Performance analysis of amplify-and-forward multiple-relay MIMO systems with ZF reception

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***Abstract*—This paper deals with performance analysis of a cooperative multiple-input multiple-output (MIMO) network with spatial multiplexing, wherein multiple half-duplex relays perform noncoherent (i.e., without channel state information) amplify-and-forward (AF) relaying and the destination employs a linear zero-forcing (ZF) equalizer. The correlation among the noise samples at the destination, the non-Gaussian nature of the dual-hop channel, and the fact that the relays are located in different positions significantly complicate the performance analysis of the system. In the high signal-to-noise ratio regime, we derive a simple and accurate approximation of the symbol error probability at the destination. In the special case of a relaying cluster, such a result allows to discuss the best placement of the relays and the performance gain of cooperation over the direct (i.e., without relaying) transmission. The theoretical analysis is validated by comparison with semi-analytical Monte Carlo simulations.**

***Index Terms*—Amplify-and-forward (AF) relaying, linear zero-forcing (ZF) equalization, multiple-input multiple-output (MIMO) systems, spatial multiplexing.**

I. INTRODUCTION

In fading channels, the use of multiple-input multiple-output (MIMO) systems is a well-known approach to achieve diversity and/or multiplexing gains [1]. In recent years, a significant attention has also been devoted to cooperative diversity techniques [2], due to their ability to improve coverage and quality of service through relaying, both in infrastructure-based [3], [4] and *ad-hoc* [5]–[7] wireless networks. Among cooperative techniques [8], the amplify-and-forward (AF) relaying protocol allows one to gain most of the benefits of cooperation without a significant increase in system complexity [9]–[11]: in this protocol, the relays receive data from the source during the first time slot (*first hop*), and forward an amplified version of the received signal during the second time slot (*second hop*), while the source remains silent. Moreover, albeit suboptimal, linear designs of cooperative MIMO transceivers have received a great deal of attention [12]–[16], since they offer a good trade-off between performance and complexity.

Although the aforementioned designs allow one to keep computational requirements low, they require that full channel state information (CSI) about the first and second hops of the MIMO cooperative transmission are known to all the cooperating nodes: in practice, acquisition of such a large amount of CSI might lead to an unsustainable waste of communication resources, especially in cooperative networks with multiple MIMO relays. According to these drawbacks, all the works [12]–[16] and the related theoretical performance analyses [17]–[20] deal only with the single-relay case.

In this paper, we consider the performance analysis, in terms of symbol error probability (SEP), of a cooperative system, where multiple MIMO AF relays simultaneously transmit in the second hop, and the destination adopts linear zero-forcing (ZF) equalization. To avoid incurring significant signaling overhead, a *non-coherent* [21]–[23] or *blind* [24] relaying system is considered, where CSI is not available at the source and relays. Although noncoherent relaying simplifies the analysis, since precoding at the source and relays does not depend on the channel coefficients, there are two additional problems with respect to existing theoretical studies [17]–[20]: (i) the correlation between noise samples at the destination, due to the simultaneous relay transmission; (ii) the different locations of the relays, which imply different variances of the fading channel coefficients. Relying on the strong law of large numbers [25] and on properties of the linear combination of Wishart matrices with unequal covariances [26], we develop an approximation of the noise correlation matrix that allows us to derive an accurate yet simple approximation of the SEP in the high-SNR regime, for an arbitrary number and locations of relays. According to [21], we show that noncoherent schemes cannot achieve distributed spatial diversity, but only coding and multiplexing gain. Additionally, by specializing our results in the case of a *relaying cluster*, for which the distances between the different relays are negligible with respect to the distance between the source and the destination, we discuss the placement of the cluster that maximizes the coding gain and, moreover, we present a comparison between cooperative and direct transmissions.

II. SYSTEM MODEL AND BASIC ASSUMPTIONS

The network of Fig. 1 is composed by a source-destination pair, which cannot communicate directly due to large shadowing effects, and $N_{\rm C}$ half-duplex AF relays, with no relay-to-relay communication. All nodes are perfectly synchronized

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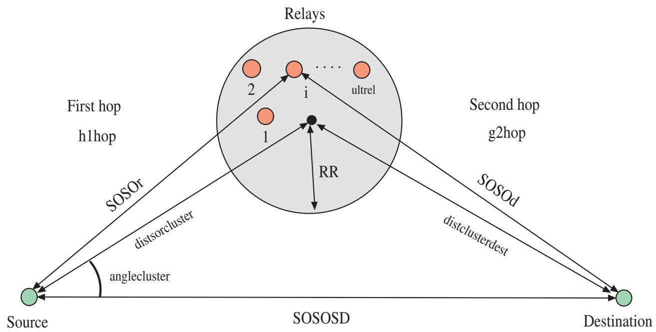

Fig. 1. The considered network configuration.

and equipped with multiple antennas, with the number of antennas at source, relays, and destination denoted by N_{S}, N_{R}, and N_{D}, respectively. A frequency-flat quasi-static fading channel model is assumed for each pair of antennas, with $\mathbf{H}_i \in \mathbb{C}^{N_{\mathrm{R}} \times N_{\mathrm{S}}}$ and $\mathbf{G}_i \in \mathbb{C}^{N_{\mathrm{D}} \times N_{\mathrm{R}}}$ denoting the first- and second-hop MIMO channels, respectively, for $i \in \{1, 2, \ldots, N_{\mathrm{C}}\}$. Instantaneous CSI is known only at the destination via training, but is unknown at both the source and relays [21]–[24].

Let $\mathbf{s} \in \mathbb{C}^{N_{\mathrm{B}}}$ gather the block of symbols to be transmitted by the source during the first time slot, we assume that **(a1):** the entries of $\mathbf{s}$ are zero-mean unit-variance circularly [27] symmetric complex random variables (RVs), which assume Q independent and identically distributed (i.i.d.) quadrature amplitude modulation (QAM) equiprobable values. The symbol block $\mathbf{s}$ is processed by a full-column rank precoding matrix $\mathbf{F}_0 \in \mathbb{C}^{N_{\mathrm{S}} \times N_{\mathrm{B}}}$, thus obtaining $\tilde{\mathbf{s}} = \mathbf{F}_0\, \mathbf{s} \in \mathbb{C}^{N_{\mathrm{S}}}$, with $\mathrm{tr}(\mathbf{F}_0^{\mathrm{H}}\, \mathbf{F}_0) = 1$, which ensures that $\mathbb{E}[\|\tilde{\mathbf{s}}\|^2] = 1$. During the second time slot, the received signal at the ith relay is given by $\mathbf{z}_i = \mathbf{H}_i\, \tilde{\mathbf{s}} + \mathbf{w}_i$, where $\mathbf{w}_i \in \mathbb{C}^{N_{\mathrm{R}}}$ accounts for noise. At the ith relay, the vector $\mathbf{z}_i$ is scaled by $\sqrt{\alpha_i}$ and processed by a nonsingular forwarding matrix $\mathbf{F}_i \in \mathbb{C}^{N_{\mathrm{R}} \times N_{\mathrm{R}}}$, hence yielding $\tilde{\mathbf{z}}_i \triangleq \sqrt{\alpha_i}\, \mathbf{F}_i\, \mathbf{z}_i \in \mathbb{C}^{N_{\mathrm{R}}}$, with $\mathrm{tr}(\mathbf{F}_i^{\mathrm{H}}\, \mathbf{F}_i) = N_{\mathrm{R}}$. The signal at the destination is $\mathbf{r} = \mathbf{C}\, \mathbf{s} + \mathbf{v}$, where

$$\mathbf{C} \triangleq \left(\sum_{i=1}^{N_{\mathrm{C}}} \sqrt{\alpha_i}\, \mathbf{G}_i\, \mathbf{F}_i\, \mathbf{H}_i \right) \mathbf{F}_0 \in \mathbb{C}^{N_{\mathrm{D}} \times N_{\mathrm{B}}} \quad (1)$$

$$\mathbf{v} \triangleq \sum_{i=1}^{N_{\mathrm{C}}} \sqrt{\alpha_i}\, \mathbf{G}_i\, \mathbf{F}_i\, \mathbf{w}_i + \mathbf{d} \in \mathbb{C}^{N_{\mathrm{D}}} \quad (2)$$

denote the dual-hop channel matrix and the overall noise at the destination, respectively, with $\mathbf{d} \in \mathbb{C}^{N_{\mathrm{D}}}$ representing the noise at the destination. The following customary assumptions are made: **(a2):** $\mathbf{w}_i$ and $\mathbf{d}$ are zero-mean circularly symmetric complex Gaussian (ZMCSCG) RVs, whose variance is σ^2; **(a3):** the entries of $\mathbf{H}_i$ and $\mathbf{G}_i$ are i.i.d. ZMCSCG RVs having variance $\sigma^2_{\mathrm{h},i} \triangleq (d_{\mathrm{SD}}/d_{\mathrm{SR}_i})^{\eta}$ and $\sigma^2_{\mathrm{g},i} \triangleq (d_{\mathrm{SD}}/d_{\mathrm{R}_i\mathrm{D}})^{\eta}$, respectively, where d_{SD} is the distance between the source and destination, d_{SR_i} is the distance between the source and the ith relay, $d_{\mathrm{R}_i\mathrm{D}}$ is the distance between the ith relay and the destination, and $\eta \geq 2$ is the path-loss exponent; **(a4):** the channel matrix $\mathbf{C} \in \mathbb{C}^{N_{\mathrm{D}} \times N_{\mathrm{B}}}$ is full-column rank, i.e., $N_{\mathrm{D}} \geq N_{\mathrm{B}}$ and $\mathrm{rank}(\mathbf{C}) = N_{\mathrm{B}}$, with probability one.

When the source and relays have no CSI, a sensible choice [28], [29] is to assume that $\mathbf{F}_0$ and $\{\mathbf{F}_i\}_{i=1}^{N_{\mathrm{C}}}$ are scaled semi-unitary matrices, i.e., $\mathbf{F}_0^{\mathrm{H}}\, \mathbf{F}_0 = \mathbf{I}_{N_{\mathrm{B}}}/N_{\mathrm{B}}$ and $\mathbf{F}_i^{\mathrm{H}}\, \mathbf{F}_i = \mathbf{F}_i\, \mathbf{F}_i^{\mathrm{H}} = \mathbf{I}_{N_{\mathrm{R}}}$: in this case, $\mathbf{F}_0$ is employed to adjust the multiplexing rate and $\{\mathbf{F}_i\}_{i=1}^{N_{\mathrm{C}}}$ are used to avoid possible rank deficiency of $\mathbf{C}$ [see **(a4)**]. Each scaling factor α_i is chosen so as to satisfy the average power scaling constraint $\mathbb{E}[\|\tilde{\mathbf{z}}_i\|^2] = N_{\mathrm{R}}\, P_i$, where P_i is the power available at the ith relay for each antenna, thus obtaining $\alpha_i = \gamma\, P_i/(1 + \gamma\, \sigma^2_{\mathrm{h},i})$, with $\gamma \triangleq \sigma^{-2}$ denoting the average signal-to-noise ratio (SNR) per transmitted symbol block.

III. Theoretical performance analysis

Let us introduce $\mathbf{G} \triangleq [\mathbf{G}_1, \mathbf{G}_2, \ldots, \mathbf{G}_{N_{\mathrm{C}}}] \in \mathbb{C}^{N_{\mathrm{D}} \times (N_{\mathrm{C}} N_{\mathrm{R}})}$ and $\mathbf{H} \triangleq [\mathbf{H}_1^{\mathrm{T}}, \mathbf{H}_2^{\mathrm{T}}, \ldots, \mathbf{H}_{N_{\mathrm{C}}}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{C}^{(N_{\mathrm{C}} N_{\mathrm{R}}) \times N_{\mathrm{S}}}$; by applying the conditional expectation rule, the error probability $\mathrm{P}_{\mathrm{coop},n}(e)$ in detecting the nth entry of $\mathbf{s}$ can be calculated as $\mathrm{P}_{\mathrm{coop},n}(e) = \mathbb{E}_{\mathbf{H}}\{\mathbb{E}_{\mathbf{G}|\mathbf{H}}[\mathrm{P}_{\mathrm{coop},n}(e|\mathbf{G}, \mathbf{H})]\}$, where $\mathrm{P}_{\mathrm{coop},n}(e|\mathbf{G}, \mathbf{H})$ represents the SEP conditioned on $\mathbf{H}$ and $\mathbf{G}$. The conditional SEP $\mathrm{P}_{\mathrm{coop},n}(e|\mathbf{H}) \triangleq \mathbb{E}_{\mathbf{G}|\mathbf{H}}[\mathrm{P}_{\mathrm{coop},n}(e|\mathbf{G}, \mathbf{H})]$ is given [30] by

$$\mathrm{P}_{\mathrm{coop},n}(e\,|\,\mathbf{H}) = \frac{2\,b}{\pi} \int_0^{\pi/2} \Phi_{\mathrm{SINR}_n} \left(-\frac{u}{\sin^2 x}; \mathbf{H} \right) \mathrm{d}x \quad (3)$$

where the parameters $b \triangleq 2(1 - 1/\sqrt{Q})$ and $u \triangleq 3/[2(Q-1)]$ depend on the cardinality of the symbol constellation, whereas $\Phi_{\mathrm{SINR}_n}(y; \mathbf{H}) \triangleq \mathbb{E}_{\mathbf{G}|\mathbf{H}}\left[\exp\left(y\, \mathrm{SINR}_n\right)\right]$ is the moment generating function (MGF), conditioned on $\mathbf{H}$, of the signal-to-interference-plus-noise ratio $\mathrm{SINR}_n \triangleq 1/\left\{\mathbf{C}^{\dagger}\, \mathbf{K}_{\mathbf{vv}}\, \mathbf{C}^{\dagger \mathrm{H}}\right\}_{nn}$, with $\mathbf{K}_{\mathbf{vv}} \triangleq \mathbb{E}[\mathbf{v}\, \mathbf{v}^{\mathrm{H}}\,|\mathbf{G}]$ denoting the correlation matrix of the noise at the output of the ZF equalizer.

A. SINR approximation

It can be readily shown that $\mathbf{K}_{\mathbf{vv}}$ can be decomposed as $\mathbf{K}_{\mathbf{vv}} = \sigma^2\, (\mathbf{S}_{N_{\mathrm{C}}} + \mathbf{I}_{N_{\mathrm{D}}})$, with $\mathbf{S}_{N_{\mathrm{C}}} \triangleq \sum_{i=1}^{N_{\mathrm{C}}} \alpha_i\, \mathbf{G}_i\, \mathbf{G}_i^{\mathrm{H}}$ being a sum of complex central Wishart matrices [31] $\alpha_i\, \mathbf{G}_i\, \mathbf{G}_i^{\mathrm{H}}$, each one having N_{R} degrees of freedom and covariance matrix $\alpha_i^2\, \sigma^2_{g,i}\, \mathbf{I}_{N_{\mathrm{D}}}$, respectively. The random matrix $\mathbf{S}_{N_{\mathrm{C}}}$ has mean $\mathbf{M}_{N_{\mathrm{C}}} \triangleq \mathbb{E}[\mathbf{S}_{N_{\mathrm{C}}}] = \varrho\, N_{\mathrm{R}}\, \mathbf{I}_{N_{\mathrm{D}}}$, with $\varrho \triangleq \sum_{i=1}^{N_{\mathrm{C}}} \alpha_i\, \sigma^2_{\mathrm{g},i} > 0$, and variance given by

$$\begin{aligned} \mathrm{VAR}(\mathbf{S}_{N_{\mathrm{C}}}) &\triangleq \mathbb{E}\left[\|\mathbf{S}_{N_{\mathrm{C}}} - \mathbf{M}_{N_{\mathrm{C}}}\|^2\right] = \\ &- 2\,\varrho\, N_{\mathrm{R}} \sum_{i=1}^{N_{\mathrm{C}}} \alpha_i\, \mathbb{E}\left[\mathrm{tr}\left(\mathbf{G}_i\, \mathbf{G}_i^{\mathrm{H}}\right)\right] + \varrho^2\, {N_{\mathrm{R}}}^2\, N_{\mathrm{D}} \\ &+ \sum_{i=1}^{N_{\mathrm{C}}} \sum_{\substack{k=1 \\ k \neq i}}^{N_{\mathrm{C}}} \alpha_i\, \alpha_k\, \mathbb{E}\left[\mathrm{tr}\left(\mathbf{G}_i\, \mathbf{G}_i^{\mathrm{H}}\, \mathbf{G}_k\, \mathbf{G}_k^{\mathrm{H}}\right)\right] \\ &+ \sum_{i=1}^{N_{\mathrm{C}}} \alpha_i^2\, \mathbb{E}\left\{\mathrm{tr}\left[\left(\mathbf{G}_i\, \mathbf{G}_i^{\mathrm{H}}\right)^2\right]\right\} = N_{\mathrm{R}}\, {N_{\mathrm{D}}}^2 \sum_{i=1}^{N_{\mathrm{C}}} \alpha_i^2\, \sigma^4_{\mathrm{g},i} \end{aligned} \quad (4)$$

where we additionally observed [31] that, for complex central Wishart matrices, $\mathbb{E}\{\mathrm{tr}[(\mathbf{G}_i\, \mathbf{G}_i^{\mathrm{H}})^2]\} = \sigma^4_{\mathrm{g},i}\, N_{\mathrm{R}}\, N_{\mathrm{D}}\, (N_{\mathrm{R}} + N_{\mathrm{D}})$. When the number of relays N_{C} is sufficiently large, matrix $\mathbf{S}_{N_{\mathrm{C}}}$ converges *almost surely* to the constant matrix $\mathbf{M}_{N_{\mathrm{C}}}$, as stated by the following lemma:

Lemma 1: Under the assumption that

$$\sum_{i=1}^{+\infty} \frac{\alpha_i^2\, \sigma_{\text{g},i}^4}{i^2} < +\infty \tag{5}$$

the matrix sequence $\{\alpha_i\, \mathbf{G}_i\, \mathbf{G}_i^{\text{H}}\}_{i=1}^{N_\text{C}}$ obeys the strong law of large numbers, i.e., $\forall \epsilon > 0$ and $\forall \Delta > 0$, there is $N > 0$ such that, $\forall K > 0$, the probability of the simultaneous fulfilment of the K inequalities $\|\mathbf{S}_{N_\text{C}} - \mathbf{M}_{N_\text{C}}\|/N_\text{C} < \epsilon$ is greater than or equal to $1 - \Delta$, for $N_\text{C} \in \{N, N+1, \ldots, N+K\}$.

Proof: The proof, which is omitted for brevity, relies on a generalization of the Kolmogorov's inequality [25] to the case of random matrices. ■

Fulfilment of (5) depends on the sequence $\{\alpha_i^2\, \sigma_{\text{g},i}^4\}_{i=1}^{+\infty}$ whose general term can be upper bounded as

$$\begin{aligned} \alpha_i^2\, \sigma_{\text{g},i}^4 &= \gamma^2\, P_i^2\, \left[\sigma_{\text{g},i}^2/(1+\gamma\, \sigma_{\text{h},i}^2)\right]^2 \\ &\le \gamma^2\, P_{\text{max}}^2\, \left[\sigma_{\text{g},i}^2/(1+\gamma\, \sigma_{\text{h},i}^2)\right]^2 \end{aligned} \tag{6}$$

where $P_{\text{max}} > 0$ is the maximum (finite) power available at each relay. Thus, one can argue that (5) is trivially satisfied when the average path losses do not depend on i, i.e., $\sigma_{\text{h},i}^2 = (d_{\text{SD}}/d_{\text{SR}_i})^\eta = \sigma_\text{h}^2$ and $\sigma_{\text{g},i}^2 = (d_{\text{SD}}/d_{\text{R}_i\text{D}})^\eta = \sigma_\text{g}^2$: in this case, the sum of the series in (5) is upper bounded by $(\pi^2\, \alpha_{\text{max}}^2\, \sigma_\text{g}^4)/6$, with $\alpha_{\text{max}} \triangleq (\gamma\, P_{\text{max}})/(1+\gamma\, \sigma_\text{h}^2)$. From a physical viewpoint, this happens when the relays form a relaying cluster, wherein the distances between the different relays are negligible with respect to the distance d_{SD} between the source and destination. When the cooperative relays do not belong to a cluster, the general term of $\{\alpha_i^2\, \sigma_{\text{g},i}^4\}_{i=1}^{+\infty}$ can be further upper bounded as

$$\begin{aligned} \alpha_i^2\, \sigma_{\text{g},i}^4 &\le \gamma^2\, P_{\text{max}}^2\, \left[\sigma_{\text{g},\text{max}}^2/(1+\gamma\, \sigma_{\text{h},\text{min}}^2)\right]^2 \\ &< P_{\text{max}}^2\, (d_{\text{SR}_{\text{max}}}/d_{\text{R}_{\text{min}}\text{D}})^{2\eta} \end{aligned} \tag{7}$$

with $\sigma_{\text{h},\text{min}}^2 \triangleq (d_{\text{SD}}/d_{\text{SR}_{\text{max}}})^\eta$ and $\sigma_{\text{g},\text{max}}^2 \triangleq (d_{\text{SD}}/d_{\text{R}_{\text{min}}\text{D}})^\eta$, where $d_{\text{SR}_{\text{max}}}$ is the distance of the farthest relay from the source, i.e., $d_{\text{SR}_{\text{max}}} \triangleq \max_{i\in\{1,2,\ldots,N_\text{C}\}} d_{\text{SR}_i}$, and $d_{\text{R}_{\text{min}}\text{D}}$ is the distance of the nearest relay to the destination, i.e., $d_{\text{R}_{\text{min}}\text{D}} \triangleq \min_{i\in\{1,2,\ldots,N_\text{C}\}} d_{\text{R}_i\text{D}}$. Hence, for any value of γ, a *sufficient condition* for fulfilment of (5) is that $d_{\text{SR}_{\text{max}}}/d_{\text{R}_{\text{min}}\text{D}} \le \chi$, for some finite constant $0 < \chi < +\infty$, which, besides requiring that $d_{\text{SR}_{\text{max}}}$ is finite, imposes that $d_{\text{R}_{\text{min}}\text{D}}$ must be sufficiently large, i.e., the relays cannot be too close to the destination; in this case, the sum of the series in (5) is upper bounded by $(\pi^2\, P_{\text{max}}^2\, \chi^{2\eta})/6$.

B. SEP approximation

Roughly speaking, Lemma 1 states that, with high probability, $\|\mathbf{S}_{N_\text{C}} - \mathbf{M}_{N_\text{C}}\|/N_\text{C}$ remains small for all $N_\text{C} \ge N$, provided that (5) holds. Therefore, if N_C is sufficiently large, by replacing $\mathbf{S}_{N_\text{C}}$ with $\mathbf{M}_{N_\text{C}}$, thus obtaining $\mathbf{K}_{\mathbf{vv}} \approx \sigma^2\, (\varrho\, N_\text{R} + 1)\, \mathbf{I}_{N_\text{D}}$, the SINR is approximated as

$$\text{SINR}_n \approx \frac{\gamma}{\beta}\, \{(\mathbf{C}^\text{H}\, \mathbf{C})^{-1}\}_{nn}^{-1} \tag{8}$$

where $\beta \triangleq \varrho\, N_\text{R} + 1$. At this point, we state the main result of the paper:

Theorem 1: Let $N_\text{R} \ge N_\text{B}$ and $N_\text{C}\, N_\text{R} > N_\text{D}$. In the high-SNR region, i.e., for $\gamma \gg 1$, one has

$$\begin{aligned} \text{P}_{\text{coop},n}(e) &\approx \overline{\text{P}}_{\text{coop}}(e) \\ &\triangleq \Upsilon_{\text{coop}} \left[\frac{3\,\gamma \displaystyle\sum_{i=1}^{N_\text{C}} (\alpha_i\, \sigma_{\text{h},i}^2\, \sigma_{\text{g},i}^2)^2}{2\, N_\text{B}\, (Q-1)\, \beta \displaystyle\sum_{i=1}^{N_\text{C}} \alpha_i\, \sigma_{\text{h},i}^2\, \sigma_{\text{g},i}^2} \right]^{-(N_\text{D}-N_\text{B}+1)} \end{aligned} \tag{9}$$

where

$$\Upsilon_{\text{coop}} \triangleq 2\, \Theta(N_\text{D} - N_\text{B} + 1) \left(1 - \frac{1}{\sqrt{Q}}\right) \frac{(N_\text{C}\, N_\text{R} - N_\text{D} - 1)!}{(N_\text{C}\, N_\text{R} - N_\text{B})!}. \tag{10}$$

with $\Theta(v) \triangleq (2/\pi) \int_0^{\pi/2} (\sin^2 x)^v\, \mathrm{d}x$, $v \in \mathbb{R}$.

Proof: See Appendix A. ■

As a first remark, we highlight that $\overline{\text{P}}_{\text{coop}}(e)$ turns out to be independent of the symbol index n. Furthermore, it is worth noting that, in the high-SNR regime, the scaling factor $\alpha_i \approx P_i/\sigma_{\text{h},i}^2$ becomes independent of γ. Consequently, $\{\alpha_i\, \sigma_{\text{h},i}^2\, \sigma_{\text{g},i}^2\}_{i=1}^{N_\text{C}}$ and β in (9) turn out to be independent of γ and, thus, the SEP can be simply approximated as $\overline{\text{P}}_{\text{coop}}(e) \approx (C_{\text{coop}}\, \gamma)^{-D}$, where $D \triangleq N_\text{D} - N_\text{B} + 1$ is the diversity order, whereas the coding gain C_{coop} is given by

$$C_{\text{coop}} \triangleq \frac{3\, \Upsilon_{\text{coop}}^{-1/D} \displaystyle\sum_{i=1}^{N_\text{C}} P_i^2\, \sigma_{\text{g},i}^4}{2\, N_\text{B}\, (Q-1) \left(N_\text{R} \displaystyle\sum_{i=1}^{N_\text{C}} P_i\, \sigma_{\text{h},i}^{-2}\, \sigma_{\text{g},i}^2 + 1\right) \displaystyle\sum_{i=1}^{N_\text{C}} P_i\, \sigma_{\text{g},i}^2}. \tag{11}$$

It is noteworthy that, as stated in [21], the diversity order of the system does not depend on the number of relays N_C: such a result stems from the fact that the forwarding matrices are channel-independent. Moreover, we highlight that the SEP is independent of the precoding and forwarding matrices $\{\mathbf{F}_i\}_{i=0}^{N_\text{C}}$, i.e., all AF MIMO cooperative systems with linear equalization at the destination and (semi-)unitary precoding and forwarding matrices exhibit the same SEP given by (9).

C. Optimal placement of the relaying cluster

To streamline the subsequent discussions, we assume hereinafter that the relays use the same maximum transmitting power,[1] i.e., $P_i = P_{\text{max}}$, $\forall i \in \{1, 2, \ldots, N_\text{C}\}$. Furthermore, we focus on the case when the relays form a relaying cluster (see Fig. 1) and, thus, the average path losses are approximately independent of i, i.e., $\sigma_{\text{h},i}^2 \approx \sigma_\text{h}^2 \triangleq (d_{\text{SD}}/d_{\text{SC}})^\eta$ and $\sigma_{\text{g},i}^2 \approx \sigma_\text{g}^2 \triangleq (d_{\text{SD}}/d_{\text{CD}})^\eta$, where d_{SC} is the source-cluster distance and d_{CD} is the cluster-destination distance.[2] Particularizing

[1]Joint optimization of the relays' transmitting powers $P_1, P_2 \ldots, P_{N_\text{C}}$, which is a complicated task in the considered decentralized framework, is outside the scope of this paper.

[2]We will show by numerical simulations in Section IV that results in this case turn out to be valuable design guidelines even when the average path losses are different, provided that the distances between the different relays are sufficiently small with respect to the distance between the source and destination.

(11) to the case when $P_i = P_{\text{max}}$, $\sigma^2_{\text{h},i} \approx \sigma^2_{\text{h}}$, and $\sigma^2_{\text{g},i} \approx \sigma^2_{\text{g}}$, $\forall i \in \{1, 2, \ldots, N_{\text{C}}\}$, one has

$$C_{\text{coop}} \approx \frac{3\, P_{\text{max}}\, \Upsilon_{\text{coop}}^{-1/D}\, \sigma^2_{\text{g}}\, \sigma^2_{\text{h}}}{2\, N_{\text{B}}\, (Q-1)\, \left(N_{\text{C}}\, N_{\text{R}}\, P_{\text{max}}\, \sigma^2_{\text{g}} + \sigma^2_{\text{h}}\right)} \,. \tag{12}$$

We say that the relaying cluster is optimally placed if the coding gain C_{coop} in (12) is maximized. Our aim is to calculate the optimal values of d_{SC} and d_{CD} that maximize the coding gain (12). Assuming d_{SD} fixed and applying the Carnot's cosine law to the triangle in Fig. 1, one obtains that

$$d_{\text{CD}} = \sqrt{d^2_{\text{SC}} + d^2_{\text{SD}} - 2\, d_{\text{SC}}\, d_{\text{SD}}\, \cos(\theta)} \tag{13}$$

where $\theta \in [0, 2\pi)$ is the angle between the source-cluster and source-destination directions. At this point, the problem boils down to computing d_{SC} and θ. It is easily shown that C_{coop} in (12) reaches its maximum value when

$$d^{\eta-1}_{\text{SC}} + N_{\text{C}}\, N_{\text{R}}\, P_{\text{max}}\, d^{\eta-2}_{\text{CD}}\, [d_{\text{SC}} - d_{\text{SD}}\, \cos(\theta)] = 0 \quad \text{and} \quad \sin(\theta) = 0 \,. \tag{14}$$

The latter equation imposes that $\theta = \theta_{\text{opt}} = 0$, i.e., the relaying cluster lies on the line joining the source and destination, and, thus, $d_{\text{CD}} = |d_{\text{SD}} - d_{\text{SC}}|$; in this case, one can infer, from the former equation, that d_{SD} is necessarily greater than d_{SC}, i.e., $d_{\text{SD}} > d_{\text{SC}}$, and thus

$$d_{\text{SC}} = (d_{\text{SC}})_{\text{opt}} = \frac{d_{\text{SD}}}{1 + \sqrt[\eta-1]{N_{\text{C}}\, N_{\text{R}}\, P_{\text{max}}}} \quad \text{and} \quad d_{\text{CD}} = (d_{\text{CD}})_{\text{opt}} = d_{\text{SD}} - (d_{\text{SC}})_{\text{opt}} \,. \tag{15}$$

It is apparent from (15) that $0 < (d_{\text{SC}})_{\text{opt}} < d_{\text{SD}}$. More specifically, in the case where $P_{\text{max}}\, N_{\text{C}}\, N_{\text{R}} = 1$, which implies that $(d_{\text{SC}})_{\text{opt}} = d_{\text{SD}}/2$, the coding gain is maximized when the cluster is equidistant from the source and destination. Moreover, when $P_{\text{max}}\, N_{\text{C}}\, N_{\text{R}} > 1$, it results that $(d_{\text{SC}})_{\text{opt}} < d_{\text{SD}}/2$, i.e., the maximum value of C_{coop} in (12) is achieved if the relaying cluster is closer to the source than the destination. Finally, if $P_{\text{max}}\, N_{\text{C}}\, N_{\text{R}} < 1$, then $(d_{\text{SC}})_{\text{opt}} > d_{\text{SD}}/2$, i.e., the coding gain reaches its maximum when the cluster is closer to the destination than the source. In these last two cases, the best location of the cluster is also dictated by the path-loss exponent η.

D. Comparison between cooperative and direct transmissions

It is interesting to compare (9) with the SEP of the direct transmission between the source and destination, i.e., when there are no relays. In this case, it can be verified [30] that, for $\gamma \gg 1$, the SEP in detecting the nth symbol s_n, in the case of direct transmission with ZF equalization at the destination, can be approximated as

$$\text{P}_{\text{dir},n}(e) \approx \overline{\text{P}}_{\text{dir}}(e) \triangleq (C_{\text{dir}}\, \gamma)^{-D} \tag{16}$$

where $C_{\text{dir}} \triangleq 3\, \Upsilon_{\text{dir}}^{-1/D} / [2\, N_{\text{B}}\, (Q-1)]$ represents the coding gain of the direct transmission, with

$$\Upsilon_{\text{dir}} \triangleq 2\, \Theta(N_{\text{D}} - N_{\text{B}} + 1)\, (1 - 1/\sqrt{Q}) \,. \tag{17}$$

Comparison with (9) reveals that the cooperative scheme and direct transmission achieve the same diversity order, but exhibit different coding gains. The ratio between such coding gains can be expressed as

$$\xi \triangleq \frac{C_{\text{coop}}}{C_{\text{dir}}} = \frac{\Omega \displaystyle\sum_{i=1}^{N_{\text{C}}} P_i^2\, \sigma^4_{\text{g},i}}{\left(N_{\text{R}} \displaystyle\sum_{i=1}^{N_{\text{C}}} P_i\, \sigma^{-2}_{\text{h},i}\, \sigma^2_{\text{g},i} + 1\right) \displaystyle\sum_{i=1}^{N_{\text{C}}} P_i\, \sigma^2_{\text{g},i}} \tag{18}$$

where $\Omega \triangleq \sqrt[D]{(N_{\text{C}}\, N_{\text{R}} - N_{\text{B}})! / (N_{\text{C}}\, N_{\text{R}} - N_{\text{D}} - 1)!} \geq 1$. Henceforth, the cooperative transmission is advantageous with respect to the direct one if $\xi > 1$. By substituting in (18) the simplified expression of C_{coop} given by (12), the ratio ξ becomes

$$\xi \approx \frac{\Omega\, P_{\text{max}}\, \sigma^2_{\text{g}}\, \sigma^2_{\text{h}}}{N_{\text{C}}\, N_{\text{R}}\, P_{\text{max}}\, \sigma^2_{\text{g}} + \sigma^2_{\text{h}}} \,. \tag{19}$$

If $N_{\text{C}}\, N_{\text{R}}\, P_{\text{max}}\, \sigma^2_{\text{g}} \ll \sigma^2_{\text{h}}$, which happens when the cluster is much closer to the source than the destination, then $\xi \approx \Omega\, P_{\text{max}}\, \sigma^2_{\text{g}}$, which unveils that the performance comparison between cooperative and direct transmissions does not depend on the first-hop path loss; in this case, the cooperative scheme outperforms the direct one if $\Omega\, P_{\text{max}}\, \sigma^2_{\text{g}} > 1$ and, thus, recalling that $\Omega \geq 1$, it suffices that $P_{\text{max}} \gg 1$, i.e., the maximum transmitting power at the relays has to be sufficiently large. On the other hand, if $N_{\text{C}}\, N_{\text{R}}\, P_{\text{max}}\, \sigma^2_{\text{g}} \gg \sigma^2_{\text{h}}$, which happens when the relaying cluster is much closer to the destination than the source, we obtain from (19) that $\xi \approx (\Omega\, \sigma^2_{\text{h}})/(N_{\text{C}}\, N_{\text{R}})$, which does not depend on the second-hop path loss. We observe that, in the latter case, the condition $\Omega\, \sigma^2_{\text{h}} > N_{\text{C}}\, N_{\text{R}}$, which ensures that the cooperative scheme is advantageous with respect to the direct one, is independent of P_{max}.

IV. NUMERICAL PERFORMANCE ANALYSIS

To corroborate and complete our theoretical results, we compare the high-SNR approximations $\overline{\text{P}}_{\text{coop}}(e)$ and $\overline{\text{P}}_{\text{dir}}(e)$ with the value of $\text{P}_{\text{coop},n}(e)$ and $\text{P}_{\text{dir},n}(e)$, respectively, calculated by a semi-analytical approach, where the averages (with respect to all the relevant fading channels) are evaluated through 10^6 independent Monte Carlo runs, with each run employing a different configuration of relays' positions, a different set of fading channels, and a different set of random scaled unitary matrices $\{\mathbf{F}_i\}_{i=0}^{N_{\text{C}}}$. Additionally, we report the average bit error rate (ABER) performances of both cooperative and direct transmissions when the MMSE equalizer is used at the destination, calculated by a semi-analytical approach similar to that used for the ZF case. Cooperative and direct schemes are referred to as "Coop" and "Dir" in the plots, respectively.

We consider the planar network topology depicted in Fig. 1, with the source located at the origin of the reference system, and the destination positioned at $(d_{\text{SD}}, 0)$, with $d_{\text{SD}} = 1$; moreover, the N_{C} relays are randomly and independently distributed in the circle of radius r, whose center is identified by the polar coordinates (d_{SC}, θ). A Gray-labeled QAM modulation with $Q = 4$ is considered, with $N_{\text{B}} = N_{\text{S}}$. All the channels are generated according to assumption **(a3)**, with $\eta = 3$, and all the relays transmit in the second hop with the same power level $P_i = P_{\text{max}} = 1$, $\forall i \in \{1, 2, \ldots, N_{\text{C}}\}$.

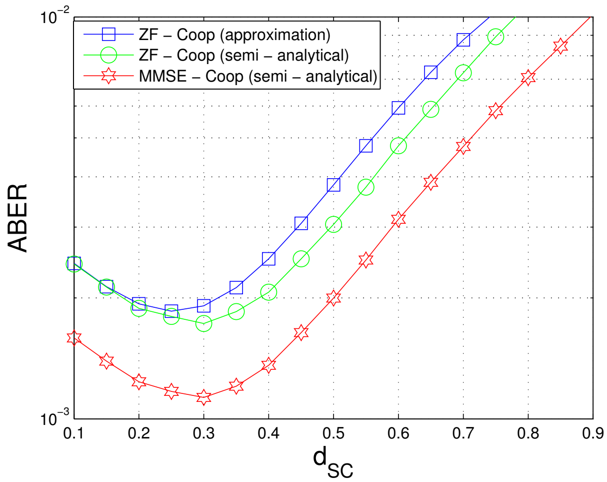

Fig. 2. ABER versus the distance d_{SC} between the source and the cluster for $\theta = 0$ (Example 1).

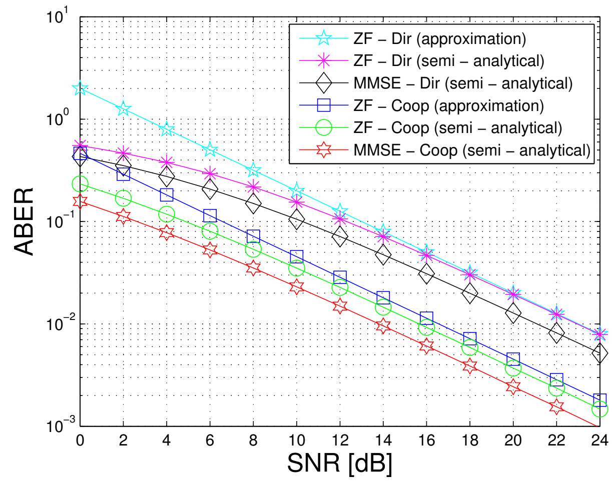

Fig. 4. ABER versus the SNR for $N_{\mathrm{C}} = 2$ relays in the case of the $(N_{\mathrm{S}}, N_{\mathrm{R}}, N_{\mathrm{D}}) = (2, 2, 2)$ antenna configuration (Example 2).

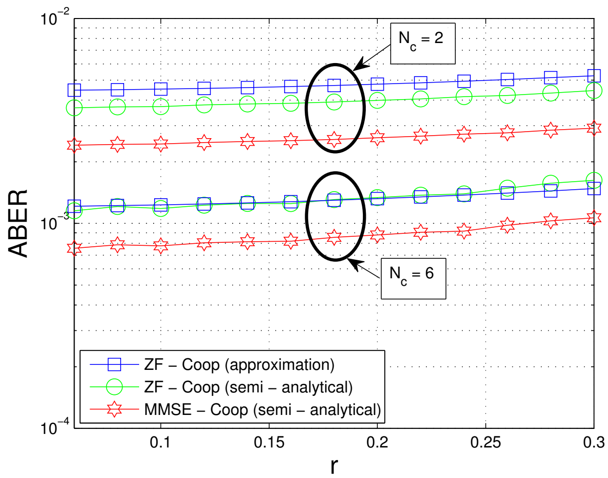

Fig. 3. ABER versus the cluster's radius r for two different values of the number $N_{\mathrm{C}} \in \{2, 6\}$ of relays (Example 1).

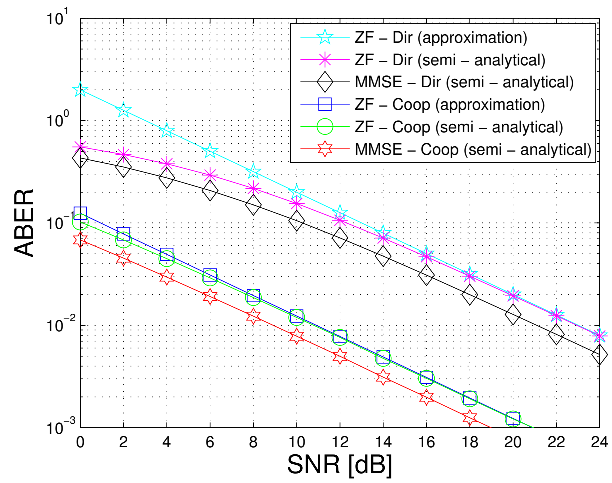

Fig. 5. ABER versus the SNR for $N_{\mathrm{C}} = 6$ relays in the case of the $(N_{\mathrm{S}}, N_{\mathrm{R}}, N_{\mathrm{D}}) = (2, 2, 2)$ antenna configuration (Example 2).

Example 1: ABER performance for different values of relaying cluster's geometrical parameters

In this example, we consider the $(N_{\mathrm{S}}, N_{\mathrm{R}}, N_{\mathrm{D}}) = (2, 2, 2)$ antenna configuration (diversity order $D = 1$), by setting $\theta = \theta_{\mathrm{opt}} = 0$ and SNR $= 20$ dB. Specifically, we depict in Fig. 2 the ABER performances of the considered cooperative transmission schemes as a function of the distance d_{SC}, with $r = 0.1$ and $N_{\mathrm{C}} = 4$, whereas Fig. 3 reports the ABER performances of the same schemes as a function of cluster's radius r for two values of $N_{\mathrm{C}} \in \{2, 6\}$, with $d_{\mathrm{SC}} = (d_{\mathrm{SC}})_{\mathrm{opt}}$ being the optimal source-cluster distance given by (15). Results of Figs. 2 and 3 evidence that, as expected, the MMSE equalizer slightly outperforms the ZF one, with a performance gain that does not significantly depend on the position and radius of the relaying cluster, provided that $r \ll d_{\mathrm{SD}}$. All the curves in Figs. 2 and 3 show a good agreement between the semi-analytical performance of the ZF equalizer and its approximation (9). With reference to the ZF case, the ABER approximation curve in Fig. 2 has a minimum point at $d_{\mathrm{SC}} = 0.25$, which corresponds to a global maximum of the coding gain. By setting $d_{\mathrm{SD}} = 1$, $P_{\max} = 1$, $N_{\mathrm{C}} = 4$, $N_{\mathrm{R}} = 2$, and $\eta = 3$ in (15), one obtains $(d_{\mathrm{SC}})_{\mathrm{opt}} = 0.2612$, thus further confirming that the results of Section III are valid also for a cluster whose radius r is not infinitesimal, provided that $r \ll d_{\mathrm{SD}}$.

Example 2: ABER performance versus SNR for different number of antennas and relays

In this example, we report the ABER performances of both the considered direct and cooperative transmission schemes as a function of the SNR for two different values of $N_{\mathrm{C}} \in \{2, 6\}$, with $d_{\mathrm{SC}} = (d_{\mathrm{SC}})_{\mathrm{opt}}$, $\theta = \theta_{\mathrm{opt}} = 0$, and $r = 0.1$. Figs. 4 and 5 refer to the $(N_{\mathrm{S}}, N_{\mathrm{R}}, N_{\mathrm{D}}) = (2, 2, 2)$ antenna configuration, whereas Figs. 6 and 7 refer to at the $(N_{\mathrm{S}}, N_{\mathrm{R}}, N_{\mathrm{D}}) = (2, 2, 3)$ antenna configuration. In accordance with the results of Section III, the ABER curves for the cooperative and direct schemes exhibit the same diversity order ($D = 1$

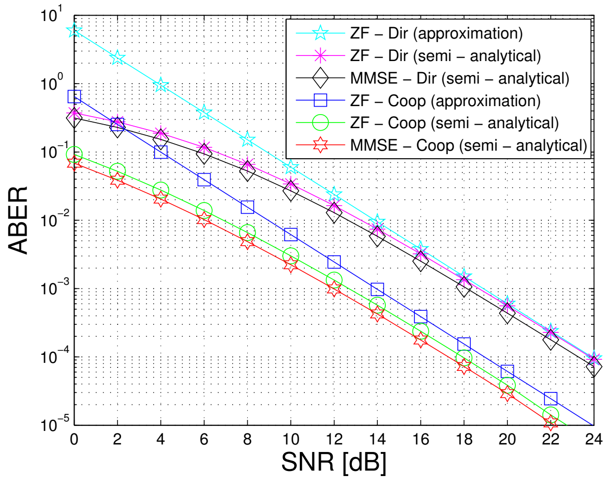

Fig. 6. ABER versus the SNR for $N_\mathrm{C} = 2$ relays in the case of the $(N_\mathrm{S}, N_\mathrm{R}, N_\mathrm{D}) = (2, 2, 3)$ antenna configuration (Example 2).

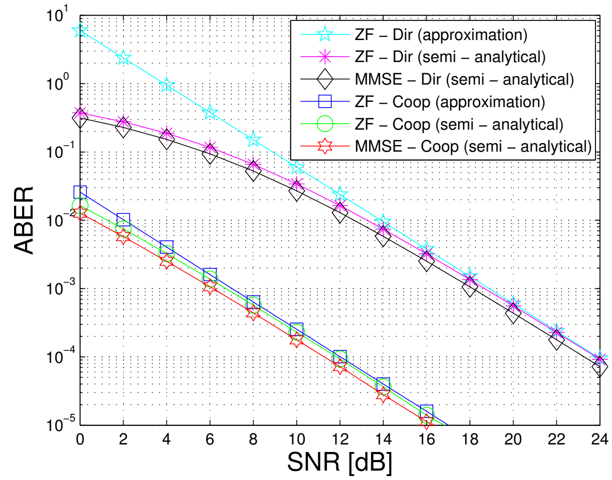

Fig. 7. ABER versus the SNR for $N_\mathrm{C} = 6$ relays in the case of the $(N_\mathrm{S}, N_\mathrm{R}, N_\mathrm{D}) = (2, 2, 3)$ antenna configuration (Example 2).

in Figs. 4 and 5, $D = 2$ in Figs. 6 and 7), but different coding gains. Specifically, with reference to ZF equalization and the $(N_\mathrm{S}, N_\mathrm{R}, N_\mathrm{D}) = (2, 2, 2)$ antenna configuration, let $\Delta\mathrm{SNR}(N_\mathrm{C}) > 0$ be the SNR gain (in dB) of the cooperative scheme over its corresponding direct counterpart, evaluated for convenience at the ABER value of $8 \cdot 10^{-3}$, it results from Figs. 4 and 5 that $\Delta\mathrm{SNR}(2) = 6$ dB and $\Delta\mathrm{SNR}(6) = 12$ dB, respectively. Such gains can also be reported in terms of the ratio between the coding gain C_coop of the cooperative scheme and the coding gain C_dir of the direct transmission as $\xi_\text{semi-an} \triangleq C_\mathrm{coop}/C_\mathrm{dir} = 10^{\frac{\Delta\mathrm{SNR}(N_\mathrm{C})}{10}}$. Interestingly, such a performance gain can be reliably predicted by resorting to (19), which refers to a relaying cluster with infinitesimal radius, i.e., $r \to 0$: for instance, in the case of $N_\mathrm{C} = 6$, one has that $\xi_\text{semi-an} = 15.8489$ (12 dB); by setting $P_\mathrm{max} = 1$, $N_\mathrm{C} = 6$, $N_\mathrm{R} = N_\mathrm{S} = N_\mathrm{D} = 2$, $\eta = 3 \Rightarrow D = 1$, $(d_\mathrm{SC})_\mathrm{opt} = 0.2240$, $\sigma_\mathrm{h}^2 = (1/0.2240)^3 = 88.9725$, and $\sigma_\mathrm{g}^2 = [1/(1-0.2240)]^3 = 2.1400$ in (18), one gets $\xi \approx 16.6068$ (12.2 dB).

V. Conclusions

An accurate approximation of the average SEP in the high-SNR regime was obtained for a spatially multiplexed dual-hop MIMO network, with multiple AF nonorthogonal relays and linear ZF equalization at the destination. With reference to a relaying cluster, we derived simple formulas that allow to optimally place the relays and assess the performance gain of the cooperative scheme over direct transmission.

Appendix A
Proof of Theorem 1

It can be shown [32] that, conditioned on $\mathbf{H}$, the RV $\mathrm{SINR}_n \approx (\gamma/\beta) \cdot \{(\mathbf{C}^\mathrm{H}\,\mathbf{C})^{-1}\}_{nn}^{-1}$ is Gamma distributed, with shape parameter $k = N_\mathrm{D} - N_\mathrm{B} + 1$ and scale parameter $\theta = (\gamma/\beta)\, \Sigma_n$, where $\Sigma_n \triangleq \{[\mathbf{F}_0^\mathrm{H}\, \widetilde{\mathbf{H}}^\mathrm{H} (\mathbf{\Omega} \otimes \mathbf{I}_{N_\mathrm{R}})\, \widetilde{\mathbf{H}}\, \mathbf{F}_0]^{-1}\}_{nn}^{-1}$ and the entries of $\widetilde{\mathbf{H}} \triangleq [\,\mathrm{diag}(\sigma_{\mathrm{h},1}^{-1}, \sigma_{\mathrm{h},2}^{-1}, \ldots, \sigma_{\mathrm{h},N_\mathrm{C}}^{-1}) \otimes \mathbf{I}_{N_\mathrm{R}}]\, \mathbf{H} \in \mathbb{C}^{(N_\mathrm{C} N_\mathrm{R}) \times N_\mathrm{S}}$ are i.i.d. ZMCSCG random variables having unit variance, with $\otimes$ being the Kronecker product and $\mathbf{\Omega} \triangleq \mathrm{diag}(\alpha_1\, \sigma_{\mathrm{h},1}^2\, \sigma_{\mathrm{g},1}^2, \alpha_2\, \sigma_{\mathrm{h},2}^2\, \sigma_{\mathrm{g},2}^2, \ldots, \alpha_{N_\mathrm{C}}\, \sigma_{\mathrm{h},N_\mathrm{C}}^2\, \sigma_{\mathrm{g},N_\mathrm{C}}^2) \in \mathbb{R}^{N_\mathrm{C} \times N_\mathrm{C}}$. Thus, one has

$$\Phi_{\mathrm{SINR}_n}\left(-\frac{u}{\sin^2 x}; \mathbf{H}\right) \approx \frac{1}{\left(1 + \frac{u\,\gamma}{\beta}\frac{\Sigma_n}{\sin^2 x}\right)^{N_\mathrm{D}-N_\mathrm{B}+1}} \approx (\sin^2 x)^{N_\mathrm{D}-N_\mathrm{B}+1} \left[\frac{u\,\gamma}{\beta}\,\Sigma_n\right]^{-(N_\mathrm{D}-N_\mathrm{B}+1)} \quad (20)$$

where the last approximation holds for $\gamma \gg 1$. Substituting (20) in (3) and recalling the expressions of b and u, one has that $\mathrm{P}_{\mathrm{coop},n}(e\,|\,\mathbf{H})$, conditioned on $\mathbf{H}$, can be approximated as follows

$$\mathrm{P}_{\mathrm{coop},n}(e\,|\,\mathbf{H}) \approx 2\,\Theta(N_\mathrm{D} - N_\mathrm{B} + 1)\left(1 - \frac{1}{\sqrt{Q}}\right) \cdot \left[\frac{3}{2(Q-1)}\,\frac{\gamma}{\beta}\,\Sigma_n\right]^{-(N_\mathrm{D}-N_\mathrm{B}+1)} \quad (21)$$

where $\Theta(v) \triangleq (2/\pi)\int_0^{\pi/2} (\sin^2 x)^v\, \mathrm{d}x$, with $v \in \mathbb{R}$.

At this point, eq. (21) has to be averaged with respect to $\mathbf{H} \in \mathbb{C}^{N_\mathrm{C} N_\mathrm{R} \times N_\mathrm{S}}$ or, equivalently, $\widetilde{\mathbf{H}}$. To this end, by partitioning $\widetilde{\mathbf{H}}$ as $\widetilde{\mathbf{H}} \triangleq [\widetilde{\mathbf{H}}_1^\mathrm{T}, \widetilde{\mathbf{H}}_2^\mathrm{T}, \ldots, \widetilde{\mathbf{H}}_{N_\mathrm{C}}^\mathrm{T}]^\mathrm{T}$, with $\widetilde{\mathbf{H}}_i \in \mathbb{C}^{N_\mathrm{R} \times N_\mathrm{S}}$, one obtains from the expression of Σ_n that $\mathbf{F}_0^\mathrm{H}\, \widetilde{\mathbf{H}}^\mathrm{H}\, (\mathbf{\Omega} \otimes \mathbf{I}_{N_\mathrm{R}})\, \widetilde{\mathbf{H}}\, \mathbf{F}_0 = \sum_{i=1}^{N_\mathrm{C}} \mathbf{A}_i^\mathrm{H}\, \mathbf{A}_i$, where $\mathbf{A}_i \triangleq \sqrt{\omega_i}\;\, \widetilde{\mathbf{H}}_i\, \mathbf{F}_0 \in \mathbb{C}^{N_\mathrm{R} \times N_\mathrm{B}}$, with $\omega_i \triangleq \alpha_i\, \sigma_{\mathrm{h},i}^2\, \sigma_{\mathrm{g},i}^2$ being the ith diagonal entries of $\mathbf{\Omega}$. It is readily seen that, for $i \in \{1, 2, \ldots, N_\mathrm{C}\}$, the matrix $\mathbf{A}_i^\mathrm{H}\, \mathbf{A}_i$ has a complex central Wishart distribution [31] with N_R degrees of freedom and covariance matrix $(\omega_i/N_\mathrm{B})\, \mathbf{I}_{N_\mathrm{B}}$, provided that $N_\mathrm{R} \geq N_\mathrm{B}$. By using a result in [26, Sec. 3], the matrix $\mathbf{F}_0^\mathrm{H}\, \widetilde{\mathbf{H}}^\mathrm{H}\, (\mathbf{\Omega} \otimes \mathbf{I}_{N_\mathrm{R}})\, \widetilde{\mathbf{H}}\, \mathbf{F}_0$ approximately has a complex central Wishart distribution with $N_\mathrm{R}\, (\sum_{i=1}^{N_\mathrm{C}} \omega_i)^2 / (\sum_{i=1}^{N_\mathrm{C}} \omega_i^2)$ degrees of freedom and covariance matrix ${N_\mathrm{B}}^{-1} (\sum_{i=1}^{N_\mathrm{C}} \omega_i^2)/(\sum_{i=1}^{N_\mathrm{C}} \omega_i)\, \mathbf{I}_{N_\mathrm{B}}$. Therefore, provided that $N_\mathrm{C}\, N_\mathrm{R} > N_\mathrm{D}$, one has [32] that $\Sigma_n \sim \mathrm{Gamma}(k, \theta)$, with $k = N_\mathrm{C}\, N_\mathrm{R} - N_\mathrm{B} + 1$ and $\theta = {N_\mathrm{B}}^{-1} (\sum_{i=1}^{N_\mathrm{C}} \omega_i^2)/(\sum_{i=1}^{N_\mathrm{C}} \omega_i)$, and, consequently, $1/\Sigma_n$ is distributed as an inverse Gamma random variable. Following

[4], after some calculations, one obtains

$$\mathbb{E}_{\mathbf{H}}\left[(\Sigma_n)^{-(N_{\rm D}-N_{\rm B}+1)}\right] = \frac{(N_{\rm C}\,N_{\rm R}-N_{\rm D}-1)!}{(N_{\rm C}\,N_{\rm R}-N_{\rm B})!} \cdot \left(\frac{N_{\rm B}\sum_{i=1}^{N_{\rm C}}\omega_i}{\sum_{i=1}^{N_{\rm C}}\omega_i^2}\right)^{N_{\rm D}-N_{\rm B}+1} \quad (22)$$

from which, averaging (21) with respect to $\mathbf{H}$ and recalling the expression of ω_i, one gets (9).

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