}(\mathbf{R}_{\mathbf{zz}}^{-1}\, \boldsymbol{\mathcal{C}}_j\, \boldsymbol{\mathcal{Q}}_{j,\mathrm{WL}}^\dagger\, \boldsymbol{\mathcal{C}}_j^H) \\ &= \sigma_v^2\, \mathrm{trace}\left[\left(\mathbf{U}_s \boldsymbol{\Lambda}_s^{-1} \mathbf{U}_s^H + \sigma_v^{-2} \mathbf{U}_n \mathbf{U}_n^H\right) \boldsymbol{\mathcal{C}}_j\, \boldsymbol{\mathcal{Q}}_{j,\mathrm{WL}}^\dagger\, \boldsymbol{\mathcal{C}}_j^H\right] \\ &= \sigma_v^2\, \mathrm{trace}(\mathbf{U}_s \boldsymbol{\Lambda}_s^{-1} \mathbf{U}_s^H\, \boldsymbol{\mathcal{C}}_j\, \boldsymbol{\mathcal{Q}}_{j,\mathrm{WL}}^\dagger\, \boldsymbol{\mathcal{C}}_j^H) \\ &\quad + \mathrm{trace}(\mathbf{U}_n \mathbf{U}_n^H\, \boldsymbol{\mathcal{C}}_j\, \boldsymbol{\mathcal{Q}}_{j,\mathrm{WL}}^\dagger\, \boldsymbol{\mathcal{C}}_j^H)\,. \end{aligned} \tag{37}$$

Therefore, for $\sigma_v^2 \to 0$, observing that $\boldsymbol{\Lambda}_s^{-1} \to \boldsymbol{\Omega}_{\mathrm{WL}}^{-1}$ and using also the trace properties, one has

$$\begin{aligned} &\lim_{\sigma_v^2 \to 0} \sigma_v^2\, \mathrm{trace}(\mathbf{R}_{\mathbf{zz}}^{-1}\, \boldsymbol{\mathcal{C}}_j\, \boldsymbol{\mathcal{Q}}_{j,\mathrm{WL}}^\dagger\, \boldsymbol{\mathcal{C}}_j^H) \\ &\qquad \mathrm{trace}(\boldsymbol{\mathcal{Q}}_{j,\mathrm{WL}}^\dagger \underbrace{\boldsymbol{\mathcal{C}}_j^H\, \mathbf{U}_n\, \mathbf{U}_n^H\, \boldsymbol{\mathcal{C}}_j}_{\boldsymbol{\mathcal{Q}}_{j,\mathrm{WL}}}) = 2L_j - 1\,, \end{aligned} \tag{38}$$

where we refer to Appendix C for a formal proof of the result $\mathrm{trace}(\boldsymbol{\mathcal{Q}}_{j,\mathrm{WL}}^\dagger \boldsymbol{\mathcal{Q}}_{j,\mathrm{WL}}) = 2L_j - 1$. In addition, as $\sigma_v^2 \to 0$, it can be easily checked that $\gamma_{j,\mathrm{WL}}\, \sigma_v^2 \to 1$ and $\zeta_{j,\mathrm{WL}} \to 1$. Consequently, accounting for (34)–(36) and (38), the SINR behavior in the high-SNR region of the WL-SMI-CE and WL-SUB-CE receivers is (approximately) governed by

$$\begin{aligned} \mathrm{SINR}_{j,\text{WL-SMI-CE}} &\triangleq \mathrm{SINR}(\mathbf{f}_{j,\text{WL-SMI-CE}}) \\ &= \frac{\mathrm{SINR}_{j,\text{WL-MOE}}}{1 + \frac{2(N-L_j)}{K}\, \mathrm{SINR}_{j,\text{WL-MOE}}}\,, \end{aligned} \tag{39}$$

$$\begin{aligned} \mathrm{SINR}_{j,\text{WL-SUB-CE}} &\triangleq \mathrm{SINR}(\mathbf{f}_{j,\text{WL-SUB-CE}}) \\ &= \frac{\mathrm{SINR}_{j,\text{WL-MOE}}}{1 + \frac{J-1}{K}\, \mathrm{SINR}_{j,\text{WL-MOE}}}\,, \end{aligned} \tag{40}$$

which are directly comparable[8] to [1, eq. (44)] and [1, eq. (47)]. Our simulation results show that (39) and (40) accurately predict the SINR performances of the WL-SMI-CE and WL-SUB-CE receivers not only in the high-SNR regime, but also for moderate values of the SNR, wherein many systems of practical interest are envisioned to operate. A first exam of the obtained expression shows that, for $K \to +\infty$, both receivers attain the maximum SINR equal to $\mathrm{SINR}_{j,\text{WL-MOE}}$. A more interesting comparison is between (39)–(40) and the corresponding ones derived in [1] in the known-channel case. For the WL-SUB receiver, such a comparison shows that the SINR when the channel is estimated is the same as that obtained when the channel is known, namely, for moderate-to-high values of the SNR, the WL-SUB-CE receiver (approximately) pays no penalty w.r.t. its counterpart employing the exact channel. Such a result indirectly shows the reliability of the considered subspace-based CE procedure, which simultaneously exploits the channel information contained in both $\mathbf{R}_{\mathbf{rr}}$ and $\mathbf{R}_{\mathbf{rr}^*}$ by jointly processing the received vector $\mathbf{r}(k)$ and its conjugate version $\mathbf{r}^*(k)$. Surprisingly enough, the SINR of the WL-SMI-CE turns out to be even better than that of the corresponding WL-SMI receiver with known channel: as a matter of fact, this phenomenon is well-known in the array processing literature (see e.g. [31], [34]), where it is sometimes referred to as *signature mismatch*, and its effects vanish only when $K \to +\infty$. As a by-product, eqs. (39) and (40), together with their corresponding ones derived in [1] in the known-channel case, provide the SINR assessment of the signature mismatch problem, thereby showing the simplicity and insightfulness of our SINR formulas. For a finite sample-size K, indeed, accounting for [1, eq. (44)] and (39), the SINR degradation due to signature mismatch in the high-SNR region is given by

$$\lim_{\sigma_v^2 \to 0} \frac{\mathrm{SINR}_{j,\text{WL-SMI}}}{\mathrm{SINR}_{j,\text{WL-SMI-CE}}} = \frac{2(N - L_j)}{2\,N - 1} < 1\,, \tag{41}$$

which increases with the channel length. Another interesting conclusion that can be drawn from (39) and (40) is that, not differently from the case [1] where the channel is known, the data-estimated receivers exhibit a SINR saturation effect, for vanishingly small noise. Indeed, when $\sigma_v^2 \to 0$ and $\mathbf{H}$ is full-column rank ($J \leq 2\,N$), it has been shown in [1,

[8] Eq. (40), as well as the subsequent (50), is similar to [34, eq. (38)], which was derived for the L-MV projection beamformer by considering however only the effects of ACM estimation.

Subsection IV-A] that $\text{SINR}_{j,\text{WL-MOE}}$ grows without bound. Thus, for $J \leq 2(N - L_j) < 2N$, accounting for (39) and (40), we get

$$\lim_{\sigma_v^2 \to 0} \text{SINR}_{j,\text{WL-SMI-CE}} = \frac{K}{2(N - L_j)} \, , \tag{42}$$

$$\lim_{\sigma_v^2 \to 0} \text{SINR}_{j,\text{WL-SUB-CE}} = \frac{K}{J - 1} \, , \tag{43}$$

which show that, in the high-SNR regime, the performance of the WL-SMI-CE receiver does not depend on the number of users J, but it depends on the processing gain N as well as on the channel length L_j of user j, whereas the performance of the WL-SUB-CE receiver is independent of both the processing gain N and the channel length L_j, while depending on the number of users J.

IV. Comparison between L- and WL-MOE receivers with channel estimation

As done in [1] in the case of known channel, it is interesting to compare the SINR performances of the data-estimated WL-MOE receivers with CE based on (16) against the data-estimated L-MOE receivers with CE based on the algorithm of [17]. Similarly to the WL-MOE one, the synthesis of the L-MOE receiver given by (9) or (11) involves estimation from the received data of $\mathbf{R_{rr}}$ in (9) or its EVD in (8), as well as an accurate estimate of $\boldsymbol{\phi}_j$ in both cases. Under mild conditions, a consistent estimate of $\mathbf{R_{rr}}$ is given by

$$\widehat{\mathbf{R}}_{\mathbf{rr}} = \frac{1}{K} \sum_{k=1}^{K} \mathbf{r}(k) \, \mathbf{r}^H(k) \, . \tag{44}$$

Let $\mathbf{V}_n \in \mathbb{C}^{N \times (N-J)}$ collect the eigenvectors associated with the eigenvalue σ_v^2 of $\mathbf{R_{rr}}$, under condition **(c1)**, channel estimation can be blindly carried out [17] by exploiting the orthogonality between the signal subspace $\mathcal{R}(\boldsymbol{\Phi})$ and the noise subspace $\mathcal{R}^{\perp}(\boldsymbol{\Phi}) \equiv \mathcal{R}(\mathbf{V}_n)$, thus obtaining

$$\mathbf{V}_n^H \, \boldsymbol{\phi}_j = \mathbf{V}_n^H \, \mathbf{C}_j \, \mathbf{g}_j = \mathbf{0}_{N-J} \, , \quad \forall j \in \{1, ..., J\} \, , \tag{45}$$

where we have also used (13). In this case, (45) uniquely characterizes the channel coefficients for each user iff the following condition holds: **(c4)** the null space of $\mathbf{V}_n^H \, \mathbf{C}_j$ has dimension one or, equivalently,[9] $\text{rank}(\mathbf{V}_n^H \, \mathbf{C}_j) = L_j - 1$. A discussion about condition **(c4)** is made in [17]. If condition **(c4)** is satisfied, then an arbitrary unit-norm vector $\mathbf{g}'_j \in \mathbb{C}^{L_j}$ satisfies (45) iff $\mathbf{g}'_j = e^{i \, \vartheta_j} \, \mathbf{g}_j$, with $\vartheta_j \in [0, 2\pi)$ and $\forall j \in \{1, 2, \ldots, J\}$. It is noteworthy that fulfillment of condition **(c4)** requires that the number of rows of the matrix $\mathbf{V}_n^H \, \mathbf{C}_j \in \mathbb{C}^{(N-J) \times L_j}$ be greater than or equal to its number of columns, i.e., $N - J \geq L_j \iff J \leq N - L_j$, and, hence, from the point of view[10] of the jth user, the maximum number $J_{\text{max,L}} = N - L_j$ of users that can be supported by the system is smaller than the number N of users when the channel is assumed to be perfectly known. Observe that the maximum number of allowable users for the linear case is exactly one-half of the corresponding number for the WL case. In practice, when $J \leq N - L_j$ both blind L and WL receivers can be utilized, whereas for $N - L_j < J \leq 2(N - L_j)$ only the blind WL receivers can work (note that the above limitations are mainly due to the considered blind channel identification procedure). Hereinafter, we assume that condition **(c4)** is satisfied. When $\mathbf{R_{rr}}$ (and hence $\mathbf{V}_n$) is estimated from a finite sample size, a channel estimate $\widehat{\mathbf{g}}_j$ can be obtained by solving (45) in the least-squares sense:

$$\begin{aligned} \widehat{\mathbf{g}}_j &= \underset{\mathbf{x} \in \mathbb{C}^{L_j}}{\text{argmin}} \, \| \widehat{\mathbf{V}}_n^H \, \mathbf{C}_j \, \mathbf{x} \|^2 \\ &= \underset{\mathbf{x} \in \mathbb{C}^{L_j}}{\text{argmin}} \left(\mathbf{x}^H \, \mathbf{C}_j^H \, \widehat{\mathbf{V}}_n \, \widehat{\mathbf{V}}_n^H \, \mathbf{C}_j \, \mathbf{x} \right) \, , \text{ subject to } \|\mathbf{x}\|^2 = 1 \, , \end{aligned} \tag{46}$$

where the matrix $\widehat{\mathbf{V}}_n \in \mathbb{C}^{N \times (N-J)}$ is the sample estimate of $\mathbf{V}_n$. The solution [36] of (46) is the eigenvector associated with the smallest eigenvalue of the matrix $\widehat{\boldsymbol{\mathcal{Q}}}_{j,\text{L}} \triangleq \mathbf{C}_j^H \, \widehat{\mathbf{V}}_n \, \widehat{\mathbf{V}}_n^H \, \mathbf{C}_j \in \mathbb{C}^{L_j \times L_j}$. By substituting in (9) and (11) the sample ACM (44) and its EVD, respectively, as well as the resulting estimate $\widehat{\boldsymbol{\phi}}_j = \alpha_j \, \mathbf{C}_j \, \widehat{\mathbf{g}}_j$ of the signature $\boldsymbol{\phi}_j$ [see (13)] in both, we obtain the L-SMI-CE and L-SUB-CE receivers defined as

$$\mathbf{w}_{j,\text{L-SMI-CE}} \triangleq (\widehat{\boldsymbol{\phi}}_j^H \, \widehat{\mathbf{R}}_{\mathbf{rr}}^{-1} \, \widehat{\boldsymbol{\phi}}_j)^{-1} \, \widehat{\mathbf{R}}_{\mathbf{rr}}^{-1} \, \widehat{\boldsymbol{\phi}}_j \, , \tag{47}$$

$$\mathbf{w}_{j,\text{L-SUB-CE}} \triangleq (\widehat{\boldsymbol{\phi}}_j^H \, \widehat{\mathbf{V}}_s \, \widehat{\boldsymbol{\Upsilon}}_s^{-1} \, \widehat{\mathbf{V}}_s^H \, \widehat{\boldsymbol{\phi}}_j)^{-1} \, \widehat{\mathbf{V}}_s \, \widehat{\boldsymbol{\Upsilon}}_s^{-1} \, \widehat{\mathbf{V}}_s^H \, \widehat{\boldsymbol{\phi}}_j \, . \tag{48}$$

As for WL receivers, while (9) and (11) are perfectly equivalent, their estimated counterparts (47) and (48) are different, even when $\widehat{\boldsymbol{\phi}}_j = \boldsymbol{\phi}_j$. The performance analysis of the L-SMI-CE and L-SUB-CE receivers is complicated from the fact that the SINR (19) must again be evaluated but, differently from the WL ones, linear receivers do not exhibit the CS property, since $\mathbf{f}_{j,2} = \mathbf{0}_N$ in (2). Such an analysis is similar in principle to the one carried out in [28]–[30], but the approach adopted here leads to more easily interpretable results, which are directly comparable with those obtained in the WL case, at the cost of a minimal loss in accuracy. To avoid a burdensome treatment, we will report only the final results and defer their synthetic proofs to Appendix D. By assuming that both α_j and ϑ_j are known exactly, it turns out that, in the high-SNR regime, the output SINR of the L-SMI-CE and L-SUB-CE receivers can be approximately written as

$$\begin{aligned} \text{SINR}_{j,\text{L-SMI-CE}} &\triangleq \text{SINR}(\mathbf{w}_{j,\text{L-SMI-CE}}) \\ &= \frac{\text{SINR}_{j,\text{L-MOE}}}{1 + \frac{N+J-L_j-1}{2K} \, \text{SINR}_{j,\text{L-MOE}}} \, , \end{aligned} \tag{49}$$

$$\begin{aligned} \text{SINR}_{j,\text{L-SUB-CE}} &\triangleq \text{SINR}(\mathbf{w}_{j,\text{L-SUB-CE}}) \\ &= \frac{\text{SINR}_{j,\text{L-MOE}}}{1 + \frac{J-1}{K} \, \text{SINR}_{j,\text{L-MOE}}} \, . \end{aligned} \tag{50}$$

Our simulation results show that the SINR performances of the L-SMI-CE and L-SUB-CE receivers are accurately described by (49) and (50) even when the SNR assumes moderate values. Due to the similarity between the SINR expressions obtained

[9] The dimension of the null space of $\mathbf{V}_n^H \, \mathbf{C}_j \in \mathbb{C}^{(N-J) \times L_j}$ is equal to $L_j - \text{rank}(\mathbf{V}_n^H \mathbf{C}_j)$.

[10] Following footnote 5, the maximum number of users that can be supported by the system is given by $J \leq (N - L_{\text{max}})$.

for L and WL receivers, most observations of Subsection III-A regarding the comparison between receivers with or without CE apply also in this case. Summarizing, the SINR of the L-SUB-CE receiver turns out to be (approximately) equal to that of the L-SUB one. Moreover, due to the mentioned signature mismatch problem [31], [34], the SINR of the L-SMI receiver with known channel is worse than that of the corresponding L-SMI-CE receiver: indeed, for a finite sample size K, in the high-SNR regime, it results that

$$\lim_{\sigma_v^2 \to 0} \frac{\text{SINR}_{j,\text{L-SMI}}}{\text{SINR}_{j,\text{L-SMI-CE}}} = \frac{N+J-L_j-1}{N+J-2} < 1 \, . \tag{51}$$

Additionally, similarly to the WL case, the data-estimated linear receivers exhibit a SINR saturation effect, for $\sigma_v^2 \to 0$. In this case, if $\mathbf{\Phi}$ is full-column rank ($J \leq N$), it is readily verified that $\text{SINR}_{j,\text{L-MOE}} \to +\infty$. Henceforth, for $J \leq N - L_j < N$, accounting for (49) and (50), one obtains

$$\lim_{\sigma_v^2 \to 0} \text{SINR}_{j,\text{L-SMI-CE}} = \frac{2K}{N+J-L_j-1} \, , \tag{52}$$

$$\lim_{\sigma_v^2 \to 0} \text{SINR}_{j,\text{L-SUB-CE}} = \frac{K}{J-1} \, , \tag{53}$$

which show that, at high SNR, the performance of the L-SMI-CE receiver depends on the processing gain N and the number of users J, as well as on the channel length L_j of the jth user, whereas the performance of the L-SUB-CE receiver is independent of both the processing gain N and the channel length L_j, while depending on the number of users J.

At this point, we are able to establish a direct comparison between L- and WL-MOE receivers with CE, focusing our attention to the case $J \leq J_{\text{max,L}} = N - L_j$, wherein both L- and WL-MOE receivers with CE can work [note indeed that the WL-MOE with CE can accommodate up to $J_{\text{max,WL}} = 2(N - L_j)$ users]. By comparing (40) and (50) for the subspace-based receivers, it turns out that $\text{SINR}_{j,\text{WL-SUB-CE}} \geq \text{SINR}_{j,\text{L-SUB-CE}}$ for any value of K. Instead, for the SMI-based receivers [see (39) and (49)], it results that $\text{SINR}_{j,\text{WL-SMI-CE}} \geq \text{SINR}_{j,\text{L-SMI-CE}}$ only when $K \geq K_{\text{min}}$, where

$$K_{\text{min}} \triangleq \frac{3(N-L_j)-J+1}{2(\text{SINR}_{j,\text{L-MOE}}^{-1} - \text{SINR}_{j,\text{WL-MOE}}^{-1})} \tag{54}$$

is a threshold sample size, that is, similarly to the known channel case described in [1], the WL-SMI-CE receiver assures a performance advantage only by processing a sufficient number of samples.[11] Finally, for $J \leq N - L_j$, as regards the comparison between the saturation SINRs (i.e., the SINRs for $\sigma_v^2 \to 0$) of the L- and WL-MOE receivers with CE, it can be observed from (42)–(43) and (52)–(53) that the value for the L-SMI-CE receiver is better than the corresponding value for WL-SMI-CE, whereas the saturation SINRs for the subspace-based receivers are exactly coincident.

V. Numerical Examples

In this section, Monte Carlo simulations are presented, aimed at validating and extending our performance analysis. We consider a DS-CDMA system with $\alpha_1 = \alpha_2 = \cdots = \alpha_J = 1$ and $N = 16$. The J users employ unit-norm (i.e., $\|\mathbf{c}_j\| = 1$) random signatures $\mathbf{c}_j$, whose entries are i.i.d. random variables assuming equiprobable values in the complex set $\{\pm 1/\sqrt{2N}, \pm i/\sqrt{2N}\}$, with $\mathbf{c}_{j_1}$ and $\mathbf{c}_{j_2}$ statistically independent of each other for $j_1 \neq j_2 \in \{1, 2, \ldots, J\}$. The channel lengths are $L_j = 5$, $\forall j \in \{1, 2, \ldots, J\}$, i.e., they are equal for all the users, and, as in [24], [30], the entries of the unit-norm channel vectors $\mathbf{g}_j$ are randomly and independently drawn with equal power from a zero-mean complex circular (or proper) Gaussian process. The symbol and noise sequences are generated according to assumptions **(a1)** and **(a2)**, and the SNR is defined as $1/\sigma_v^2$. In each simulation, we carry out 10^4 independent Monte Carlo runs, with each run employing a different set of spreading sequences, channel vectors, symbol sequences and noise. In all simulations, we assume that the users have identical powers, i.e. there is perfect power control, and, without loss of generality, that the desired user is the first one, i.e., $j = 1$. Note that, in the considered scenario, the maximum number of users that can be accommodated by the receivers with CE is equal to $J_{\text{max,L}} = 11$ for the L-MOE receivers and $J_{\text{max,WL}} = 22$ for the WL-MOE receivers. To extensively compare WL-MOE and L-MOE receivers, we assume that the number of users J satisfies the first, more stringent condition, exception made for the second experiment, where we evaluate the performances as a function of J.

Example 1: in this experiment, we evaluate the average SINR (ASINR) as a function of SNR for the WL-MOE (Fig. 1) and L-MOE (Fig. 2) receivers (both with and without CE), for $J = 10$ users and a sample size equal to $K = 500$ symbols. For the sake of comparison, we also report the ASINR of the exact (i.e., data-independent) WL-MOE and L-MOE receivers given by (6) and (9), respectively. All the curves show a good agreement between simulation and analytical results. Looking in detail at Fig. 1, the simulation results confirm the theoretical prediction that the two subspace versions of the WL-MOE receiver (with or without CE) exhibit practically the same performances, whereas the WL-SMI-CE receiver performs slightly better than the WL-SMI one (with known channel), since the latter is penalized by the signature mismatch phenomenon; in particular, the asymptotic (for SNR $\to +\infty$) difference between the ASINR curves of the WL-SMI-CE and WL-SMI receivers is about 1.5 dB, which is in good agreement with the value theoretically predicted by (41). Similar considerations apply to Fig. 2, where the asymptotic gain of the L-SMI-CE receiver over the L-SMI one (with known channel) is about 1 dB, as correctly predicted by (51). As regards the comparison between WL-MOE and L-MOE receivers, results of Figs. 1 and 2 allow us to extend an important conclusion of our previous work [1], relative to the underloaded case (i.e., $J \leq N$): although the exact WL-MOE receiver generally exhibits a SINR gain over the L-MOE one also when $J \leq J_{\text{max,L}}$, in practice, due to SINR saturation effects, the subspace implementations of the WL-MOE and L-MOE receivers exhibit the same performances, whereas the L-SMI receivers (both with and without CE) outperform their WL-SMI counterparts.

[11] A comparison with [1, eq. (53)] shows that, in the estimated-channel case, the value of K_{min} is slightly lower.

Fig. 1. ASINR versus SNR for WL-MOE receivers ($J = 10$ users and $K = 500$ symbols).

Fig. 2. ASINR versus SNR for L-MOE receivers ($J = 10$ users and $K = 500$ symbols).

Fig. 3. ASINR versus number of users J for WL-MOE receivers (SNR = 15 dB and $K = 500$ symbols).

Fig. 4. ASINR versus number of users J for L-MOE receivers (SNR = 15 dB and $K = 500$ symbols).

Example 2: in this experiment, we evaluate the ASINR as a function of the number of users J for the WL-MOE (Fig. 3) and L-MOE (Fig. 4) receivers (both with and without CE), for a sample size equal to $K = 500$ symbols and SNR = 15 dB. Since the subspace-based CE procedure poses a strict limit of $J_{\max,\mathrm{WL}} = 22$ users for the WL-MOE receivers and $J_{\max,\mathrm{L}} = 11$ for the L-MOE receivers with CE, the performances of the receivers with CE are not reported (i.e., the corresponding curves are truncated) for values of J exceeding these limits. Besides confirming again a good agreement between simulation and analytical results, the curves for the WL-MOE receivers (Fig. 3) show that the performance advantage of the WL-SUB receiver over the WL-SMI one (both with and without CE) progressively decreases as J increases, becoming negligible in correspondence of about $J = 20$ users for the receivers with CE, and $J = 30$ users for the receivers with known channel. It is worthwhile to observe, moreover, that when J approaches the upper limit $J_{\max,\mathrm{WL}} = 22$ for CE, the performances of the WL-MOE receivers with CE degrade rapidly, suffering from a clear threshold effect. Similar considerations apply to Fig. 4, where, however, the ASINR curves of the L-MOE receivers are more closely spaced and the performance advantage of the L-SUB receiver over the L-SMI one becomes negligible in correspondence of about $J = 10$ users for the receivers with CE, and $J = 14$ users for the receivers with known channel. A careful comparison between the performances of WL-MOE and L-MOE receivers shows again that the largest advantage in using WL-MOE receivers is obtained in the "overloaded" region, i.e., when $11 \leq J \leq 22$ for the receivers with CE (where the L-MOE receivers cannot operate at all), and when $16 \leq J \leq 32$ for the receivers with known channel (where the L-MOE receivers, although capable of operating, exhibit poor performances).

Example 3: in this last experiment, we report the ASINR as a function of the sample size K for the WL-MOE (Fig. 5) and L-MOE (Fig. 6) receivers (both with and without CE),

Fig. 5. ASINR versus sample size K for WL-MOE receivers ($J = 10$ users and SNR = 20 dB).

Fig. 6. ASINR versus sample size K for L-MOE receivers ($J = 10$ users and SNR = 20 dB).

for $J = 10$ users and SNR = 20 dB. The ASINR values of the exact (i.e, data-independent) WL-MOE and L-MOE receivers, in this scenario, are equal to 21.5 and 19.2 dB, respectively, and obviously do not depend on K. The simulation and analytical results are again in good agreement, and, as expected, the accuracy of the formulas (39)–(40) and (49)–(50) improves as K increases. In particular, Fig. 5 shows that the two versions of the WL-SUB receivers (with or without CE) exhibit almost the same performances, outperforming the WL-SMI-CE receiver by about 2 dB, and the WL-SMI one (with known channel) by about 3 dB, for all considered values of K. Instead, the ASINR curves of the L-MOE receivers (see Fig. 6) are more closely spaced, exhibiting only marginal differences in performances between the various receivers. By comparing Figs. 5 and 6, it can be seen that the two WL-SUB receivers (with or without CE) outperform the corresponding L-SUB ones, for all the considered values of K. In contrast, the WL-SMI receiver (with known channel) again performs worse than its linear counterpart for all values of K (in this case the threshold sample size evaluated as in [1] is $K_{\text{min}} = 3844$, thus larger than the maximum value of $K = 2500$ considered in the simulations), whereas the performances of the WL-SMI-CE receiver approaches those of the L-SMI-CE one for K approaching 2500, which agrees very well with the value $K_{\text{min}} = 2428$ predicted by (54).

VI. Conclusions

We presented a comprehensive performance comparison between different versions of the L- and WL-MOE receivers with blind CE, when both the ACM and the channel impulse response of the desired user are estimated from a finite sample-size. The analysis extends our previous study [1] and the obtained formulas are fully supported by computer simulation results. The answers to the two questions put forward in the abstract are the following ones. With reference to their subspace-based implementations, for moderate-to-high values of the SNR, errors in estimating the L-SUB-CE and WL-SUB-CE receivers are essentially due to ACM estimation. The same is not true for the L-SMI-CE and WL-SMI-CE receivers, implemented by using the sample ACM directly, for which CE errors undesirably combine with ACM errors (signature mismatch phenomenon); however, compared with the known-channel case, CE errors adversely affect the SINR performances of L-SMI-CE and WL-SMI-CE receivers in a similar way. In conclusion, when considering finite sample-size implementation, the more sophisticated subspace-based implementation is an effective method to assure that the WL-MOE receiver (with or without CE) significantly outperform (for low-to-moderate values of the SNR) its linear counterpart. In this case, for a given channel length, the WL-MOE receiver allows one to work with an increased number of users J, which makes it a viable choice in heavily-congested DS-CDMA networks. A future interesting development is the extension of our analysis to other robust multiuser detectors, e.g., those belonging to the family of diagonal loading methods.

Appendix
Proofs

A. *Proof of Lemma 1*

It is shown in [1, Proof of Lemma 2] that, for moderate-to-high values of the sample size, i.e., $K \geq 6\,N$, the sample ACM (12) can be decomposed as $\widehat{\mathbf{R}}_{\mathbf{zz}} = \mathbf{R}_{\mathbf{zz}} + \boldsymbol{\delta}\mathbf{R}_{\mathbf{zz}}$, where $\boldsymbol{\delta}\mathbf{R}_{\mathbf{zz}} \triangleq \mathbf{h}_j\,\widehat{\mathbf{r}}^H_{\mathbf{q}_j b_j} + \widehat{\mathbf{r}}_{\mathbf{q}_j b_j}\mathbf{h}_j^H \in \mathbb{C}^{2N\times 2N}$, with $\widehat{\mathbf{r}}_{\mathbf{q}_j b_j} \triangleq \frac{1}{K}\sum_{k=0}^{K-1}\mathbf{q}_j(k)\,b_j(k)$. Consequently, $\widehat{\mathbf{R}}_{\mathbf{zz}}^{-1}$ admits the first-order approximation $\widehat{\mathbf{R}}_{\mathbf{zz}}^{-1} \approx \mathbf{R}_{\mathbf{zz}}^{-1} - \mathbf{R}_{\mathbf{zz}}^{-1}\,\boldsymbol{\delta}\mathbf{R}_{\mathbf{zz}}\,\mathbf{R}_{\mathbf{zz}}^{-1}$.

First, let us consider the SMI-CE implementation (17) of the WL-MOE receiver. Substituting the previous approximation of $\widehat{\mathbf{R}}_{\mathbf{zz}}^{-1}$ and $\widehat{\mathbf{h}}_j = \mathbf{h}_j + \boldsymbol{\delta}\mathbf{h}_j$ in (17), after some algebraic manipulations, one obtains the first-order approximation of the weight vector

$$\mathbf{f}_{j,\text{WL-SMI-CE}} \approx \mathbf{f}_{j,\text{WL-MOE}} \underbrace{-\mathbf{P}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{zz}}^{-1}\,\boldsymbol{\delta}\mathbf{R}_{\mathbf{zz}}\,\mathbf{f}_{j,\text{WL-MOE}}}_{\delta\mathbf{f}^{(1)}_{j,\text{WL-SMI-CE}}}$$

$$+\underbrace{(\mathbf{h}_j^H\,\mathbf{R}_{\mathbf{zz}}^{-1}\,\mathbf{h}_j)^{-1}\,\mathbf{R}_{\mathbf{zz}}^{-1}\,\boldsymbol{\delta}\mathbf{h}_j - 2\,\mathrm{Re}(\mathbf{f}_{j,\text{WL-MOE}}^H\,\boldsymbol{\delta}\mathbf{h}_j)\,\mathbf{f}_{j,\text{WL-MOE}}}_{\boldsymbol{\delta}\mathbf{f}^{(2)}_{j,\text{WL-SMI-CE}}}$$

$$= \mathbf{f}_{j,\text{WL-MOE}} + \boldsymbol{\delta}\mathbf{f}^{(1)}_{j,\text{WL-SMI-CE}} + \boldsymbol{\delta}\mathbf{f}^{(2)}_{j,\text{WL-SMI-CE}}\,, \quad (55)$$

with $\mathbf{P}_{j,\text{WL}} \triangleq \mathbf{I}_{2N} - (\mathbf{h}_j^H\,\mathbf{R}_{\mathbf{zz}}^{-1}\,\mathbf{h}_j)^{-1}\,\mathbf{R}_{\mathbf{zz}}^{-1}\,\mathbf{h}_j\,\mathbf{h}_j^H = \mathbf{I}_{2N} - (\mathbf{h}_j^H\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}^{-1}\,\mathbf{h}_j)^{-1}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}^{-1}\,\mathbf{h}_j\,\mathbf{h}_j^H \in \mathbb{C}^{2N\times 2N}$. Observe that, by virtue of (6), the matrix $\mathbf{P}_{j,\text{WL}}$ can be equivalently expressed as $\mathbf{P}_{j,\text{WL}} = \mathbf{I}_{2N} - \mathbf{f}_j\,\mathbf{h}_j^H$. Substituting the expression of $\boldsymbol{\delta}\mathbf{R}_{\mathbf{zz}}$ in $\boldsymbol{\delta}\mathbf{f}^{(1)}_{j,\text{WL-SMI-CE}}$, and observing that $\mathbf{P}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{zz}}^{-1}\,\mathbf{h}_j = \mathbf{0}_{2N}$, $\mathbf{h}_j^H\mathbf{f}_{j,\text{WL-MOE}} = 1$ and $\mathbf{P}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{zz}}^{-1} = \mathbf{P}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}^{-1}$, one has

$$\delta\mathbf{f}^{(1)}_{j,\text{WL-SMI-CE}} = -\underbrace{\mathbf{P}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}^{-1}}_{\boldsymbol{\Gamma}_{j,\text{WL}}\in\mathbb{C}^{2N\times 2N}}\,\widehat{\mathbf{r}}_{\mathbf{q}_j b_j} = -\boldsymbol{\Gamma}_{j,\text{WL}}\,\widehat{\mathbf{r}}_{\mathbf{q}_j b_j}\,. \quad (56)$$

Since both $\mathbf{f}_{j,\text{WL-MOE}}^H$ and $\boldsymbol{\delta}\mathbf{h}_j$ exhibit the CS property, the scalar $\mathbf{f}_{j,\text{WL-MOE}}^H\,\boldsymbol{\delta}\mathbf{h}_j$ is real and, thus, $\mathrm{Re}(\mathbf{f}_{j,\text{WL-MOE}}^H\,\boldsymbol{\delta}\mathbf{h}_j)\,\mathbf{f}_{j,\text{WL-MOE}} = (\mathbf{f}_{j,\text{WL-MOE}}^H\,\boldsymbol{\delta}\mathbf{h}_j)\,\mathbf{f}_{j,\text{WL-MOE}} = (\mathbf{f}_{j,\text{WL-MOE}}\,\mathbf{f}_{j,\text{WL-MOE}}^H)\,\boldsymbol{\delta}\mathbf{h}_j$. Consequently,

$$\delta\mathbf{f}^{(2)}_{j,\text{WL-SMI-CE}} = \underbrace{\left[(\mathbf{h}_j^H\,\mathbf{R}_{\mathbf{zz}}^{-1}\,\mathbf{h}_j)^{-1}\,\mathbf{R}_{\mathbf{zz}}^{-1} - 2\,\mathbf{f}_{j,\text{WL-MOE}}\,\mathbf{f}_{j,\text{WL-MOE}}^H\right]}_{\boldsymbol{\Delta}_{j,\text{WL}}\in\mathbb{C}^{2N\times 2N}}\,\boldsymbol{\delta}\mathbf{h}_j = \boldsymbol{\Delta}_{j,\text{WL}}\,\boldsymbol{\delta}\mathbf{h}_j\,. \quad (57)$$

At this point, we focus attention on the SUB-CE implementation (18) of the WL-MOE receiver. When the EVD is applied to the sample ACM $\widehat{\mathbf{R}}_{\mathbf{zz}}$ given by (12), for a sufficiently large sample size K, the matrices $\widehat{\mathbf{U}}_s$ and $\widehat{\boldsymbol{\Lambda}}_s$ can be decomposed [30], [37] as $\widehat{\mathbf{U}}_s = \mathbf{U}_s + \boldsymbol{\delta}\mathbf{U}_s$ and $\widehat{\boldsymbol{\Lambda}}_s = \boldsymbol{\Lambda}_s + \boldsymbol{\delta}\boldsymbol{\Lambda}_s$, where $\boldsymbol{\delta}\mathbf{U}_s$ and $\boldsymbol{\delta}\boldsymbol{\Lambda}_s$ represent the resulting perturbation in the estimated signal subspace, whose norm is of the order of $\|\boldsymbol{\delta}\mathbf{R}_{\mathbf{zz}}\|$. Moreover, it results [30], [37] that $\boldsymbol{\delta}\mathbf{U}_s \approx \mathbf{U}_n\mathbf{U}_n^H\boldsymbol{\delta}\mathbf{R}_{\mathbf{zz}}\,\mathbf{U}_s\,\boldsymbol{\Omega}_{\text{WL}}^{-1}$, with $\boldsymbol{\Omega}_{\text{WL}} \triangleq \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_J) \in \mathbb{R}^{J\times J}$, and $\boldsymbol{\delta}\boldsymbol{\Lambda}_s \approx \mathbf{U}_s^H\boldsymbol{\delta}\mathbf{R}_{\mathbf{zz}}\,\mathbf{U}_s$. Consequently, we can write

$$\widehat{\mathbf{U}}_s\,\widehat{\boldsymbol{\Lambda}}_s^{-1}\,\widehat{\mathbf{U}}_s^H \approx \mathbf{U}_s\,\boldsymbol{\Lambda}_s^{-1}\,\mathbf{U}_s^H + \mathbf{U}_s\,\boldsymbol{\Lambda}_s^{-1}\,\boldsymbol{\delta}\mathbf{U}_s^H - \mathbf{U}_s\,\boldsymbol{\Lambda}_s^{-1}\,\boldsymbol{\delta}\boldsymbol{\Lambda}_s\,\boldsymbol{\Lambda}_s^{-1}\,\mathbf{U}_s^H + \boldsymbol{\delta}\mathbf{U}_s\,\boldsymbol{\Lambda}_s^{-1}\,\mathbf{U}_s^H\,. \quad (58)$$

Observe that, since $\mathbf{U}_n^H\,\mathbf{h}_j = \mathbf{0}_{2N-J}$, one has $\boldsymbol{\delta}\mathbf{U}_s^H\,\mathbf{h}_j = \mathbf{0}_J$. Hence, using (58), accounting for the first-order perturbations of $\mathbf{U}_s$ and $\boldsymbol{\Lambda}_s$, and remembering that $\widehat{\mathbf{h}}_j = \mathbf{h}_j + \boldsymbol{\delta}\mathbf{h}_j$, one obtains

$$\widehat{\mathbf{h}}_j^H\,\widehat{\mathbf{U}}_s\,\widehat{\boldsymbol{\Lambda}}_s^{-1}\,\widehat{\mathbf{U}}_s^H\widehat{\mathbf{h}}_j \approx \mathbf{h}_j^H\,\mathbf{U}_s\,\boldsymbol{\Lambda}_s^{-1}\,\mathbf{U}_s^H\,\mathbf{h}_j - \mathbf{h}_j^H\,\mathbf{U}_s\,\boldsymbol{\Lambda}_s^{-1}\,\boldsymbol{\delta}\boldsymbol{\Lambda}_s\,\boldsymbol{\Lambda}_s^{-1}\,\mathbf{U}_s^H\,\mathbf{h}_j + 2\,\mathrm{Re}[\mathbf{h}_j^H\,\mathbf{U}_s\,\boldsymbol{\Lambda}_s^{-1}\,\mathbf{U}_s^H\,\boldsymbol{\delta}\mathbf{h}_j]\,. \quad (59)$$

Substituting (58) and (59) in (18), after some tedious but straightforward algebra, the first-order approximation of the weight vector can be concisely written as

$$\mathbf{f}_{j,\text{WL-SUB-CE}} \approx \mathbf{f}_{j,\text{WL-MOE}} + \boldsymbol{\delta}\mathbf{f}^{(1)}_{j,\text{WL-SUB-CE}} + \boldsymbol{\delta}\mathbf{f}^{(2)}_{j,\text{WL-SUB-CE}}\,, \quad (60)$$

where

$$\boldsymbol{\delta}\mathbf{f}^{(1)}_{j,\text{WL-SUB-CE}} \triangleq -\Big\{\mathbf{P}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{zz}}^{-1}\,\boldsymbol{\delta}\mathbf{R}_{\mathbf{zz}} - \mathbf{U}_n\,\mathbf{U}_n^H\,\boldsymbol{\delta}\mathbf{R}_{\mathbf{zz}} \cdot \left[\sigma_v^{-2}\,\mathbf{I}_{2N} + \mathbf{U}_s\,\boldsymbol{\Omega}_{\text{WL}}^{-1}\,\mathbf{U}_s^H\right]\Big\}\,\mathbf{f}_{j,\text{WL-MOE}}\,, \quad (61)$$

$$\boldsymbol{\delta}\mathbf{f}^{(2)}_{j,\text{WL-SUB-CE}} \triangleq (\mathbf{h}_j^H\mathbf{R}_{\mathbf{zz}}^{-1}\,\mathbf{h}_j)^{-1}\left[\mathbf{R}_{\mathbf{zz}}^{-1} - \sigma_v^{-2}\,\mathbf{U}_n\,\mathbf{U}_n^H\right]\boldsymbol{\delta}\mathbf{h}_j - 2\,\mathrm{Re}(\mathbf{f}_{j,\text{WL-MOE}}^H\,\boldsymbol{\delta}\mathbf{h}_j)\,\mathbf{f}_{j,\text{WL-MOE}}\,. \quad (62)$$

Then, substituting the expression of the perturbation $\boldsymbol{\delta}\mathbf{R}_{\mathbf{zz}}$ in (61), remembering again that $\mathbf{U}_n^H\,\mathbf{h}_j = \mathbf{0}_{2N-J}$, $\mathbf{h}_j^H\mathbf{f}_{j,\text{WL-MOE}} = 1$, $\mathbf{P}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{zz}}^{-1}\,\mathbf{h}_j = \mathbf{0}_{2N}$ and $\mathbf{P}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{zz}}^{-1} = \mathbf{P}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}^{-1}$, one gets

$$\boldsymbol{\delta}\mathbf{f}^{(1)}_{j,\text{WL-SUB-CE}} = -\underbrace{\left(\mathbf{P}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}^{-1} - \gamma_{j,\text{WL}}\,\mathbf{U}_n\mathbf{U}_n^H\right)}_{\boldsymbol{\Gamma}_{j,\text{WL}}\in\mathbb{C}^{2N\times 2N}}\,\widehat{\mathbf{r}}_{\mathbf{q}_j b_j} = -\boldsymbol{\Gamma}_{j,\text{WL}}\,\widehat{\mathbf{r}}_{\mathbf{q}_j b_j}\,, \quad (63)$$

with $\gamma_{j,\text{WL}} \triangleq \sigma_v^{-2} + (\mathbf{h}_j^H\,\mathbf{R}_{\mathbf{zz}}^{-1}\,\mathbf{h}_j)^{-1}\mathbf{h}_j^H\mathbf{U}_s\,\boldsymbol{\Omega}_{\text{WL}}^{-1}\,\mathbf{U}_s^H\,\mathbf{R}_{\mathbf{zz}}^{-1}\,\mathbf{h}_j$. Moreover, using again the fact that $\mathrm{Re}(\mathbf{f}_{j,\text{WL-MOE}}^H\,\boldsymbol{\delta}\mathbf{h}_j)\,\mathbf{f}_{j,\text{WL-MOE}} = (\mathbf{f}_{j,\text{WL-MOE}}\,\mathbf{f}_{j,\text{WL-MOE}}^H)\,\boldsymbol{\delta}\mathbf{h}_j$ and observing that, by virtue of the EVD properties, $\mathbf{R}_{\mathbf{zz}}^{-1} - \sigma_v^{-2}\,\mathbf{U}_n\,\mathbf{U}_n^H = \mathbf{U}_s\,\boldsymbol{\Lambda}_s^{-1}\,\mathbf{U}_s^H$, the perturbation term (62) can be rewritten as in (64) shown at the top of the next page.

B. Proof of Lemma 2

For a sufficiently large sample size K, when the EVD is applied to $\widehat{\mathbf{R}}_{\mathbf{zz}} \triangleq \mathbf{R}_{\mathbf{zz}} + \boldsymbol{\delta}\mathbf{R}_{\mathbf{zz}}$, where $\boldsymbol{\delta}\mathbf{R}_{\mathbf{zz}} = \mathbf{h}_j\,\widehat{\mathbf{r}}_{\mathbf{q}_j b_j}^H + \widehat{\mathbf{r}}_{\mathbf{q}_j b_j}\mathbf{h}_j^H \in \mathbb{C}^{2N\times 2N}$, with $\widehat{\mathbf{r}}_{\mathbf{q}_j b_j} \triangleq \frac{1}{K}\sum_{k=0}^{K-1}\mathbf{q}_j(k)\,b_j(k)$, the matrix $\widehat{\mathbf{U}}_n$ can be decomposed [30], [37] as $\widehat{\mathbf{U}}_n = \mathbf{U}_n + \boldsymbol{\delta}\mathbf{U}_n$ and the perturbation in the estimated noise subspace has the following form $\boldsymbol{\delta}\mathbf{U}_n \approx -\mathbf{U}_s\,\boldsymbol{\Omega}_{\text{WL}}^{-1}\,\mathbf{U}_s^H\boldsymbol{\delta}\mathbf{R}_{\mathbf{zz}}\,\mathbf{U}_n$, with $\boldsymbol{\Omega}_{\text{WL}} \triangleq \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_J) \in \mathbb{R}^{J\times J}$. By substituting the expression of $\boldsymbol{\delta}\mathbf{R}_{\mathbf{zz}}$ and noticing that $\mathbf{U}_n^H\,\mathbf{h}_j = \mathbf{0}_{2N-J}$, one obtains

$$\boldsymbol{\delta}\mathbf{U}_n \approx -\mathbf{U}_s\,\boldsymbol{\Omega}_{\text{WL}}^{-1}\,\mathbf{U}_s^H\,\mathbf{h}_j\,\widehat{\mathbf{r}}_{\mathbf{q}_j b_j}^H\,\mathbf{U}_n\,. \quad (65)$$

The perturbation $\boldsymbol{\delta}\mathbf{U}_n$ induces an error in the channel estimate $\widehat{\boldsymbol{\varrho}}_j$ given by (16), which assumes the form $\widehat{\boldsymbol{\varrho}}_j = \boldsymbol{\varrho}_j + \boldsymbol{\delta}\boldsymbol{\varrho}_j$, where $\boldsymbol{\delta}\boldsymbol{\varrho}_j$ represents the CE error. Remembering that $\widehat{\mathbf{h}}_j = \mathbf{h}_j + \boldsymbol{\delta}\mathbf{h}_j = \widetilde{\alpha}_j\,\boldsymbol{\mathcal{C}}_j\,\mathbf{T}_j\,\widehat{\boldsymbol{\varrho}}_j$ is the estimate of the signature $\mathbf{h}_j = \widetilde{\alpha}_j\,\boldsymbol{\mathcal{C}}_j\,\mathbf{T}_j\,\boldsymbol{\varrho}_j$, one easily gets $\boldsymbol{\delta}\mathbf{h}_j = \widetilde{\alpha}_j\,\boldsymbol{\mathcal{C}}_j\,\mathbf{T}_j\,\boldsymbol{\delta}\boldsymbol{\varrho}_j$. According to (15), the channel vector $\boldsymbol{\varrho}_j$ is the unique eigenvector corresponding to the null eigenvalue of $\mathbf{T}_j^H\,\boldsymbol{\mathcal{Q}}_{j,\text{WL}}\,\mathbf{T}_j \in \mathbb{C}^{2L_j\times 2L_j}$, with $\boldsymbol{\mathcal{Q}}_{j,\text{WL}} \triangleq \boldsymbol{\mathcal{C}}_j^H\,\mathbf{U}_n\,\mathbf{U}_n^H\,\boldsymbol{\mathcal{C}}_j \in \mathbb{C}^{2L_j\times 2L_j}$. The sample estimate $\widehat{\boldsymbol{\mathcal{Q}}}_{j,\text{WL}} = \boldsymbol{\mathcal{C}}_j^H\,\widehat{\mathbf{U}}_n\,\widehat{\mathbf{U}}_n^H\,\boldsymbol{\mathcal{C}}_j$ of matrix $\boldsymbol{\mathcal{Q}}_{j,\text{WL}}$ can be decomposed as $\widehat{\boldsymbol{\mathcal{Q}}}_{j,\text{WL}} = \boldsymbol{\mathcal{Q}}_{j,\text{WL}} + \boldsymbol{\delta}\boldsymbol{\mathcal{Q}}_{j,\text{WL}}$ where, accounting for (65), the perturbation $\boldsymbol{\delta}\boldsymbol{\mathcal{Q}}_{j,\text{WL}}$ has the form

$$\begin{aligned}\boldsymbol{\delta}\boldsymbol{\mathcal{Q}}_{j,\text{WL}} &\approx \boldsymbol{\mathcal{C}}_j^H\,\boldsymbol{\delta}\mathbf{U}_n\,\mathbf{U}_n^H\,\boldsymbol{\mathcal{C}}_j + \boldsymbol{\mathcal{C}}_j^H\,\mathbf{U}_n\,\boldsymbol{\delta}\mathbf{U}_n^H\,\boldsymbol{\mathcal{C}}_j\\ &= -\boldsymbol{\mathcal{C}}_j^H\,\mathbf{U}_s\,\boldsymbol{\Omega}_{\text{WL}}^{-1}\,\mathbf{U}_s^H\,\mathbf{h}_j\,\widehat{\mathbf{r}}_{\mathbf{q}_j b_j}^H\,\mathbf{U}_n\,\mathbf{U}_n^H\,\boldsymbol{\mathcal{C}}_j\\ &\quad - \boldsymbol{\mathcal{C}}_j^H\,\mathbf{U}_n\,\mathbf{U}_n^H\,\widehat{\mathbf{r}}_{\mathbf{q}_j b_j}\,\mathbf{h}_j^H\,\mathbf{U}_s\,\boldsymbol{\Omega}_{\text{WL}}^{-1}\,\mathbf{U}_s^H\,\boldsymbol{\mathcal{C}}_j\,. \quad (66)\end{aligned}$$

Based on (15), one has $\mathbf{T}_j^H\,\widehat{\boldsymbol{\mathcal{Q}}}_{j,\text{WL}}\,\mathbf{T}_j\,\widehat{\boldsymbol{\varrho}}_j = \mathbf{T}_j^H\,(\boldsymbol{\mathcal{Q}}_{j,\text{WL}} + \boldsymbol{\delta}\boldsymbol{\mathcal{Q}}_{j,\text{WL}})\,\mathbf{T}_j\,(\boldsymbol{\varrho}_j + \boldsymbol{\delta}\boldsymbol{\varrho}_j) \approx \mathbf{T}_j^H\,\boldsymbol{\mathcal{Q}}_{j,\text{WL}}\,\mathbf{T}_j\,\boldsymbol{\delta}\boldsymbol{\varrho}_j +$

$$\boldsymbol{\delta}\mathbf{f}^{(2)}_{j,\text{WL-SUB-CE}} = \underbrace{\left[(\mathbf{h}_j^H\,\mathbf{R}_{\mathbf{zz}}^{-1}\,\mathbf{h}_j)^{-1}\,\mathbf{U}_s\,\boldsymbol{\Lambda}_s^{-1}\,\mathbf{U}_s^H - 2\,\mathbf{f}_{j,\text{WL-MOE}}\,\mathbf{f}_{j,\text{WL-MOE}}^H\right]}_{\boldsymbol{\Delta}_{j,\text{WL}}\in\mathbb{C}^{2N\times 2N}}\,\boldsymbol{\delta}\mathbf{h}_j = \boldsymbol{\Delta}_{j,\text{WL}}\,\boldsymbol{\delta}\mathbf{h}_j \tag{64}$$

$\mathbf{T}_j^H\,\boldsymbol{\delta}\boldsymbol{\mathcal{Q}}_{j,\text{WL}}\,\mathbf{T}_j\,\boldsymbol{\varrho}_j \approx \mathbf{0}_{2L_j}$, which implies that $\mathbf{T}_j^H\,\boldsymbol{\mathcal{Q}}_{j,\text{WL}}\,\mathbf{T}_j\,\boldsymbol{\delta}\boldsymbol{\varrho}_j \approx -\mathbf{T}_j^H\,\boldsymbol{\delta}\boldsymbol{\mathcal{Q}}_{j,\text{WL}}\,\mathbf{T}_j\,\boldsymbol{\varrho}_j$, whose minimal-norm least-squares solution [38] is given by

$$\begin{aligned}\boldsymbol{\delta}\boldsymbol{\varrho}_j &\approx -(\mathbf{T}_j^H\,\boldsymbol{\mathcal{Q}}_{j,\text{WL}}\,\mathbf{T}_j)^{\dagger}\,\mathbf{T}_j^H\,\boldsymbol{\delta}\boldsymbol{\mathcal{Q}}_{j,\text{WL}}\,\mathbf{T}_j\,\boldsymbol{\varrho}_j\\ &= -\mathbf{T}_j^H\,\boldsymbol{\mathcal{Q}}_{j,\text{WL}}^{\dagger}\,\mathbf{T}_j\,\mathbf{T}_j^H\,\boldsymbol{\delta}\boldsymbol{\mathcal{Q}}_{j,\text{WL}}\,\mathbf{T}_j\,\boldsymbol{\varrho}_j\\ &= -\mathbf{T}_j^H\,\boldsymbol{\mathcal{Q}}_{j,\text{WL}}^{\dagger}\,\boldsymbol{\delta}\boldsymbol{\mathcal{Q}}_{j,\text{WL}}\,\mathbf{T}_j\,\boldsymbol{\varrho}_j\,,\end{aligned} \tag{67}$$

since $\mathbf{T}_j$ is unitary. Substituting (66) in (67) and observing that, due to (15), $\mathbf{U}_n^H\,\boldsymbol{\mathcal{C}}_j\,\mathbf{T}_j\,\boldsymbol{\varrho}_j = \mathbf{0}_{2N-J}$, one has

$$\begin{aligned}\boldsymbol{\delta}\boldsymbol{\varrho}_j \approx \mathbf{T}_j^H\,\boldsymbol{\mathcal{Q}}_{j,\text{WL}}^{\dagger}\,\boldsymbol{\mathcal{C}}_j^H\,\mathbf{U}_n\,\mathbf{U}_n^H\,\widehat{\mathbf{r}}_{\mathbf{q}_j b_j}&\\ \cdot\,(\mathbf{h}_j^H\,\mathbf{U}_s\,\boldsymbol{\Omega}_{\text{WL}}^{-1}\,\mathbf{U}_s^H\,\boldsymbol{\mathcal{C}}_j\,\mathbf{T}_j\,\boldsymbol{\varrho}_j)\,,&\end{aligned} \tag{68}$$

from which we finally have

$$\begin{aligned}\boldsymbol{\delta}\mathbf{h}_j &= \widetilde{\alpha}_j\,\boldsymbol{\mathcal{C}}_j\,\mathbf{T}_j\,\boldsymbol{\delta}\boldsymbol{\varrho}_j\\ &= (\mathbf{h}_j^H\,\mathbf{U}_s\,\boldsymbol{\Omega}_{\text{WL}}^{-1}\,\mathbf{U}_s^H\,\mathbf{h}_j)\,\boldsymbol{\mathcal{C}}_j\,\boldsymbol{\mathcal{Q}}_{j,\text{WL}}^{\dagger}\,\boldsymbol{\mathcal{C}}_j^H\,\mathbf{U}_n\,\mathbf{U}_n^H\,\widehat{\mathbf{r}}_{\mathbf{q}_j b_j}\,.\end{aligned} \tag{69}$$

C. *Evaluation of* $\text{trace}(\boldsymbol{\Sigma}_{j,\text{WL}}^H\,\mathbf{R}_{\mathbf{zz}}\,\boldsymbol{\Sigma}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j})$

Initially, we will proceed in a unified manner by treating the SMI and SUB cases jointly. Since $\boldsymbol{\Sigma}_{j,\text{WL}} = -\boldsymbol{\Gamma}_{j,\text{WL}} + \boldsymbol{\Delta}_{j,\text{WL}}\,\boldsymbol{\Pi}_{j,\text{WL}}$ (see Lemma 3), using the linearity property of the trace operator and observing that $\boldsymbol{\Delta}_{j,\text{WL}}$ is Hermitian (see Lemma 1), we can write

$$\begin{aligned}&\text{trace}(\boldsymbol{\Sigma}_{j,\text{WL}}^H\,\mathbf{R}_{\mathbf{zz}}\,\boldsymbol{\Sigma}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j})\\ &\quad= \text{trace}(\boldsymbol{\Gamma}_{j,\text{WL}}^H\,\mathbf{R}_{\mathbf{zz}}\,\boldsymbol{\Gamma}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j})\\ &\quad- \text{trace}(\boldsymbol{\Pi}_{j,\text{WL}}^H\,\boldsymbol{\Delta}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{zz}}\,\boldsymbol{\Gamma}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j})\\ &\quad- \text{trace}(\boldsymbol{\Gamma}_{j,\text{WL}}^H\,\mathbf{R}_{\mathbf{zz}}\,\boldsymbol{\Delta}_{j,\text{WL}}\,\boldsymbol{\Pi}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j})\\ &\quad+ \text{trace}(\boldsymbol{\Pi}_{j,\text{WL}}^H\,\boldsymbol{\Delta}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{zz}}\,\boldsymbol{\Delta}_{j,\text{WL}}\,\boldsymbol{\Pi}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j})\,.\end{aligned} \tag{70}$$

By invoking the properties of the trace operator, it follows that

$$\begin{aligned}&\text{trace}(\boldsymbol{\Gamma}_{j,\text{WL}}^H\,\mathbf{R}_{\mathbf{zz}}\,\boldsymbol{\Delta}_{j,\text{WL}}\,\boldsymbol{\Pi}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j})\\ &\quad= \text{trace}^*(\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}\,\boldsymbol{\Pi}_{j,\text{WL}}^H\,\boldsymbol{\Delta}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{zz}}\,\boldsymbol{\Gamma}_{j,\text{WL}})\\ &\quad= \text{trace}^*(\boldsymbol{\Pi}_{j,\text{WL}}^H\,\boldsymbol{\Delta}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{zz}}\,\boldsymbol{\Gamma}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j})\,,\end{aligned} \tag{71}$$

which shows that the third summand in (70) is the conjugate version of the second one. Moreover, remembering that $\mathbf{R}_{\mathbf{zz}} = \mathbf{h}_j\,\mathbf{h}_j^H + \mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}$ and $\mathbf{U}_n^H\,\mathbf{h}_j = \mathbf{0}_{2N-J}$, and accounting for the expressions of $\boldsymbol{\Gamma}_{j,\text{WL}}$ (see Lemma 1) and $\boldsymbol{\Pi}_{j,\text{WL}}$ (see Lemma 2), it can be directly verified that $\mathbf{R}_{\mathbf{zz}}\,\boldsymbol{\Gamma}_{j,\text{WL}} = \mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}\,\boldsymbol{\Gamma}_{j,\text{WL}}$ and $\boldsymbol{\Pi}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j} = \boldsymbol{\Pi}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{zz}}$. Thus, the first summand in (70) becomes $\text{trace}(\boldsymbol{\Gamma}_{j,\text{WL}}^H\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}\,\boldsymbol{\Gamma}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j})$, whereas the fourth one reduces to $\text{trace}[(\boldsymbol{\Delta}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{zz}}\,\boldsymbol{\Delta}_{j,\text{WL}})\,(\boldsymbol{\Pi}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{zz}}\,\boldsymbol{\Pi}_{j,\text{WL}}^H)]$, where we have also used the properties of the trace operator again. This last trace can be further expanded by replacing $\boldsymbol{\Pi}_{j,\text{WL}}$ with its expression given in Lemma 2: in particular, using $\mathbf{R}_{\mathbf{zz}} = \mathbf{U}_s\,\boldsymbol{\Lambda}_s\,\mathbf{U}_s^H + \sigma_v^2\,\mathbf{U}_n\,\mathbf{U}_n^H$, remembering that $\mathbf{U}_n^H\,\mathbf{U}_n = \mathbf{I}_{2N-J}$ and $\mathbf{U}_n^H\,\mathbf{U}_s = \mathbf{O}_{(2N-J)\times J}$, and observing that, on the basis of the Moore-Penrose conditions [38], $\boldsymbol{\mathcal{Q}}_{j,\text{WL}}^{\dagger}\,\boldsymbol{\mathcal{Q}}_{j,\text{WL}}\,\boldsymbol{\mathcal{Q}}_{j,\text{WL}}^{\dagger} = \boldsymbol{\mathcal{Q}}_{j,\text{WL}}^{\dagger}$, one has $\boldsymbol{\Pi}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{zz}}\,\boldsymbol{\Pi}_{j,\text{WL}}^H = \sigma_v^2\,(\mathbf{h}_j^H\,\mathbf{U}_s\,\boldsymbol{\Omega}_{\text{WL}}^{-1}\,\mathbf{U}_s^H\,\mathbf{h}_j)^2\,\boldsymbol{\mathcal{C}}_j\,\boldsymbol{\mathcal{Q}}_{j,\text{WL}}^{\dagger}\,\boldsymbol{\mathcal{C}}_j^H$. Consequently, substituting also the expression of $\boldsymbol{\Pi}_{j,\text{WL}}$ in the second summand of (70), we get

$$\begin{aligned}&\text{trace}(\boldsymbol{\Sigma}_{j,\text{WL}}^H\,\mathbf{R}_{\mathbf{zz}}\,\boldsymbol{\Sigma}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j})\\ &= \text{trace}(\boldsymbol{\Gamma}_{j,\text{WL}}^H\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}\,\boldsymbol{\Gamma}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}) - 2\,(\mathbf{h}_j^H\,\mathbf{U}_s\,\boldsymbol{\Omega}_{\text{WL}}^{-1}\,\mathbf{U}_s^H\,\mathbf{h}_j)\\ &\cdot\,\text{Re}[\text{trace}(\mathbf{U}_n\,\mathbf{U}_n^H\,\boldsymbol{\mathcal{C}}_j\,\boldsymbol{\mathcal{Q}}_{j,\text{WL}}^{\dagger}\,\boldsymbol{\mathcal{C}}_j^H\,\boldsymbol{\Delta}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{zz}}\,\boldsymbol{\Gamma}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j})]\\ &\qquad+ \sigma_v^2\,(\mathbf{h}_j^H\,\mathbf{U}_s\,\boldsymbol{\Omega}_{\text{WL}}^{-1}\,\mathbf{U}_s^H\,\mathbf{h}_j)^2\\ &\qquad\cdot\,\text{trace}[(\boldsymbol{\Delta}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{zz}}\,\boldsymbol{\Delta}_{j,\text{WL}})\,(\boldsymbol{\mathcal{C}}_j\,\boldsymbol{\mathcal{Q}}_{j,\text{WL}}^{\dagger}\,\boldsymbol{\mathcal{C}}_j^H)]\,.\end{aligned} \tag{72}$$

At this point, we have to consider the SMI and SUB cases separately. Let us start from the SMI case, for which $\boldsymbol{\Gamma}_{j,\text{WL}} = \mathbf{P}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}^{-1}$ and $\boldsymbol{\Delta}_{j,\text{WL}} = (\mathbf{h}_j^H\mathbf{R}_{\mathbf{zz}}^{-1}\mathbf{h}_j)^{-1}\mathbf{R}_{\mathbf{zz}}^{-1} - 2\,\mathbf{f}_j\mathbf{f}_j^H$. Following [1], it can be shown that $\text{trace}(\boldsymbol{\Gamma}_{j,\text{WL}}^H\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}\,\boldsymbol{\Gamma}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}) = 2\,N - 1$. As regards the second summand in (72), we observe that $\boldsymbol{\Delta}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{zz}}\,\boldsymbol{\Gamma}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j} = (\mathbf{h}_j^H\mathbf{R}_{\mathbf{zz}}^{-1}\mathbf{h}_j)^{-1}\,(\mathbf{P}_{j,\text{WL}} - \mathbf{f}_j\,\mathbf{h}_j^H)\,\mathbf{P}_{j,\text{WL}} = (\mathbf{h}_j^H\mathbf{R}_{\mathbf{zz}}^{-1}\mathbf{h}_j)^{-1}\,\mathbf{P}_{j,\text{WL}}$, where the second equality follows by noticing that $\mathbf{h}_j^H\,\mathbf{P}_{j,\text{WL}} = \mathbf{0}_{2N}^T$ and $\mathbf{P}_{j,\text{WL}}^2 = \mathbf{P}_{j,\text{WL}}$. Henceforth, observing that $\mathbf{P}_{j,\text{WL}}\,\mathbf{U}_n = \mathbf{U}_n$ and using the trace properties, the second summand in (72) becomes

$$\begin{aligned}&-2\,\zeta_{j,\text{WL}}\,\text{Re}[\text{trace}(\mathbf{U}_n\,\mathbf{U}_n^H\,\boldsymbol{\mathcal{C}}_j\,\boldsymbol{\mathcal{Q}}_{j,\text{WL}}^{\dagger}\,\boldsymbol{\mathcal{C}}_j^H\,\mathbf{P}_{j,\text{WL}})]\\ &\quad= -2\,\zeta_{j,\text{WL}}\,\text{Re}[\text{trace}(\mathbf{P}_{j,\text{WL}}\,\mathbf{U}_n\,\mathbf{U}_n^H\,\boldsymbol{\mathcal{C}}_j\,\boldsymbol{\mathcal{Q}}_{j,\text{WL}}^{\dagger}\,\boldsymbol{\mathcal{C}}_j^H)]\\ &\quad= -2\,\zeta_{j,\text{WL}}\,\text{Re}[\text{trace}(\boldsymbol{\mathcal{Q}}_{j,\text{WL}}^{\dagger}\,\underbrace{\boldsymbol{\mathcal{C}}_j^H\,\mathbf{U}_n\,\mathbf{U}_n^H\,\boldsymbol{\mathcal{C}}_j}_{\boldsymbol{\mathcal{Q}}_{j,\text{WL}}})]\\ &\quad= -2\,\zeta_{j,\text{WL}}\,(2\,L_j - 1)\,,\end{aligned} \tag{73}$$

with $\zeta_{j,\text{WL}} \triangleq (\mathbf{h}_j^H\,\mathbf{R}_{\mathbf{zz}}^{-1}\,\mathbf{h}_j)^{-1}\,\mathbf{h}_j^H\,\mathbf{U}_s\,\boldsymbol{\Omega}_{\text{WL}}^{-1}\,\mathbf{U}_s^H\,\mathbf{h}_j > 0$, where the last equality comes from the fact that $\boldsymbol{\mathcal{Q}}_{j,\text{WL}}^{\dagger}\,\boldsymbol{\mathcal{Q}}_{j,\text{WL}}$ is the orthogonal projector onto the subspace $\mathcal{R}(\boldsymbol{\mathcal{Q}}_{j,\text{WL}}^H) \equiv \mathcal{R}(\boldsymbol{\mathcal{Q}}_{j,\text{WL}})$ (see the Moore definition of the generalized inverse [38]) and, hence,[12] $\text{trace}(\boldsymbol{\mathcal{Q}}_{j,\text{WL}}^{\dagger}\,\boldsymbol{\mathcal{Q}}_{j,\text{WL}}) = \text{rank}(\boldsymbol{\mathcal{Q}}_{j,\text{WL}}) = \text{rank}(\mathbf{U}_n^H\,\boldsymbol{\mathcal{C}}_j)$, while $\text{rank}(\mathbf{U}_n^H\,\boldsymbol{\mathcal{C}}_j) = 2\,L_j - 1$ by virtue of condition **(c3)**. Considering the third summand in (72), we note that $\boldsymbol{\Delta}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{zz}}\,\boldsymbol{\Delta}_{j,\text{WL}} = \mathbf{R}_{\mathbf{zz}}^{-1}(\mathbf{R}_{\mathbf{zz}}\,\boldsymbol{\Delta}_{j,\text{WL}})^2 = (\mathbf{h}_j^H\mathbf{R}_{\mathbf{zz}}^{-1}\mathbf{h}_j)^{-2}\,\mathbf{R}_{\mathbf{zz}}^{-1}\,(\mathbf{P}_{j,\text{WL}}^H - \mathbf{h}_j\,\mathbf{f}_j^H)^2$ which, using the facts that $(\mathbf{P}_{j,\text{WL}}^H)^2 = \mathbf{P}_{j,\text{WL}}^H$, $\mathbf{P}_{j,\text{WL}}^H\,\mathbf{h}_j = \mathbf{0}_{2N}$, $\mathbf{f}_j^H\,\mathbf{P}_{j,\text{WL}}^H = \mathbf{0}_{2N}^T$ and $\mathbf{P}_{j,\text{WL}}^H + \mathbf{h}_j\,\mathbf{f}_j^H = \mathbf{I}_{2N}$, ends up to $\boldsymbol{\Delta}_{j,\text{WL}}\,\mathbf{R}_{\mathbf{zz}}\,\boldsymbol{\Delta}_{j,\text{WL}} = (\mathbf{h}_j^H\mathbf{R}_{\mathbf{zz}}^{-1}\mathbf{h}_j)^{-2}\,\mathbf{R}_{\mathbf{zz}}^{-1}$. Consequently, the third summand in (72)

[12] If χ is an eigenvalue of the orthogonal projector $\boldsymbol{\mathcal{Q}}_{j,\text{WL}}^{\dagger}\,\boldsymbol{\mathcal{Q}}_{j,\text{WL}}$, then $\chi \in \{0,1\}$.

assumes the form $\sigma_v^2\,\zeta_{j,\mathrm{WL}}^2\,\mathrm{trace}(\mathbf{R}_{\mathbf{zz}}^{-1}\,\boldsymbol{\mathcal{C}}_j\,\boldsymbol{\mathcal{Q}}_{j,\mathrm{WL}}^{\dagger}\,\boldsymbol{\mathcal{C}}_j^H)$. Thus, we have proven (35).

Let us consider now the SUB case, wherein

$$\boldsymbol{\Delta}_{j,\mathrm{WL}} = (\mathbf{h}_j^H \mathbf{R}_{\mathbf{zz}}^{-1}\mathbf{h}_j)^{-1}\mathbf{U}_s\boldsymbol{\Lambda}_s^{-1}\mathbf{U}_s^H - 2\,\mathbf{f}_j\mathbf{f}_j^H\,, \tag{74}$$

$$\boldsymbol{\Gamma}_{j,\mathrm{WL}} = \mathbf{P}_{j,\mathrm{WL}}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}^{-1} - \gamma_{j,\mathrm{WL}}\,\mathbf{U}_n\mathbf{U}_n^H\,, \tag{75}$$

with $\gamma_{j,\mathrm{WL}} \triangleq \sigma_v^{-2} + (\mathbf{h}_j^H\,\mathbf{R}_{\mathbf{zz}}^{-1}\,\mathbf{h}_j)^{-1}\mathbf{h}_j^H\mathbf{U}_s\,\boldsymbol{\Omega}_{\mathrm{WL}}^{-1}\,\mathbf{U}_s^H\mathbf{R}_{\mathbf{zz}}^{-1}\,\mathbf{h}_j$. Following [1], it can be shown that

$$\mathrm{trace}(\boldsymbol{\Gamma}_{j,\mathrm{WL}}^H\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}\boldsymbol{\Gamma}_{j,\mathrm{WL}}\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}) = (J-1) + (2\,N-J)\,|1-\gamma_{j,\mathrm{WL}}\,\sigma_v^2|^2\,. \tag{76}$$

As to the second summand in (72), since $\mathbf{R}_{\mathbf{zz}} = \mathbf{h}_j\,\mathbf{h}_j^H + \mathbf{R}_{\mathbf{q}_j\mathbf{q}_j} = \mathbf{U}_s\,\boldsymbol{\Lambda}_s\,\mathbf{U}_s^H + \sigma_v^2\,\mathbf{U}_n\,\mathbf{U}_n^H$, with $\mathbf{U}_n^H\,\mathbf{h}_j = \mathbf{0}_{2N-J}$, $\mathbf{U}_n^H\,\mathbf{U}_n = \mathbf{I}_{2N-J}$, $\mathbf{U}_s^H\,\mathbf{U}_s = \mathbf{I}_J$, $\mathbf{U}_n^H\,\mathbf{U}_s = \mathbf{O}_{(2N-J)\times J}$ and $\mathbf{U}_n\,\mathbf{U}_n^H + \mathbf{U}_s\,\mathbf{U}_s^H = \mathbf{I}_{2N}$, we obtain that $\boldsymbol{\Gamma}_{j,\mathrm{WL}}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j} = \mathbf{P}_{j,\mathrm{WL}} - \gamma_{j,\mathrm{WL}}\,\mathbf{U}_n\mathbf{U}_n^H\mathbf{R}_{\mathbf{zz}} = \mathbf{P}_{j,\mathrm{WL}} - \gamma_{j,\mathrm{WL}}\,\sigma_v^2\,\mathbf{U}_n\mathbf{U}_n^H$ and $\boldsymbol{\Delta}_{j,\mathrm{WL}}\,\mathbf{R}_{\mathbf{zz}} = (\mathbf{h}_j^H\mathbf{R}_{\mathbf{zz}}^{-1}\mathbf{h}_j)^{-1}\,(\mathbf{P}_{j,\mathrm{WL}} - \mathbf{f}_j\,\mathbf{h}_j^H - \mathbf{U}_n\mathbf{U}_n^H)$. Consequently, we get

$$\boldsymbol{\Delta}_{j,\mathrm{WL}}\,\mathbf{R}_{\mathbf{zz}}\,\boldsymbol{\Gamma}_{j,\mathrm{WL}}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j} = (\mathbf{h}_j^H\mathbf{R}_{\mathbf{zz}}^{-1}\mathbf{h}_j)^{-1}\,(\mathbf{P}_{j,\mathrm{WL}} - \mathbf{U}_n\,\mathbf{U}_n^H), \tag{77}$$

where we have used the facts that $\mathbf{P}_{j,\mathrm{WL}}^2 = \mathbf{P}_{j,\mathrm{WL}}$, $\mathbf{h}_j^H\,\mathbf{P}_{j,\mathrm{WL}} = \mathbf{0}_{2N}^T$, $\mathbf{U}_n^H\,\mathbf{P}_{j,\mathrm{WL}} = \mathbf{U}_n^H$, $\mathbf{h}_j^H\,\mathbf{U}_n = \mathbf{0}_{2N-J}^T$ and $\mathbf{P}_{j,\mathrm{WL}}\,\mathbf{U}_n = \mathbf{U}_n$. Therefore, observing again that $\mathbf{P}_{j,\mathrm{WL}}\,\mathbf{U}_n = \mathbf{U}_n$, $\mathbf{U}_n^H\,\mathbf{U}_n = \mathbf{I}_{2N-J}$ and using the trace properties, the second summand in (72) simplifies to

$$\begin{aligned} &-2\,\zeta_{j,\mathrm{WL}}\,\Big\{\mathrm{Re}\Big[\mathrm{trace}(\mathbf{U}_n\,\mathbf{U}_n^H\,\boldsymbol{\mathcal{C}}_j\,\boldsymbol{\mathcal{Q}}_{j,\mathrm{WL}}^{\dagger}\,\boldsymbol{\mathcal{C}}_j^H\,\mathbf{P}_{j,\mathrm{WL}}) \\ &\quad -\mathrm{trace}(\mathbf{U}_n\,\mathbf{U}_n^H\,\boldsymbol{\mathcal{C}}_j\,\boldsymbol{\mathcal{Q}}_{j,\mathrm{WL}}^{\dagger}\,\boldsymbol{\mathcal{C}}_j^H\,\mathbf{U}_n\,\mathbf{U}_n^H)\Big]\Big\} \\ &= -2\,\zeta_{j,\mathrm{WL}}\,\Big\{\mathrm{Re}\Big[\mathrm{trace}(\boldsymbol{\mathcal{Q}}_{j,\mathrm{WL}}^{\dagger}\,\boldsymbol{\mathcal{Q}}_{j,\mathrm{WL}}) \\ &\quad -\mathrm{trace}(\boldsymbol{\mathcal{Q}}_{j,\mathrm{WL}}^{\dagger}\,\boldsymbol{\mathcal{Q}}_{j,\mathrm{WL}})\Big]\Big\} = 0\,. \end{aligned} \tag{78}$$

With reference to the third summand in (72), we note that $\boldsymbol{\Delta}_{j,\mathrm{WL}}\,\mathbf{R}_{\mathbf{zz}}\,\boldsymbol{\Delta}_{j,\mathrm{WL}} = \mathbf{R}_{\mathbf{zz}}^{-1}(\mathbf{R}_{\mathbf{zz}}\,\boldsymbol{\Delta}_{j,\mathrm{WL}})^2 = (\mathbf{h}_j^H\mathbf{R}_{\mathbf{zz}}^{-1}\mathbf{h}_j)^{-2}\,\mathbf{R}_{\mathbf{zz}}^{-1}\,(\mathbf{P}_{j,\mathrm{WL}}^H - \mathbf{h}_j\,\mathbf{f}_j^H - \mathbf{U}_n\mathbf{U}_n^H)^2$ which, exploiting the EVD $\mathbf{R}_{\mathbf{zz}} = \mathbf{U}_s\,\boldsymbol{\Lambda}_s\,\mathbf{U}_s^H + \sigma_v^2\,\mathbf{U}_n\,\mathbf{U}_n^H$ and its related properties (as done for the second summand), and using the facts that $(\mathbf{P}_{j,\mathrm{WL}}^H)^2 = \mathbf{P}_{j,\mathrm{WL}}^H$, $\mathbf{f}_j^H\,\mathbf{P}_{j,\mathrm{WL}}^H = \mathbf{0}_{2N}^T$, $\mathbf{U}_n^H\,\mathbf{P}_{j,\mathrm{WL}}^H = \mathbf{U}_n^H$, $\mathbf{P}_{j,\mathrm{WL}}^H\,\mathbf{h}_j = \mathbf{0}_{2N}$, $\mathbf{U}_n^H\,\mathbf{h}_j = \mathbf{0}_{2N-J}$, $\mathbf{P}_{j,\mathrm{WL}}^H\,\mathbf{U}_n = \mathbf{U}_n$, $\mathbf{f}_j^H\,\mathbf{U}_n = \mathbf{0}_{2N-J}^T$ and $\mathbf{P}_{j,\mathrm{WL}}^H + \mathbf{h}_j\,\mathbf{f}_j^H = \mathbf{I}_{2N}$, boils down to $\boldsymbol{\Delta}_{j,\mathrm{WL}}^H\,\mathbf{R}_{\mathbf{zz}}\,\boldsymbol{\Delta}_{j,\mathrm{WL}} = (\mathbf{h}_j^H\mathbf{R}_{\mathbf{zz}}^{-1}\mathbf{h}_j)^{-2}\,(\mathbf{R}_{\mathbf{zz}}^{-1} - \sigma_v^{-2}\,\mathbf{U}_n\,\mathbf{U}_n^H)$. Consequently, the third summand in (72) assumes the form

$$\begin{aligned} &\sigma_v^2\,\zeta_{j,\mathrm{WL}}^2\,\mathrm{trace}(\mathbf{R}_{\mathbf{zz}}^{-1}\,\boldsymbol{\mathcal{C}}_j\,\boldsymbol{\mathcal{Q}}_{j,\mathrm{WL}}^{\dagger}\,\boldsymbol{\mathcal{C}}_j^H) \\ &\quad - \zeta_{j,\mathrm{WL}}^2\,\underbrace{\mathrm{trace}(\mathbf{U}_n\,\mathbf{U}_n^H\,\boldsymbol{\mathcal{C}}_j\,\boldsymbol{\mathcal{Q}}_{j,\mathrm{WL}}^{\dagger}\,\boldsymbol{\mathcal{C}}_j^H)}_{\mathrm{trace}(\boldsymbol{\mathcal{Q}}_{j,\mathrm{WL}}^{\dagger}\,\boldsymbol{\mathcal{Q}}_{j,\mathrm{WL}})} \\ &= \sigma_v^2\,\zeta_{j,\mathrm{WL}}^2\,\mathrm{trace}(\mathbf{R}_{\mathbf{zz}}^{-1}\,\boldsymbol{\mathcal{C}}_j\,\boldsymbol{\mathcal{Q}}_{j,\mathrm{WL}}^{\dagger}\,\boldsymbol{\mathcal{C}}_j^H) - \zeta_{j,\mathrm{WL}}^2(2\,L_j-1)\,. \end{aligned} \tag{79}$$

Thus, we have also proven (36).

D. Performance analysis of the L-SMI-CE and L-SUB-CE receivers

Let us denote with $\widehat{\mathbf{w}}_j$ *any* data-estimated L-MOE receiver, i.e, $\widehat{\mathbf{w}}_j = \mathbf{w}_{j,\text{L-SMI-CE}}$ or $\widehat{\mathbf{w}}_j = \mathbf{w}_{j,\text{L-SUB-CE}}$, and set $\mathbf{w}_j = \mathbf{w}_{j,\text{L-MOE}}$ for simplicity. When a linear data-estimated receiver $\widehat{\mathbf{w}}_j$ is employed [i.e., $\mathbf{f}_{j,1} = \widehat{\mathbf{w}}_j$ and $\mathbf{f}_{j,2} = \mathbf{0}_N$ in (2)], accounting for (4), (5) assumes the form

$$\mathrm{SINR}(\widehat{\mathbf{w}}_j) = \frac{\mathrm{E}^2\{\mathrm{Re}[\widehat{\mathbf{w}}_j^H\boldsymbol{\phi}_j]\}}{\mathrm{E}\{\mathrm{Re}^2[\widehat{\mathbf{w}}_j^H\mathbf{p}_j(k)]\} + \mathrm{Var}\{\mathrm{Re}[\widehat{\mathbf{w}}_j^H\boldsymbol{\phi}_j]\}}\,. \tag{80}$$

It is important to observe that, differently from the WL case, the real parts in (80) cannot be omitted, since $\widehat{\mathbf{w}}_j^H\boldsymbol{\phi}_j$ and $\widehat{\mathbf{w}}_j^H\mathbf{p}_j(k)$ are in general complex-valued quantities. This fact significantly complicates the analysis with respect to the WL case. Assume that $\widehat{\mathbf{w}}_j = \mathbf{w}_j + \boldsymbol{\delta}\mathbf{w}_j$, where $\boldsymbol{\delta}\mathbf{w}_j$ is a *small* (i.e., $\|\boldsymbol{\delta}\mathbf{w}_j\| \ll 1$) *zero-mean* perturbation term, and let $\mathrm{E}_{\boldsymbol{\delta}\mathbf{w}_j}[\cdot]$ be the average w.r.t. $\boldsymbol{\delta}\mathbf{w}_j$. Thus, one has

$$\mathrm{E}_{\boldsymbol{\delta}\mathbf{w}_j}[\mathrm{Re}\{\widehat{\mathbf{w}}_j^H\boldsymbol{\phi}_j\}] = \mathrm{Re}\{\mathrm{E}_{\boldsymbol{\delta}\mathbf{w}_j}[\widehat{\mathbf{w}}_j^H\boldsymbol{\phi}_j]\} = 1\,, \tag{81}$$

and

$$2\,\mathrm{E}_{\boldsymbol{\delta}\mathbf{w}_j}[\mathrm{Re}^2\{\widehat{\mathbf{w}}_j^H\boldsymbol{\phi}_j\}] = \mathrm{E}_{\boldsymbol{\delta}\mathbf{w}_j}[|\widehat{\mathbf{w}}_j^H\boldsymbol{\phi}_j|^2] + \mathrm{E}_{\boldsymbol{\delta}\mathbf{w}_j}[\mathrm{Re}\{(\widehat{\mathbf{w}}_j^H\boldsymbol{\phi}_j)^2\}]\,, \tag{82}$$

with $\mathrm{E}_{\boldsymbol{\delta}\mathbf{w}_j}[|\widehat{\mathbf{w}}_j^H\boldsymbol{\phi}_j|^2] = 1 + \mathrm{E}_{\boldsymbol{\delta}\mathbf{w}_j}[\boldsymbol{\delta}\mathbf{w}_j^H\boldsymbol{\phi}_j\,\boldsymbol{\phi}_j^H\,\boldsymbol{\delta}\mathbf{w}_j]$ and

$$\begin{aligned} \mathrm{E}_{\boldsymbol{\delta}\mathbf{w}_j}[\mathrm{Re}\{(\widehat{\mathbf{w}}_j^H\boldsymbol{\phi}_j)^2\}] &= \mathrm{Re}\{\mathrm{E}_{\boldsymbol{\delta}\mathbf{w}_j}[(\widehat{\mathbf{w}}_j^H\boldsymbol{\phi}_j)^2]\} \\ &= 1 + \mathrm{Re}\{\mathrm{E}_{\boldsymbol{\delta}\mathbf{w}_j}[\boldsymbol{\delta}\mathbf{w}_j^H\boldsymbol{\phi}_j\,\boldsymbol{\phi}_j^T\,\boldsymbol{\delta}\mathbf{w}_j^*]\}\,, \end{aligned} \tag{83}$$

since $\mathbf{w}_j^H\boldsymbol{\phi}_j = 1$ by (9), $\boldsymbol{\delta}\mathbf{w}_j$ is zero-mean by assumption, and $\mathrm{Re}^2(z) = \frac{1}{2}[|z|^2 + \mathrm{Re}(z^2)]$, $\forall z \in \mathbb{C}$. Consequently, it follows

$$\begin{aligned} &\mathrm{Var}\{\mathrm{Re}[\widehat{\mathbf{w}}_j^H\boldsymbol{\phi}_j]\} = \mathrm{E}_{\boldsymbol{\delta}\mathbf{w}_j}[\mathrm{Re}^2\{\widehat{\mathbf{w}}_j^H\boldsymbol{\phi}_j\}] \\ &\quad - \mathrm{E}_{\boldsymbol{\delta}\mathbf{w}_j}^2[\mathrm{Re}\{\widehat{\mathbf{w}}_j^H\boldsymbol{\phi}_j\}] = \tfrac{1}{2}\,\mathrm{E}_{\boldsymbol{\delta}\mathbf{w}_j}[\boldsymbol{\delta}\mathbf{w}_j^H\boldsymbol{\phi}_j\,\boldsymbol{\phi}_j^H\,\boldsymbol{\delta}\mathbf{w}_j] + \\ &\quad \tfrac{1}{2}\,\mathrm{Re}\{\mathrm{E}_{\boldsymbol{\delta}\mathbf{w}_j}[\boldsymbol{\delta}\mathbf{w}_j^H\boldsymbol{\phi}_j\,\boldsymbol{\phi}_j^T\,\boldsymbol{\delta}\mathbf{w}_j^*]\}\,. \end{aligned} \tag{84}$$

Similarly to the WL case, we assume that the weight vector $\widehat{\mathbf{w}}_j$ is independent from the data vector $\mathbf{p}_j(k)$. Let $\mathrm{E}_{\widehat{\mathbf{w}}_j\mathbf{p}_j}[\cdot]$ denote the joint average w.r.t. $\widehat{\mathbf{w}}_j$ and $\mathbf{p}_j(k)$, using again the identity $\mathrm{Re}^2(z) = \frac{1}{2}[|z|^2 + \mathrm{Re}(z^2)]$, $\forall z \in \mathbb{C}$, performing the average w.r.t to $\mathbf{p}_j(k)$, and recalling that, due to assumptions **(a1)** and **(a2)**, the vector $\mathbf{p}_j(k)$ is zero-mean, one obtains

$$\begin{aligned} &2\,\mathrm{E}_{\widehat{\mathbf{w}}_j,\mathbf{p}_j}\{\mathrm{Re}^2[\widehat{\mathbf{w}}_j^H\mathbf{p}_j(k)]\} = \mathrm{E}_{\widehat{\mathbf{w}}_j,\mathbf{p}_j}\{|\widehat{\mathbf{w}}_j^H\mathbf{p}_j(k)|^2\} \\ &\quad + \mathrm{E}_{\widehat{\mathbf{w}}_j,\mathbf{p}_j}\{\mathrm{Re}[(\widehat{\mathbf{w}}_j^H\mathbf{p}_j(k))^2]\} = \mathbf{w}_j^H\mathbf{R}_{\mathbf{p}_j\mathbf{p}_j}\,\mathbf{w}_j \\ &\quad + \mathrm{E}_{\boldsymbol{\delta}\mathbf{w}_j}\{\boldsymbol{\delta}\mathbf{w}_j^H\mathbf{R}_{\mathbf{p}_j\mathbf{p}_j}\,\boldsymbol{\delta}\mathbf{w}_j\} + \mathrm{Re}(\mathbf{w}_j^H\mathbf{R}_{\mathbf{p}_j\mathbf{p}_j^*}\,\mathbf{w}_j^*) \\ &\quad + \mathrm{Re}\{\mathrm{E}_{\boldsymbol{\delta}\mathbf{w}_j}[\boldsymbol{\delta}\mathbf{w}_j^H\mathbf{R}_{\mathbf{p}_j\mathbf{p}_j^*}\,\boldsymbol{\delta}\mathbf{w}_j^*]\}\,. \end{aligned} \tag{85}$$

At this point, noticing that $\mathbf{R}_{\mathbf{rr}} = \boldsymbol{\phi}_j\,\boldsymbol{\phi}_j^H + \mathbf{R}_{\mathbf{p}_j\mathbf{p}_j}$ and $\mathbf{R}_{\mathbf{rr}^*} = \boldsymbol{\phi}_j\,\boldsymbol{\phi}_j^T + \mathbf{R}_{\mathbf{p}_j\mathbf{p}_j^*}$, collecting all the above-obtained results and substituting in (80), we thus get (86) shown at the top of the next page, where we have also accounted for (10) and used the properties of the trace operator. The following Lemma gives a first-order characterization of the perturbation vector $\boldsymbol{\delta}\mathbf{w}_j$:

$$\text{SINR}(\widehat{\mathbf{w}}_j) = \frac{\text{SINR}_{j,\text{L-MOE}}}{1 + \frac{1}{2}\,\text{SINR}_{j,\text{L-MOE}} \left\{\text{trace}(\mathbf{R}_{\mathbf{rr}}\,\mathbf{R}_{\boldsymbol{\delta}\mathbf{w}_j\boldsymbol{\delta}\mathbf{w}_j}) + \text{Re}\left[\text{trace}(\mathbf{R}_{\mathbf{rr}^*}\,\mathbf{R}^*_{\boldsymbol{\delta}\mathbf{w}_j\boldsymbol{\delta}\mathbf{w}^*_j})\right]\right\}} \tag{86}$$

Lemma 4: Given the estimate $\widehat{\boldsymbol{\phi}}_j = \boldsymbol{\phi}_j + \boldsymbol{\delta\phi}_j = \alpha_j\,\mathbf{C}_j\,\widehat{\mathbf{g}}_j$ of the signature $\boldsymbol{\phi}_j$, where the channel estimate $\widehat{\mathbf{g}}_j$ is the solution of (46) and $\boldsymbol{\delta\phi}_j$ is a *small* (i.e., $\|\boldsymbol{\delta\phi}_j\| \ll 1$) *zero-mean* perturbation term, the first-order perturbation term of the L-SMI-CE and L-SUB-CE receivers can be expressed as

$$\boldsymbol{\delta}\mathbf{w}_j = \boldsymbol{\delta}\mathbf{w}_j^{(1)} + \boldsymbol{\delta}\mathbf{w}_j^{(2)}\,, \tag{87}$$

with

$$\boldsymbol{\delta}\mathbf{w}_j^{(1)} \approx -\boldsymbol{\Gamma}_{j,\text{L}}\,\widehat{\mathbf{r}}_{\mathbf{p}_j b_j}\,, \tag{88}$$

$$\boldsymbol{\delta}\mathbf{w}_j^{(2)} \approx \boldsymbol{\Delta}^{(1)}_{j,\text{L}}\,\boldsymbol{\delta\phi}_j + \boldsymbol{\Delta}^{(2)}_{j,\text{L}}\,\boldsymbol{\delta\phi}^*_j\,, \tag{89}$$

where $\boldsymbol{\delta\phi}_j \approx \boldsymbol{\Pi}_{j,\text{L}}\,\widehat{\mathbf{r}}_{\mathbf{p}_j b_j}$, with

$$\boldsymbol{\Pi}_{j,\text{L}} \triangleq (\boldsymbol{\phi}_j^H\,\mathbf{V}_s\,\boldsymbol{\Omega}_\text{L}^{-1}\,\mathbf{V}_s^H\,\boldsymbol{\phi}_j)\,\mathbf{C}_j\,\boldsymbol{\mathcal{Q}}^\dagger_{j,\text{L}}\,\mathbf{C}_j^H\,\mathbf{V}_n\,\mathbf{V}_n^H \in \mathbb{C}^{N\times N} \tag{90}$$

and the random vector $\widehat{\mathbf{r}}_{\mathbf{p}_j b_j} \triangleq \frac{1}{K}\sum_{k=1}^K \mathbf{p}_j(k)\,b_j(k) \in \mathbb{C}^N$ being the sample estimate of the cross-correlation between the disturbance vector $\mathbf{p}_j(k)$ and the desired symbol $b_j(k)$, whereas

$$\boldsymbol{\Gamma}_{j,\text{L}} \triangleq \begin{cases} \mathbf{P}_{j,\text{L}}\,\mathbf{R}^{-1}_{\mathbf{p}_j\mathbf{p}_j}\,, & \text{(L-SMI-CE)} \\ \mathbf{P}_{j,\text{L}}\,\mathbf{R}^{-1}_{\mathbf{p}_j\mathbf{p}_j} - \gamma_{j,\text{L}}\,\mathbf{V}_n\mathbf{V}_n^H\,, & \text{(L-SUB-CE)} \end{cases} \tag{91}$$

$$\boldsymbol{\Delta}^{(1)}_{j,\text{L}} \triangleq \begin{cases} (\boldsymbol{\phi}_j^H\mathbf{R}^{-1}_{\mathbf{rr}}\,\boldsymbol{\phi}_j)^{-1}\,\mathbf{R}^{-1}_{\mathbf{rr}}\,\mathbf{P}^H_{j,\text{L}}\,, & \text{(L-SMI-CE)} \\ (\boldsymbol{\phi}_j^H\mathbf{R}^{-1}_{\mathbf{rr}}\,\boldsymbol{\phi}_j)^{-1}\,\mathbf{V}_s\,\boldsymbol{\Upsilon}_s^{-1}\,\mathbf{V}_s^H\,\mathbf{P}^H_{j,\text{L}}\,, & \text{(L-SUB-CE)} \end{cases} \tag{92}$$

and $\boldsymbol{\Delta}^{(2)}_{j,\text{L}} \triangleq -\mathbf{w}_j\,\mathbf{w}_j^T$, with $\mathbf{P}_{j,\text{L}} \triangleq \mathbf{I}_N - (\boldsymbol{\phi}_j^H\,\mathbf{R}^{-1}_{\mathbf{p}_j\mathbf{p}_j}\,\boldsymbol{\phi}_j)^{-1}\,\mathbf{R}^{-1}_{\mathbf{p}_j\mathbf{p}_j}\,\boldsymbol{\phi}_j\,\boldsymbol{\phi}_j^H = \mathbf{I}_N - \mathbf{w}_j\,\boldsymbol{\phi}_j^H \in \mathbb{C}^{N\times N}$ being an oblique projection matrix [32] and $\gamma_{j,\text{L}} \triangleq \sigma_v^{-2} + (\boldsymbol{\phi}_j^H\,\mathbf{R}^{-1}_{\mathbf{rr}}\,\boldsymbol{\phi}_j)^{-1}\boldsymbol{\phi}_j^H\mathbf{V}_s\,\boldsymbol{\Omega}_\text{L}^{-1}\,\mathbf{V}_s^H\mathbf{R}^{-1}_{\mathbf{rr}}\,\boldsymbol{\phi}_j > 0$, while the diagonal matrix $\boldsymbol{\Omega}_\text{L} \triangleq \text{diag}(\mu_1, \mu_2, \ldots, \mu_J) \in \mathbb{R}^{J\times J}$ collects the nonzero eigenvalues of $\boldsymbol{\Phi}\,\boldsymbol{\Phi}^H$, and, finally, $\boldsymbol{\mathcal{Q}}_{j,\text{L}} \triangleq \mathbf{C}_j^H\,\mathbf{V}_n\,\mathbf{V}_n^H\,\mathbf{C}_j \in \mathbb{C}^{L_j\times L_j}$.

Proof: The proof can be conducted along the same lines of Appendices A and B, with the additional complication that, contrary to $\mathbf{f}_j^H\boldsymbol{\delta}\mathbf{h}_j$, the scalar product $\mathbf{w}_j^H\boldsymbol{\delta\phi}_j$ is complex rather than real. ∎

By virtue of Lemma 4, the overall perturbation of the L-SMI-CE and L-SUB-CE weight vectors can be expressed, similarly to the WL case, as a *linear* function of $\widehat{\mathbf{r}}_{\mathbf{q}_j b_j} = [\widehat{\mathbf{r}}^T_{\mathbf{p}_j b_j}, \widehat{\mathbf{r}}^H_{\mathbf{p}_j b_j}]^T$, that is, $\boldsymbol{\delta}\mathbf{w}_j = \boldsymbol{\delta}\mathbf{w}_j^{(1)} + \boldsymbol{\delta}\mathbf{w}_j^{(2)} \approx \boldsymbol{\Sigma}_{j,\text{L}}\,\widehat{\mathbf{r}}_{\mathbf{q}_j b_j}$, where $\boldsymbol{\Sigma}_{j,\text{L}} \triangleq [\boldsymbol{\Sigma}^{(1)}_{j,\text{L}}, \boldsymbol{\Sigma}^{(2)}_{j,\text{L}}]$, with $\boldsymbol{\Sigma}^{(1)}_{j,\text{L}} \triangleq -\boldsymbol{\Gamma}_{j,\text{L}} + \boldsymbol{\Delta}^{(1)}_{j,\text{L}}\,\boldsymbol{\Pi}_{j,\text{L}} \in \mathbb{C}^{N\times N}$ and $\boldsymbol{\Sigma}^{(2)}_{j,\text{L}} \triangleq \boldsymbol{\Delta}^{(2)}_{j,\text{L}}\,\boldsymbol{\Pi}^*_{j,\text{L}} \in \mathbb{C}^{N\times N}$. Therefore, the SINR in (86) assumes the form shown in (93) at the top of the next page, where, by virtue of assumptions **(a1)** and **(a2)**, we have used (see [1] for details) the fact that $\text{E}[\widehat{\mathbf{r}}_{\mathbf{q}_j b_j}\,\widehat{\mathbf{r}}^H_{\mathbf{q}_j b_j}] = \frac{1}{K}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}$ and $\text{E}[\widehat{\mathbf{r}}_{\mathbf{q}_j b_j}\,\widehat{\mathbf{r}}^T_{\mathbf{q}_j b_j}] = \frac{1}{K}\,\mathbf{J}\,\mathbf{R}^*_{\mathbf{q}_j\mathbf{q}_j}$, with $\mathbf{J} \triangleq \left[[\mathbf{O}_{N\times N}, \mathbf{I}_N]^T, [\mathbf{I}_N, \mathbf{O}_{N\times N}]^T\right]^T$. The matrix $\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}$ has a particular block structure where the lower-right block $\mathbf{R}^*_{\mathbf{p}_j\mathbf{p}_j}$ is the conjugate of the upper-left one $\mathbf{R}_{\mathbf{p}_j\mathbf{p}_j}$, and the lower-left block $\mathbf{R}^*_{\mathbf{p}_j\mathbf{p}^*_j}$ is the conjugate of the upper-right one $\mathbf{R}_{\mathbf{p}_j\mathbf{p}^*_j}$. Moreover, since $\mathbf{R}_{\mathbf{p}_j\mathbf{p}_j} = \overline{\boldsymbol{\Phi}}_j\,\overline{\boldsymbol{\Phi}}_j^H + \sigma_v^2\,\mathbf{I}_N$, $\mathbf{R}_{\mathbf{p}_j\mathbf{p}^*_j} = \overline{\boldsymbol{\Phi}}_j\,\overline{\boldsymbol{\Phi}}_j^T$ and $\mathbf{V}_n^H\,\overline{\boldsymbol{\Phi}}_j = \mathbf{O}_{(N-J)\times(J-1)}$, one has $\boldsymbol{\Pi}^*_{j,\text{L}}\,\mathbf{R}^*_{\mathbf{p}_j\mathbf{p}^*_j} = \mathbf{O}_{N\times N}$ and $\boldsymbol{\Pi}^*_{j,\text{L}}\,\mathbf{R}^*_{\mathbf{p}_j\mathbf{p}_j} = \sigma_v^2\,\boldsymbol{\Pi}^*_{j,\text{L}}$ and, hence, $\boldsymbol{\Sigma}^{(2)}_{j,\text{L}}\,\mathbf{R}^*_{\mathbf{p}_j\mathbf{p}^*_j} = \mathbf{R}_{\mathbf{p}_j\mathbf{p}^*_j}\,(\boldsymbol{\Sigma}^{(2)}_{j,\text{L}})^H = \mathbf{O}_{N\times N}$ and $\boldsymbol{\Sigma}^{(2)}_{j,\text{L}}\,\mathbf{R}^*_{\mathbf{p}_j\mathbf{p}_j} = \sigma_v^2\,\boldsymbol{\Sigma}^{(2)}_{j,\text{L}}$. By exploiting the block structure of $\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}$ and $\boldsymbol{\Sigma}_{j,\text{L}}$, it follows that

$$\begin{aligned}\text{trace}(\mathbf{R}_{\mathbf{rr}}\,\boldsymbol{\Sigma}_{j,\text{L}}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}\,\boldsymbol{\Sigma}^H_{j,\text{L}}) &= \text{trace}[(\boldsymbol{\Sigma}^{(1)}_{j,\text{L}})^H\mathbf{R}_{\mathbf{rr}}\,\boldsymbol{\Sigma}^{(1)}_{j,\text{L}}\,\mathbf{R}_{\mathbf{p}_j\mathbf{p}_j}] \\ &\quad + \sigma_v^2\,\text{trace}[(\boldsymbol{\Sigma}^{(2)}_{j,\text{L}})^H\mathbf{R}_{\mathbf{rr}}\,\boldsymbol{\Sigma}^{(2)}_{j,\text{L}}]\,.\end{aligned} \tag{94}$$

Using the expressions of $\boldsymbol{\Delta}^{(2)}_{j,\text{L}}$ and $\boldsymbol{\Pi}_{j,\text{L}}$, after simple manipulations, we get

$$\begin{aligned}&\text{trace}[(\boldsymbol{\Sigma}^{(2)}_{j,\text{L}})^H\mathbf{R}_{\mathbf{rr}}\,\boldsymbol{\Sigma}^{(2)}_{j,\text{L}}] \\ &\qquad = \zeta^2_{j,\text{L}}\,\text{trace}[\mathbf{R}^{-1}_{\mathbf{rr}}\,(\boldsymbol{\phi}_j\,\mathbf{w}_j^H)\,(\mathbf{C}_j\,\boldsymbol{\mathcal{Q}}^\dagger_{j,\text{L}}\,\mathbf{C}_j^H)]\,,\end{aligned} \tag{95}$$

where $\zeta_{j,\text{L}} \triangleq (\boldsymbol{\phi}_j^H\,\mathbf{R}^{-1}_{\mathbf{rr}}\,\boldsymbol{\phi}_j)^{-1}\,\boldsymbol{\phi}_j^H\,\mathbf{V}_s\,\boldsymbol{\Omega}_\text{L}^{-1}\,\mathbf{V}_s^H\,\boldsymbol{\phi}_j > 0$. Therefore, proceeding similarly to the WL case (see Appendix C) and accounting for [1, eq. (79)], it can be verified that, with reference to the L-SMI-CE receiver, the first trace term in (93) is given by

$$\begin{aligned}\text{trace}(\mathbf{R}_{\mathbf{rr}}\,\boldsymbol{\Sigma}_{j,\text{L}}\,\mathbf{R}_{\mathbf{p}_j\mathbf{p}_j}\,\boldsymbol{\Sigma}^H_{j,\text{L}}) &= (N-1) - 2\,\zeta_{j,\text{L}}(L_j-1) \\ &\quad + \zeta^2_{j,\text{L}}\,\sigma_v^2\,\text{trace}(\mathbf{R}^{-1}_{\mathbf{rr}}\,\mathbf{C}_j\,\boldsymbol{\mathcal{Q}}^\dagger_{j,\text{L}}\,\mathbf{C}_j^H)\,,\end{aligned} \tag{96}$$

whereas, for the L-SUB-CE receiver, one obtains

$$\begin{aligned}\text{trace}(\mathbf{R}_{\mathbf{rr}}\,\boldsymbol{\Sigma}_{j,\text{L}}\,\mathbf{R}_{\mathbf{p}_j\mathbf{p}_j}\,\boldsymbol{\Sigma}^H_{j,\text{L}}) &= (J-1) + (N-J)|1-\gamma_{j,\text{L}}\,\sigma_v^2|^2 \\ &\quad - \zeta^2_{j,\text{L}}(L_j-1) + \zeta^2_{j,\text{L}}\,\sigma_v^2\,\text{trace}(\mathbf{R}^{-1}_{\mathbf{rr}}\,\mathbf{C}_j\,\boldsymbol{\mathcal{Q}}^\dagger_{j,\text{L}}\,\mathbf{C}_j^H)\,.\end{aligned} \tag{97}$$

As regards the other trace term in (93), proceeding as done for the first one, it can be shown that

$$\begin{aligned}&\text{trace}(\mathbf{R}_{\mathbf{rr}^*}\,\boldsymbol{\Sigma}^*_{j,\text{L}}\,\mathbf{J}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}\,\boldsymbol{\Sigma}^H_{j,\text{L}}) \\ &\qquad = \text{trace}[(\boldsymbol{\Sigma}^{(1)}_{j,\text{L}})^H\,\mathbf{R}_{\mathbf{rr}^*}\,(\boldsymbol{\Sigma}^{(1)}_{j,\text{L}})^*\,\mathbf{R}^*_{\mathbf{p}_j\mathbf{p}^*_j}] \\ &\qquad\quad + 2\,\text{trace}[(\boldsymbol{\Sigma}^{(2)}_{j,\text{L}})^H\mathbf{R}_{\mathbf{rr}^*}\,(\boldsymbol{\Sigma}^{(1)}_{j,\text{L}})^*\,\mathbf{R}^*_{\mathbf{p}_j\mathbf{p}_j}]\,.\end{aligned} \tag{98}$$

Since, in addition to $\boldsymbol{\Pi}^*_{j,\text{L}}\,\mathbf{R}^*_{\mathbf{p}_j\mathbf{p}^*_j} = \mathbf{O}_{N\times N}$, the fact that $\mathbf{V}_n^H\,\overline{\boldsymbol{\Phi}}_j = \mathbf{O}_{(N-J)\times(J-1)}$ also implies that $(\boldsymbol{\Sigma}^{(1)}_{j,\text{L}})^*\,\mathbf{R}^*_{\mathbf{p}_j\mathbf{p}^*_j} = -\boldsymbol{\Gamma}_{j,\text{L}}\,\mathbf{R}^*_{\mathbf{p}_j\mathbf{p}^*_j}$, by resorting to the properties of the trace

$$\mathrm{SINR}(\widehat{\mathbf{w}}_j) = \frac{\mathrm{SINR}_{j,\text{L-MOE}}}{1+\frac{1}{2K}\,\mathrm{SINR}_{j,\text{L-MOE}}\left\{\mathrm{trace}(\mathbf{R_{rr}}\,\boldsymbol{\Sigma}_{j,\mathrm{L}}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}\,\boldsymbol{\Sigma}_{j,\mathrm{L}}^H)+\mathrm{Re}[\mathrm{trace}(\mathbf{R}_{\mathbf{rr}^*}\,\boldsymbol{\Sigma}_{j,\mathrm{L}}^*\,\mathbf{J}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}\,\boldsymbol{\Sigma}_{j,\mathrm{L}}^H)]\right\}} \quad (93)$$

operator, one has

$$\begin{aligned}\mathrm{trace}[&(\boldsymbol{\Sigma}_{j,\mathrm{L}}^{(1)})^H\,\mathbf{R}_{\mathbf{rr}^*}\,(\boldsymbol{\Sigma}_{j,\mathrm{L}}^{(1)})^*\,\mathbf{R}^*_{\mathbf{p}_j\mathbf{p}_j^*}]\\ &= \mathrm{trace}(\boldsymbol{\Gamma}_{j,\mathrm{L}}^H\,\mathbf{R}_{\mathbf{rr}^*}\,\boldsymbol{\Gamma}_{j,\mathrm{L}}^*\,\mathbf{R}^*_{\mathbf{p}_j\mathbf{p}_j^*})\\ &\quad - \mathrm{trace}(\boldsymbol{\Pi}_{j,\mathrm{L}}^H\,\boldsymbol{\Delta}_{j,\mathrm{L}}^{(1)}\,\mathbf{R}_{\mathbf{rr}^*}\,\boldsymbol{\Gamma}_{j,\mathrm{L}}^*\,\mathbf{R}^*_{\mathbf{p}_j\mathbf{p}_j^*})\\ &= \mathrm{trace}(\boldsymbol{\Gamma}_{j,\mathrm{L}}^H\,\mathbf{R}_{\mathbf{p}_j\mathbf{p}_j^*}\,\boldsymbol{\Gamma}_{j,\mathrm{L}}^*\,\mathbf{R}^*_{\mathbf{p}_j\mathbf{p}_j^*})\\ &\quad - \mathrm{trace}(\boldsymbol{\Delta}_{j,\mathrm{L}}^{(1)}\,\mathbf{R}_{\mathbf{rr}^*}\,\boldsymbol{\Gamma}_{j,\mathrm{L}}^*\,\mathbf{R}^*_{\mathbf{p}_j\mathbf{p}_j^*}\,\boldsymbol{\Pi}_{j,\mathrm{L}}^H)\\ &= \mathrm{trace}(\boldsymbol{\Gamma}_{j,\mathrm{L}}^H\,\mathbf{R}_{\mathbf{p}_j\mathbf{p}_j^*}\,\boldsymbol{\Gamma}_{j,\mathrm{L}}^*\,\mathbf{R}^*_{\mathbf{p}_j\mathbf{p}_j^*})\,, \quad (99)\end{aligned}$$

where it is verified that $\boldsymbol{\Gamma}_{j,\mathrm{L}}^H\,\mathbf{R}_{\mathbf{rr}^*} = \boldsymbol{\Gamma}_{j,\mathrm{L}}^H\,\mathbf{R}_{\mathbf{p}_j\mathbf{p}_j^*}$ and $\mathbf{R}^*_{\mathbf{p}_j\mathbf{p}_j^*}\,\boldsymbol{\Pi}_{j,\mathrm{L}}^H = \mathbf{O}_{N\times N}$. Moreover, observing that $\mathbf{R}_{\mathbf{rr}^*} = \boldsymbol{\Phi}_j\,\boldsymbol{\Phi}_j^T$ is symmetric, substituting the expression of $\boldsymbol{\Sigma}_{j,\mathrm{L}}^{(1)}$, $\boldsymbol{\Sigma}_{j,\mathrm{L}}^{(2)}$ and $\mathbf{w}_j$ [see (9)], one obtains

$$\begin{aligned}\mathrm{Re}\{&\mathrm{trace}[(\boldsymbol{\Sigma}_{j,\mathrm{L}}^{(2)})^H\mathbf{R}_{\mathbf{rr}^*}\,(\boldsymbol{\Sigma}_{j,\mathrm{L}}^{(1)})^*\,\mathbf{R}^*_{\mathbf{p}_j\mathbf{p}_j}]\}\\ &= -\sigma_v^2\,\zeta_{j,\mathrm{L}}^2\,(\boldsymbol{\phi}_j^H\mathbf{R}_{\mathbf{rr}}^{-1}\boldsymbol{\phi}_j)^{-1}\\ &\cdot\mathrm{Re}[\boldsymbol{\phi}_j^H\,\mathbf{R}_{\mathbf{rr}}^{-1}\,\mathbf{C}_j\,\boldsymbol{\mathcal{Q}}_{j,\mathrm{L}}^{\dagger}\,\mathbf{C}_j^H\mathbf{P}_{j,\mathrm{L}}\,\mathbf{R}_{\mathbf{rr}}^{-1}\,\mathbf{R}_{\mathbf{rr}^*}\,(\mathbf{R}_{\mathbf{rr}}^{-1})^*\,\boldsymbol{\phi}_j^*]\,, \quad (100)\end{aligned}$$

where we have also observed that, with reference to both L-SMI-CE and L-SUB-CE receivers, $\boldsymbol{\Pi}_{j,\mathrm{L}}\,\mathbf{R}_{\mathbf{p}_j\mathbf{p}_j}\,\boldsymbol{\Gamma}_{j,\mathrm{L}}^H\,\mathbf{R}_{\mathbf{rr}^*} = \boldsymbol{\Pi}_{j,\mathrm{L}}\,\mathbf{R}_{\mathbf{p}_j\mathbf{p}_j}\,\mathbf{R}_{\mathbf{p}_j\mathbf{p}_j}^{-1}\,\mathbf{P}_{j,\mathrm{L}}^H\,\mathbf{R}_{\mathbf{rr}^*} = \boldsymbol{\Pi}_{j,\mathrm{L}}\,\mathbf{P}_{j,\mathrm{L}}^H\,\mathbf{R}_{\mathbf{rr}^*} = \boldsymbol{\Pi}_{j,\mathrm{L}}\,\mathbf{R}_{\mathbf{rr}^*} = \mathbf{O}_{N\times N}$, since $\boldsymbol{\Pi}_{j,\mathrm{L}}\,\mathbf{P}_{j,\mathrm{L}}^H = \boldsymbol{\Pi}_{j,\mathrm{L}}$ and moreover, that $\boldsymbol{\Pi}_{j,\mathrm{L}}\,\mathbf{R}_{\mathbf{p}_j\mathbf{p}_j}\,\boldsymbol{\Pi}_{j,\mathrm{L}}^H = \sigma_v^2\,(\boldsymbol{\phi}_j^H\,\mathbf{V}_s\,\boldsymbol{\Omega}_{\mathrm{L}}^{-1}\,\mathbf{V}_s^H\,\boldsymbol{\phi}_j)^2\,\mathbf{C}_j\,\boldsymbol{\mathcal{Q}}_{j,\mathrm{L}}^{\dagger}\,\mathbf{C}_j^H$. The evaluation of the trace terms at the last hand of (99) and (100) are complicated and, to obtain manageable expressions, it is convenient to consider their asymptotic values as $\sigma_v^2 \to 0$. Using the limit formula for the generalized inverse [38], one gets

$$\begin{aligned}&\lim_{\sigma_v^2\to 0}\boldsymbol{\phi}_j^H\,\mathbf{R}_{\mathbf{rr}}^{-1}\,\mathbf{C}_j\,\boldsymbol{\mathcal{Q}}_{j,\mathrm{L}}^{+}\,\mathbf{C}_j^H\,\mathbf{P}_{j,\mathrm{L}}\,\mathbf{R}_{\mathbf{rr}}^{-1}\,\mathbf{R}_{\mathbf{rr}^*}\,(\mathbf{R}_{\mathbf{rr}}^{-1})^*\,\boldsymbol{\phi}_j^*\\ &= \mathbf{1}_j^T\,\boldsymbol{\Phi}^{\dagger}\,\mathbf{C}_j\,\boldsymbol{\mathcal{Q}}_{j,\mathrm{L}}^{+}\,\mathbf{C}_j^H\,(\boldsymbol{\Phi}^H)^{\dagger}\,(\mathbf{I}_J-\mathbf{1}_j\,\mathbf{1}_j^T)\,\mathbf{1}_j = 0\,, \quad (101)\end{aligned}$$

with $\mathbf{1}_j \triangleq [\overbrace{0,\ldots,0}^{j-1},1,0,\ldots,0]^T \in \mathbb{R}^{J\times 1}$, where we have observed that $\boldsymbol{\Phi}^{\dagger}\boldsymbol{\Phi} = \mathbf{I}_J$. Similarly, it can be verified that $\lim_{\sigma_v^2\to 0}(\boldsymbol{\phi}_j^H\mathbf{R}_{\mathbf{rr}}^{-1}\boldsymbol{\phi}_j)^{-1} = 1$ and $\lim_{\sigma_v^2\to 0}\zeta_{j,\mathrm{L}} = 1$. Henceforth, noticing that the trace term at the last hand of (99) has been evaluated in [1, Appendix E], accounting for (101), it can be shown that, with reference to both L-SMI-CE and L-SUB-CE receivers,

$$\begin{aligned}&\lim_{\sigma_v^2\to 0}\mathrm{Re}[\mathrm{trace}(\mathbf{R}_{\mathbf{rr}^*}\,\boldsymbol{\Sigma}_{j,\mathrm{L}}^*\,\mathbf{J}\,\mathbf{R}_{\mathbf{q}_j\mathbf{q}_j}\,\boldsymbol{\Sigma}_{j,\mathrm{L}}^H)] \quad (102)\\ &= \lim_{\sigma_v^2\to 0}\mathrm{Re}[\mathrm{trace}(\boldsymbol{\Gamma}_{j,\mathrm{L}}^H\,\mathbf{R}_{\mathbf{p}_j\mathbf{p}_j^*}\,\boldsymbol{\Gamma}_{j,\mathrm{L}}^*\,\mathbf{R}^*_{\mathbf{p}_j\mathbf{p}_j^*})] = J-1\,. \quad (103)\end{aligned}$$

Finally, for $\sigma_v^2 \to 0$, as in the WL case (see Subsection III-A), one has $\sigma_v^2\,\mathrm{trace}(\mathbf{R}_{\mathbf{rr}}^{-1}\,\mathbf{C}_j\,\boldsymbol{\mathcal{Q}}_{j,\mathrm{L}}^{\dagger}\,\mathbf{C}_j^H) \to L_j-1$, $\gamma_{j,\mathrm{L}}\,\sigma_v^2 \to 1$ and $\zeta_{j,\mathrm{L}} \to 1$. Thus, it follows from (96) and (97) that

$$\lim_{\sigma_v^2\to 0}\mathrm{trace}(\mathbf{R_{rr}}\boldsymbol{\Sigma}_{j,\mathrm{L}}\mathbf{R}_{\mathbf{p}_j\mathbf{p}_j}\boldsymbol{\Sigma}_{j,\mathrm{L}}^H) = \begin{cases} N-L_j\,, & \text{(L-SMI-CE)}\\ J-1\,, & \text{(L-SUB-CE)}\end{cases} \quad (104)$$

By substituting (103) and (104) into (93), eqs. (49) and (50) are easily obtained.

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