Subspace-based blind multiuser detection for quasi-synchronous MC-CDMA systems

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***Abstract*— A subspace-based linear minimum mean-square error (MMSE) multiuser detector is proposed for quasi-synchronous multicarrier code-division multiple-access (MC-CDMA) systems. The proposed receiver is totally blind, i.e., it is both channel- and delay-independent and, unlike many subspace-based algorithms, requires only an upper bound (rather than the exact knowledge) of the channel order, without needing explicit delay estimation. The effectiveness of the proposed blind approach is tested by computer simulation results, which show that it assures satisfactory performances in severe near-far scenarios, while exhibiting a lower computational complexity in comparison with existing subspace-based methods.**

***Index Terms*— Blind multiuser detection, subspace methods.**

I. Introduction

RECENTLY, among various multicarrier (MC) modulation techniques [1], two schemes have been receiving considerable interest for high data-rate applications, because of their ability to cope with interchip and intersymbol interference more effectively than in single-carrier (SC) direct-sequence (DS) code-division multiple-access (CDMA) systems: the former, referred to as MC-CDMA [2], is based on *frequency-domain* spreading; whereas the latter, referred to as MC-DS-CDMA [3], is based on *time-domain* spreading. When the receiver is implemented by means of simple *single-user* diversity-combining strategies [1], the MC-CDMA scheme can perform better than the MC-DS-CDMA one, both in the synchronous downlink [1] and asynchronous uplink [4]. However, in the uplink, single-user detection strategies tend to exhibit [5] unsatisfactory performances in the presence of severe multiple-access interference (MAI), even for *quasi-synchronous* (QS) transmissions. In prohibitive near-far scenarios, the linear minimum mean-square error (MMSE) *multiuser* detector (MUD) offers high near-far resistance, with a reasonable computational complexity; however, its synthesis requires knowledge of the desired user's channel-impulse response and transmission delay.

As happens in SC-DS-CDMA networks, the performance of the MMSE-MUD, in both MC-CDMA and MC-DS-CDMA systems, critically depends on timing and channel estimation, which can be performed by resorting to bandwidth-consuming training sequences. To avoid waste of resources, borrowing ideas from the area of multiuser detection for SC-DS-CDMA [6]–[8], a subspace-based *blind* (i.e., requiring the only knowledge of the spreading code of the desired user) MMSE-MUD for an asynchronous MC-DS-CDMA uplink was proposed in [9], which relies on a finite-length approximation of the chip waveform in order to obtain a nontrivial noise subspace. Although this approximation allows one to blindly extract the timing and channel information, the obtained blind MMSE-MUD exhibits however a limited near-far resistance. In [10] a blind subspace-based MMSE-MUD was proposed for synchronous MC-CDMA systems without *cyclic prefix* (CP), which exploits the presence of "virtual carriers", i.e., subcarriers unused in transmission: this method cannot be directly applied to both asynchronous and QS systems.

In this letter, we propose a blind subspace-based MMSE-MUD for a QS-MC-CDMA uplink, where a CP is inserted in the transmitted data and the asynchronisms of the incoming signals at the receiver are limited to only a few sampling intervals. In the considered system, the use of subspace-based techniques turns out to be particularly advantageous since: (i) as in [8] and unlike many subspace-based multiuser methods (e.g., [6]), the extraction of the signal and noise subspaces does not require *exact* knowledge of the channel order; (ii) unlike [9], the proposed blind receiver is able to assure satisfactory performance also in severe near-far scenarios; (iii) finally, the synthesis of the proposed detector is based on the observation of only *one* symbol period and does not require a search-based algorithm for initial delay estimation, allowing thus a significant reduction of the receiver complexity.

II. The MC-CDMA system model

Let us consider the baseband-equivalent of a MC-CDMA cellular system with N subcarriers. For each value[1] of $n \in \mathbb{Z}$, the information symbol $b_j(n)$ emitted by the jth user is multiplied with the *frequency-domain* spreading code $\boldsymbol{c}_j \triangleq [c_j^{(0)}, c_j^{(1)}, \ldots, c_j^{(N-1)}]^T \in \mathbb{C}^N$, with $(\cdot)^T$ denoting transpose; the resulting sequence is subject to the inverse discrete Fourier transform (IDFT), producing thus the vector $\widetilde{\boldsymbol{u}}_j(n) = \boldsymbol{W}_{\rm IDFT}\, \boldsymbol{c}_j\, b_j(n) \in \mathbb{C}^N$, where $\boldsymbol{W}_{\rm IDFT} \in \mathbb{C}^{N \times N}$ denotes the IDFT matrix. After computing the IDFT, a CP of length $L_{\rm cp} < N$ is appended at the beginning of $\widetilde{\boldsymbol{u}}_j(n)$

Manuscript received September 5, 2003; revised November 19, 2003. This work was supported by the MIUR 2002 Italian National Research Project. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Steven M. Kay.

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Digital Object Identifier S 0000-0000(00)00000-0.

[1] $\mathbb{C}$, $\mathbb{R}$ and $\mathbb{Z}$ are the fields of complex, real and integer numbers, respectively; $\mathbb{C}^n$ $[\mathbb{R}^n]$ denotes the vector-space of all n-column vectors with complex [real] coordinates; similarly, $\mathbb{C}^{n \times m}$ $[\mathbb{R}^{n \times m}]$ denotes the vector-space of all the $n \times m$ matrices with complex [real] elements.

obtaining thus the block $\boldsymbol{u}_j(n) = \boldsymbol{T}_{\rm cp}\,\boldsymbol{W}_{\rm IDFT}\,\boldsymbol{c}_j\,b_j(n) \in \mathbb{C}^P$, with $P \triangleq N + L_{\rm cp}$, where $\boldsymbol{T}_{\rm cp} \triangleq [\boldsymbol{I}_{\rm cp}^T, \boldsymbol{I}_N]^T \in \mathbb{R}^{P\times N}$, with $\boldsymbol{I}_{\rm cp} \in \mathbb{R}^{L_{\rm cp}\times N}$ obtained by drawing out the last $L_{\rm cp}$ rows of the identity $N \times N$ matrix $\boldsymbol{I}_N$. The block $\boldsymbol{u}_j(n)$ is subject to parallel-to-serial conversion, and the resulting sequence $\{u_j^{(m)}(n)\}_{m=0}^{P-1}$ feeds a linear modulator, operating at rate $1/T_c = P/T_s$, where T_s and T_c denote the symbol and the sampling period, respectively. Let J denote the number of users, assuming that ideal carrier-frequency recovery is carried out at the receiver, after sampling at the time epochs $t_{k,\ell} \triangleq k\,T_s + \ell\,T_c$, with $k \in \mathbb{Z}$ and $\ell \in \{0, 1, \ldots, P-1\}$, the received baseband signal is given by

$$\widetilde{r}^{(\ell)}(k) = \sum_{j=1}^{J}\sum_{p=0}^{2}\sum_{q=\ell-P+1}^{\ell} g_j^{(q)}(p)\,u_j^{(\ell-q)}(k-p) + \widetilde{v}^{(\ell)}(k)\,, \tag{1}$$

where $g_j^{(q)}(p) \triangleq g_{c,j}[(p\,P+q-d_j)\,T_c-\beta_j]$, with $g_{c,j}(t)$ being the *composite* impulse response of the linear time-invariant channel of the jth user, spanning L_j sampling periods, $\tau_j = d_j\,T_c + \beta_j$, with $d_j \in \{0, 1, \ldots, P-1\}$ and $\beta_j \in [0, T_c)$, represents the transmission delay of the jth user, and $\widetilde{v}^{(\ell)}(k)$ represents the additive noise.

We will assume that: **A1)** the information symbols $\{b_j(n)\}_{j=1}^J$ are mutually independent zero-mean and independent identically-distributed sequences, with equal variance σ_b^2; **A2)** the additive noise samples $\{\widetilde{v}^{(\ell)}(k)\}_{\ell=0}^{P-1}$ are modeled as mutually independent complex circular zero-mean white Gaussian processes, with variance σ_v^2, which are independent of the sequences $\{b_j(n)\}_{j=1}^J$. Finally, by introducing the vector $\widetilde{\boldsymbol{r}}(k) \triangleq [\widetilde{r}^{(0)}(k), \widetilde{r}^{(1)}(k), \ldots, \widetilde{r}^{(P-1)}(k)]^T \in \mathbb{C}^P$, we obtain

$$\widetilde{\boldsymbol{r}}(k) = \sum_{j=1}^{J}\sum_{p=0}^{2} \widetilde{\boldsymbol{G}}_j(p)\,\boldsymbol{T}_{\rm cp}\,\boldsymbol{W}_{\rm IDFT}\,\boldsymbol{c}_j\,b_j(k-p) + \widetilde{\boldsymbol{v}}(k)\,, \tag{2}$$

where $\widetilde{\boldsymbol{G}}_j(p) = \sum_{h=0}^{P-1} g_j^{(h)}(p)\,\widetilde{\boldsymbol{F}}^h + \sum_{h=1}^{P-1} g_j^{(-h)}(p)\,\widetilde{\boldsymbol{B}}^h$, with $\widetilde{\boldsymbol{F}}^h$ and $\widetilde{\boldsymbol{B}}^h$ denoting the hth power of the $P \times P$ *forward shift* and *backward shift* matrices[2], respectively, whereas $\widetilde{\boldsymbol{v}}(k) \triangleq [\widetilde{v}^{(0)}(k), \widetilde{v}^{(1)}(k), \ldots, \widetilde{v}^{(P-1)}(k)]^T \in \mathbb{C}^P$. Without loss of generality, our aim is to demodulate the first user ($j = 1$), which will be referred to as the *desired user*.

III. Blind subspace-based MMSE detection

In the uplink of a QS system, all the users attempt to synchronize their transmissions by resorting to a local reference clock [obtained, e.g., with the help of a global positioning system (GPS) device] or to a pilot signal transmitted by the base station [11]. However, due to oscillator drifts, GPS uncertainties, and the relative motion among the mobiles and the base station, the signals received by the users are still asynchronous, even though their asynchronisms are contained within a limited number of sampling intervals. Thus, for each user, the sum of the delay d_j and the channel order L_j turns out to be within one symbol period, i.e., $d_j + L_j \leq P$; in this case, it results that $\widetilde{\boldsymbol{G}}_j(2) = \mathbf{O}_{P\times P}$, with $\mathbf{O}_{P\times P}$ denoting the zero $P \times P$ matrix, and the signal model (2) becomes

$$\widetilde{\boldsymbol{r}}(k) = \sum_{j=1}^{J}\sum_{p=0}^{1} \widetilde{\boldsymbol{G}}_j(p)\,\boldsymbol{T}_{\rm cp}\,\boldsymbol{W}_{\rm IDFT}\,\boldsymbol{c}_j\,b_j(k-p) + \widetilde{\boldsymbol{v}}(k)\,, \tag{3}$$

where $\widetilde{\boldsymbol{G}}_j(0) = \sum_{h=0}^{L_j} g_j(h)\,\widetilde{\boldsymbol{F}}^{(h+d_j)}$ and $\widetilde{\boldsymbol{G}}_j(1) = \sum_{h=0}^{L_j} g_j(h)\,\widetilde{\boldsymbol{B}}^{(P-d_j-h)}$, with $g_j(h) \triangleq g_{c,j}(h\,T_c - \beta_j)$. It can be seen that the last $P - L_j - d_j$ rows of the matrix $\widetilde{\boldsymbol{G}}_j(1)$ are identically zero, that is, the interblock interference (IBI) contribution is entirely contained in the first $L_j + d_j$ components of $\widetilde{\boldsymbol{G}}_j(1)\,\boldsymbol{T}_{\rm cp}\,\boldsymbol{W}_{\rm IDFT}\,\boldsymbol{c}_j\,b_j(k-1)$. This suggests a simple detection technique to *deterministically* remove the IBI of the desired user and partially mitigate the MAI. More specifically, under the assumption that: **A3)** the CP length $L_{\rm cp}$ satisfies $L_{\rm cp} \geq \max_{j\in\{1,2,\ldots,J\}} \left[L_j + d_j + 1\right]$, the IBI contribution for *each* user can be completely discarded by dropping the first $L_{\rm cp}$ components of $\widetilde{\boldsymbol{r}}(k)$. This operation can be accomplished by defining the matrix $\boldsymbol{R}_{\rm cp} \triangleq [\mathbf{O}_{N\times L_{\rm cp}}, \boldsymbol{I}_N] \in \mathbb{R}^{N\times P}$ and forming at the receiver the product $\boldsymbol{r}(k) \triangleq \boldsymbol{R}_{\rm cp}\,\widetilde{\boldsymbol{r}}(k) \in \mathbb{C}^N$. According to assumption A3, after CP removal, one has

$$\boldsymbol{r}(k) = \sum_{j=1}^{J} \boldsymbol{G}_j(0)\,\boldsymbol{W}_{\rm IDFT}\,\boldsymbol{c}_j\,b_j(k) + \boldsymbol{v}(k)\,, \tag{4}$$

where $\boldsymbol{G}_j(0) = \sum_{h=0}^{L_j} g_j(h)\,[\boldsymbol{R}_{\rm cp}\,\widetilde{\boldsymbol{F}}^{(h+d_j)}\boldsymbol{T}_{\rm cp}]$ and $\boldsymbol{v}(k) \triangleq \boldsymbol{R}_{\rm cp}\,\widetilde{\boldsymbol{v}}(k) \in \mathbb{C}^N$. It can be verified that $\boldsymbol{G}_j(0)$ is a circulant matrix, with the first column given by $\boldsymbol{Q}_j\,\boldsymbol{g}_j \in \mathbb{C}^N$, where the full-column rank matrix $\boldsymbol{Q}_j \triangleq [\mathbf{O}_{d_j\times(L_j+1)}^T, \boldsymbol{I}_{L_j+1}, \mathbf{O}_{(N-L_j-d_j-1)\times(L_j+1)}^T]^T \in \mathbb{R}^{N\times(L_j+1)}$ accounts for the *unknown* transmission delay d_j and the vector $\boldsymbol{g}_j \triangleq [g_j(0), g_j(1), \ldots, g_j(L_j)]^T \in \mathbb{C}^{L_j+1}$ collects the *unknown* channel coefficients. Using standard eigenstructure concepts, one obtains $\boldsymbol{G}_j(0) = \boldsymbol{W}_{\rm IDFT}\,\boldsymbol{\Psi}_j\,\boldsymbol{W}_{\rm DFT}$, where $\boldsymbol{W}_{\rm DFT} \triangleq \boldsymbol{W}_{\rm IDFT}^H$, with $(\cdot)^H$ denoting the conjugate transpose, and $\boldsymbol{\Psi}_j$ is the diagonal matrix associated with the vector $\boldsymbol{\psi}_j = \sqrt{N}\,\boldsymbol{W}_{\rm DFT}\,\boldsymbol{Q}_j\,\boldsymbol{g}_j \in \mathbb{C}^N$. By substituting $\boldsymbol{G}_j(0)$ in (4) and observing that $\boldsymbol{\Psi}\,\boldsymbol{c}_j = \boldsymbol{C}_j\,\boldsymbol{\psi}_j$, where $\boldsymbol{C}_j \triangleq {\rm diag}[c_j^{(0)}, c_j^{(1)}, \ldots, c_j^{(N-1)}] \in \mathbb{C}^{N\times N}$, we ultimately obtain the concise vector model $\boldsymbol{r}(k) = \boldsymbol{\mathcal{G}}\,\boldsymbol{b}(k) + \boldsymbol{v}(k)$, where $\boldsymbol{\mathcal{G}} \triangleq [\boldsymbol{\mathcal{C}}_1\,\boldsymbol{Q}_1\,\boldsymbol{g}_1, \boldsymbol{\mathcal{C}}_2\,\boldsymbol{Q}_2\,\boldsymbol{g}_2, \ldots, \boldsymbol{\mathcal{C}}_J\,\boldsymbol{Q}_J\,\boldsymbol{g}_J] \in \mathbb{C}^{N\times J}$ and $\boldsymbol{b}(k) = [b_1(k), b_2(k), \ldots, b_J(k)]^T \in \mathbb{C}^J$, with $\boldsymbol{\mathcal{C}}_j \triangleq \sqrt{N}\cdot\boldsymbol{W}_{\rm IDFT}\,\boldsymbol{C}_j\,\boldsymbol{W}_{\rm DFT} \in \mathbb{C}^{N\times N}$.

To obtain a reliable estimate of the desired symbol $b_1(k)$, we consider a linear MMSE-MUD, which performs the linear filtering $y(k) = \boldsymbol{f}^H\boldsymbol{r}(k)$, where $\boldsymbol{f} \in \mathbb{C}^N$ is determined by minimizing the cost function ${\rm MSE} \triangleq {\rm E}[|y(k) - b_1(k)|^2]$, with ${\rm E}[\cdot]$ denoting statistical averaging. The optimal MMSE weight vector is given by $\boldsymbol{f}_{\rm mmse} = \sigma_b^2\,\boldsymbol{R}_{\boldsymbol{rr}}^{-1}\,\boldsymbol{\mathcal{C}}_1\,\boldsymbol{Q}_1\,\boldsymbol{g}_1$, where $\boldsymbol{R}_{\boldsymbol{rr}} \triangleq {\rm E}[\boldsymbol{r}(k)\,\boldsymbol{r}^H(k)] = \sigma_b^2\,\boldsymbol{\mathcal{G}}\,\boldsymbol{\mathcal{G}}^H + \sigma_v^2\,\boldsymbol{I}_N$ is the autocorrelation matrix of $\boldsymbol{r}(k)$. The MMSE-MUD can be conveniently expressed in terms of the eigenvalue decomposition (EVD) of $\boldsymbol{R}_{\boldsymbol{rr}}$. To this end, following common practice in subspace-based multiuser detection literature (see, e.g., [6]–[9]), we assume hereinafter that: **A4)** the channel matrix $\boldsymbol{\mathcal{G}}$ is full-column rank.

[2] The first column of $\widetilde{\boldsymbol{F}}$ and the first row of $\widetilde{\boldsymbol{B}}$ are given by $[0, 1, 0, \ldots, 0]^T$ and $[0, 1, 0, \ldots, 0]$, respectively, with $\widetilde{\boldsymbol{F}}^0 \triangleq \boldsymbol{I}_P$.

The EVD of $\boldsymbol{R_{rr}}$ is given by $\boldsymbol{R_{rr}} = \boldsymbol{V}_s \boldsymbol{\Sigma}_s \boldsymbol{V}_s^H + \sigma_v^2 \boldsymbol{V}_n \boldsymbol{V}_n^H$, where $\boldsymbol{V}_s \in \mathbb{C}^{N \times J}$ collects the eigenvectors associated with the J largest eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_J$ of $\boldsymbol{R_{rr}}$ (arranged in descending order), whose columns span the *signal subspace*, i.e., the column space $\mathcal{R}(\boldsymbol{\mathcal{G}})$ of $\boldsymbol{\mathcal{G}}$, while $\boldsymbol{V}_n \in \mathbb{C}^{N \times (N-J)}$ collects the eigenvectors associated with the eigenvalue σ_v^2, whose columns span the *noise subspace*, i.e., the orthogonal complement $\mathcal{R}^{\perp}(\boldsymbol{\mathcal{G}})$ in $\mathbb{C}^N$ of the signal subspace and, finally, $\boldsymbol{\Sigma}_s \triangleq \text{diag}[\lambda_1, \lambda_2, \ldots, \lambda_J]$. By substituting the EVD of $\boldsymbol{R_{rr}}$ in $\boldsymbol{f}_{\text{mmse}}$, one obtains that $\boldsymbol{f}_{\text{mmse}} = \sigma_b^2 \boldsymbol{V}_s \boldsymbol{\Sigma}_s^{-1} \boldsymbol{V}_s^H \boldsymbol{\mathcal{C}}_1 \boldsymbol{Q}_1 \boldsymbol{g}_1$. The synthesis of the MMSE-MUD requires knowledge of the transmission delay (i.e., $\boldsymbol{Q}_1$) and the channel coefficients (i.e., $\boldsymbol{g}_1$) of the desired user. However, we observe that for each user the matrix $\boldsymbol{Q}_j$ is partially known; more specifically, by exploiting the inequality $d_1 \leq (L_{\text{cp}} - L_1) - 1$, which follows from assumption A3, $\boldsymbol{Q}_j$ can be factorized as $\boldsymbol{Q}_j = \boldsymbol{\Phi} \boldsymbol{P}_j$, where $\boldsymbol{\Phi} \triangleq [\boldsymbol{I}_{L_{\text{cp}}}, \mathbf{O}^T_{(N-L_{\text{cp}}) \times L_{\text{cp}}}]^T \in \mathbb{R}^{N \times L_{\text{cp}}}$ (known) and $\boldsymbol{P}_j \triangleq [\mathbf{O}^T_{d_j \times (L_j+1)}, \boldsymbol{I}_{L_j+1}, \mathbf{O}^T_{(L_{\text{cp}}-L_j-d_j-1) \times (L_j+1)}]^T \in \mathbb{R}^{L_{\text{cp}} \times (L_j+1)}$ (unknown) are full-column rank matrices. Based on this decomposition, one has

$$\boldsymbol{f}_{\text{mmse}} = \sigma_b^2 \boldsymbol{V}_s \boldsymbol{\Sigma}_s^{-1} \boldsymbol{V}_s^H \boldsymbol{\mathcal{C}}_1 \boldsymbol{\Phi} \overline{\boldsymbol{g}}_1 \, , \tag{5}$$

where for each user the vector $\overline{\boldsymbol{g}}_j \triangleq \boldsymbol{P}_j \boldsymbol{g}_j \in \mathbb{C}^{L_{\text{cp}}}$ collects *all* the unknowns of the problem at hand.

At this point, we describe a subspace-based technique that allows one to simultaneously estimate all the unknowns (i.e., $\overline{\boldsymbol{g}}_1$) required to synthesize the MMSE detector given by (5). As it is apparent from the above considerations, unlike MC-DS-CDMA systems [9], the spectral decomposition of $\boldsymbol{R_{rr}}$ exhibits a nontrivial noise subspace $\mathcal{R}(\boldsymbol{V}_n)$. By using the orthogonality between signal and noise subspaces, the following equation can be written

$$\boldsymbol{V}_n^H \boldsymbol{\mathcal{C}}_1 \boldsymbol{Q}_1 \boldsymbol{g}_1 = \mathbf{0}_N \quad \Longleftrightarrow \quad \boldsymbol{V}_n^H \boldsymbol{\mathcal{C}}_1 \boldsymbol{\Phi} \overline{\boldsymbol{g}}_1 = \mathbf{0}_N \, , \tag{6}$$

where $\mathbf{0}_N$ denotes the N-column zero vector, which shows that $\overline{\boldsymbol{g}}_1$ belongs to the null space $\mathcal{N}(\boldsymbol{V}_n^H)$ of $\boldsymbol{V}_n^H$. Under mild conditions, equation (6) *uniquely* characterizes $\overline{\boldsymbol{g}}_1$ up to a complex multiplicative constant.

Theorem 1: Under assumptions A4 and: **A5)** the matrix $\boldsymbol{\Omega} \triangleq [\boldsymbol{\mathcal{C}}_1 \boldsymbol{\Phi}, \boldsymbol{\mathcal{C}}_2 \boldsymbol{Q}_2 \boldsymbol{g}_2, \ldots, \boldsymbol{\mathcal{C}}_J \boldsymbol{Q}_J \boldsymbol{g}_J] \in \mathbb{C}^{N \times (L_{\text{cp}}+J-1)}$ is full-column rank, an arbitrary vector $\boldsymbol{\xi} \in \mathbb{C}^{L_{\text{cp}}}$ is a solution of (6) if and only if $\boldsymbol{\xi} = \alpha_1 \overline{\boldsymbol{g}}_1$, with $\alpha_1 \in \mathbb{C}$.

Proof: The direct statement is easily proven. As to the indirect part, let $\boldsymbol{\xi} \in \mathbb{C}^{L_{\text{cp}}}$ be an arbitrary vector satisfying equation (6), i.e., $\boldsymbol{V}_n^H \boldsymbol{\mathcal{C}}_1 \boldsymbol{\Phi} \boldsymbol{\xi} = \mathbf{0}_N$. This means that $\boldsymbol{\mathcal{C}}_1 \boldsymbol{\Phi} \boldsymbol{\xi} \in \mathcal{N}(\boldsymbol{V}_n^H) = \mathcal{R}^{\perp}(\boldsymbol{V}_n)$ and, due to the uniqueness of the orthogonal complement, it follows that $\boldsymbol{\mathcal{C}}_1 \boldsymbol{\Phi} \boldsymbol{\xi} \in \mathcal{R}(\boldsymbol{\mathcal{G}})$, which implies that $\boldsymbol{\mathcal{C}}_1 \boldsymbol{\Phi} \boldsymbol{\xi}$ can be expressed as a linear combination of the columns of $\boldsymbol{\mathcal{G}}$. Thus, there exist J complex scalar $\alpha_1, \alpha_2, \ldots, \alpha_J$ such that $\boldsymbol{\mathcal{C}}_1 \boldsymbol{\Phi} \boldsymbol{\xi} = \alpha_1 \boldsymbol{\mathcal{C}}_1 \boldsymbol{\Phi} \overline{\boldsymbol{g}}_1 + \alpha_2 \boldsymbol{\mathcal{C}}_2 \boldsymbol{\Phi} \overline{\boldsymbol{g}}_2 + \ldots + \alpha_J \boldsymbol{\mathcal{C}}_J \boldsymbol{\Phi} \overline{\boldsymbol{g}}_J \iff \boldsymbol{\mathcal{C}}_1 \boldsymbol{\Phi} (\boldsymbol{\xi} - \alpha_1 \overline{\boldsymbol{g}}_1) - \alpha_2 \boldsymbol{\mathcal{C}}_2 \boldsymbol{\Phi} \overline{\boldsymbol{g}}_2 - \ldots - \alpha_J \boldsymbol{\mathcal{C}}_J \boldsymbol{\Phi} \overline{\boldsymbol{g}}_J = \mathbf{0}_N$. By invoking A5, it turns out that this equality holds if and only if $\boldsymbol{\xi} - \alpha_1 \overline{\boldsymbol{g}}_1 = \mathbf{0}_{L_{\text{cp}}}$ and $\alpha_2 = \alpha_3 = \ldots = \alpha_J = 0$ and, thus, $\boldsymbol{\xi} = \alpha_1 \overline{\boldsymbol{g}}_1$. ■

In practice, the matrix $\boldsymbol{R_{rr}}$ is estimated from the received data and, therefore, only a sample estimate $\widehat{\boldsymbol{V}}_n$ of $\boldsymbol{V}_n$ is available. In this case, the vector $\overline{\boldsymbol{g}}_1$ can be estimated by resorting (see, e.g., [7]) to the least square approach: $\widehat{\overline{\boldsymbol{g}}}_1 = \arg\min_{\overline{\boldsymbol{g}}_1} \|\widehat{\boldsymbol{V}}_n^H \boldsymbol{\mathcal{C}}_1 \boldsymbol{\Phi} \overline{\boldsymbol{g}}_1\|^2$, with $\|\cdot\|$ denoting the Frobenius norm, whose solution is given by the eigenvector associated with the smallest eigenvalue of $\boldsymbol{\Phi}^T \boldsymbol{\mathcal{C}}_1^H \widehat{\boldsymbol{V}}_n \widehat{\boldsymbol{V}}_n^H \boldsymbol{\mathcal{C}}_1 \boldsymbol{\Phi}$.

Some remarks are now in order. Firstly, assumption A5 requires that the number J of active users must obey the relation $J \leq N - L_{\text{cp}} + 1$; since, to limit the amount of introduced redundancy, the CP length L_{cp} is usually chosen not greater than $0.25\,N$, this condition implies that, in the worst case, the proposed channel identification method can handle up to $J_{\max} = 0.75\,N + 1$ users. Secondly, as in [8] and unlike [6], the proposed subspace-based algorithm does not require exact knowledge of the orders $\{L_j\}_{j=1}^J$ of the channels: it is required *only* (see assumption A3) upper bounds on the orders of the channels and delays of the users[3]. Thirdly, the QS nature of the considered system allows one to avoid the *explicit* estimation of the desired transmission delay: in fact, the estimate of $\overline{\boldsymbol{g}}_1$ automatically contains the information about both the delay and the channel coefficients. Although, in principle, this strategy may also be applied in asynchronous systems, the large uncertainty interval wherein the transmission delays can be confined significantly increases in this case the number of unknowns to be estimated, reducing thus the estimation accuracy: this suggests [6]–[9] to first estimate the unknown delay, by means of a search-based algorithm, and then to perform channel estimation. On the contrary, as shown by our simulation results, joint estimation of delay and channel coefficients does not practically affect the performances of the proposed detector, allowing thus a significant reduction of the receiver complexity[4]. Finally, the synthesis of the proposed receiver is based on the observation of only one symbol period [i.e., the vector $\boldsymbol{r}(k)$], which allows us to further reduce the computational complexity[5] with respect to that of the methods [6]–[9].

IV. Simulation results

We present the performance analysis (carried out by Monte Carlo computer simulations) of the proposed blind subspace-based MMSE-MUD (referred to as B-MMSE), together with a comparison with its non-blind counterpart (referred to as NB-MMSE), which has exact knowledge of the channel vector $\boldsymbol{g}_1$ and delay d_1 of the desired user. The QS-MC-CDMA network employs $N = 32$ subcarriers, with a cyclic prefix of length $L_{\text{cp}} = 8$, and QPSK symbol modulation; the frequency-domain spreading codes are length-32 Walsh-Hadamard sequences. The multipath channel of the jth user is $g_{c,j}(t) =$

[3]This is a reasonable assumption since: (i) in general, depending on the transmitted signal parameters (carrier frequency and bandwidth) and application (indoor or outdoor), the maximum channel multipath spread is known; (ii) for QS cellular systems, the delays of the users are confined to a small uncertainty interval, whose support can be typically predicted [11].

[4]The complexity of the algorithms [6], [7] for timing acquisition increases as the fourth-power of the processing gain; in [8], this complexity is reduced of one order of magnitude by using a QR-based method; finally, the complexity (for a given hypothesized value of the delay) of the algorithm employed in [9] increases cubically with the number of subcarriers.

[5]To construct a nontrivial noise subspace, the approaches of [6]–[9] are based on the observation of multiple consecutive symbol intervals, which significantly increases the dimension of the matrices involved.

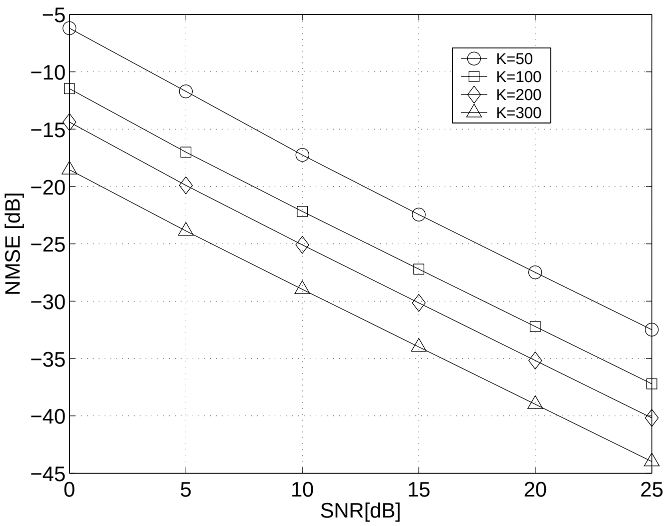

Fig. 1. Channel NMSE versus SNR for different values of the sample size.

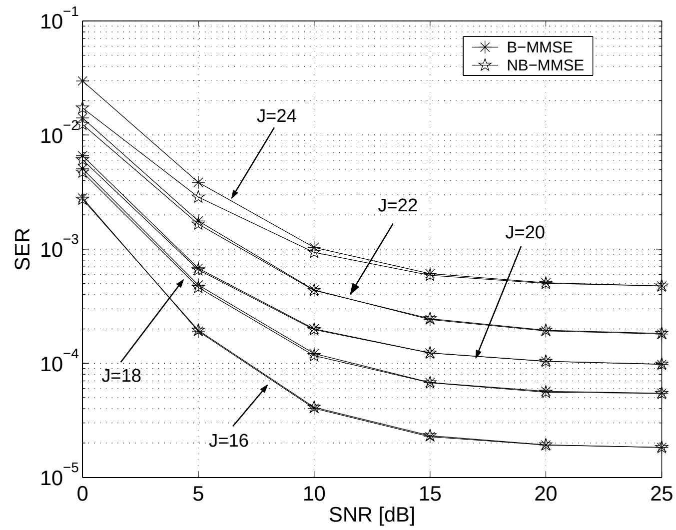

Fig. 2. SER versus SNR for different values of the number J of active users.

$\sum_{m=1}^{4} \beta_{m,j}\, \varphi_c(t-\tau_{m,j})$, for $t \in [0, L_j\, T_c)$, where $\varphi_c(t)$ is a Nyquist-shaped pulse with roll-off 0.35, the path gains $\beta_{m,j}$, for $m = 1, 2, 3, 4$, are modeled as mutually independent complex circular Gaussian zero-mean random variables, with standard deviation 0.3, whereas the corresponding propagation times $\tau_{m,j}$ are modeled as mutually independent random variables uniformly distributed over $L_j + 1 = 6$ symbol intervals. The transmission delays τ_j are uniformly distributed in $\{0, 1, \ldots, L_{\rm cp} - L_j - 1\}$. The signal-to-noise ratio (SNR) of the desired user at the detector input is defined, according to (4), as SNR $\triangleq \sigma_b^2 \, \|\boldsymbol{G}_1(0)\, \boldsymbol{W}_{\rm IDFT}\, \boldsymbol{c}_1\|^2 / {\rm E}[\, \|\boldsymbol{v}(k)\|^2\,]$. We considered a severe near-far scenario: the path gains of each user channel are adjusted so that each interfering user is 10 dB stronger than the desired user. As performance measure, we resorted to the symbol error rate (SER) at the output of the considered receivers, whereas, to analyze the estimation accuracy of the proposed identification algorithm, we used the normalized mean square error (NMSE) between the estimated vector $\widehat{\overline{\boldsymbol{g}}}_1$ and the true one $\overline{\boldsymbol{g}}_1$, defined as NMSE $\triangleq \min_\gamma \left[\|\overline{\boldsymbol{g}}_1 - \gamma\, \widehat{\overline{\boldsymbol{g}}}_1\|^2 / \|\overline{\boldsymbol{g}}_1\|^2 \right]$. After estimating the receiver weights on the basis of the given data record of length K, an independent record of $K_{\rm ser} = 10^5$ symbols is considered to evaluate the SER at the output of the considered receivers. For the proposed blind receiver, the equalized symbols are first rotated and scaled before evaluating the SER. The results are averaged over 100 independent trials, with each run using a different set of transmission delays, data e noise sequence, whereas the path gains and propagation delays for each user are randomly generated and then fixed over all runs.

Fig. 1 reports the values of NMSE, obtained with the proposed identification technique, as a function of SNR ranging from 0 to 25 dB, and for different values of the sample size $K \in \{50, 100, 200, 300\}$, with $J = 22$. Results show that the proposed method is able to successfully estimate the vector $\overline{\boldsymbol{g}}_1$ even when K is as short as 50 symbols; clearly, the estimation accuracy improves as either SNR or K increase. Thus, the proposed method demonstrates the ability of providing reliable estimates, without requiring an initial delay estimation. Finally, we reported in Fig. 2 the values of SER, as a function of SNR, and for different values of the number of active users $J \in \{16, 18, 20, 22, 24\}$, with $K = 300$ symbols. Note that the SER curves of the B-MMSE and NB-MMSE receivers are practically indistinguishable for almost all values of SNR and J, confirming again the effectiveness of the proposed estimation procedure. Moreover, results of Fig. 2 show the capability of the proposed blind detector to safely operate in a severe near-far scenario; in particular, it is worthwhile to note that, with $J = 24$ active users, the B-MMSE receiver assures a SER equal to 10^{-3} even for a SNR of 10 dB.

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