# A simple recruitment scheme of multiple nodes for cooperative MAC

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***Abstract*—Physical (PHY) layer cooperation in a wireless network allows neighboring nodes to share their communication resources in order to create a virtual antenna array by means of distributed transmission and signal processing. A novel medium access control (MAC) protocol, called CoopMAC, has been recently proposed to integrate cooperation at the PHY layer with the MAC sublayer, thereby achieving substantial throughput and delay performance improvements. CoopMAC capitalizes on the broadcast nature of the wireless channel and rate adaptation, recruiting a single relay on the fly to support the communication of a particular source-destination pair. In this paper, we propose a cross-layer rate-adaptive design that opportunistically combines the recruitment of *multiple* cooperative nodes and carrier sensing multiple access with collision avoidance. We focus on a single-source single-destination setup, and develop a randomized cooperative framework, which is referred to as randomized CoopMAC (RCoopMAC). Thanks to the randomization of the coding rule, the RCoopMAC approach enables the blind participation of multiple relays at unison relying only on the *mean* channel state information (CSI) of the potential cooperating nodes, without introducing additional signaling overhead to coordinate the relaying process. The proposed RCoopMAC scheme is not only beneficial in substantially improving the link quality and therefore the sustainable data rates but, thanks to the decentralized and agnostic coding rule, it also allows to effectively recruit multiple relays in a robust fashion, i.e., even when the required mean CSI is partially outdated.**

***Index Terms*—Cooperative wireless networks, cross-layer approach, medium access control (MAC) sublayer, physical (PHY) layer optimization, random matrix analysis, space-time block coding, spatial diversity.**

## I. INTRODUCTION

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COOPERATIVE DIVERSITY is an effective means to provide robustness at the physical (PHY) layer against the vagaries of the wireless channel, by opportunistically taking advantage of the broadcast nature of wireless communications [1]–[5]. Since cooperation allocates network resources, the PHY operations have to be coupled with the activities of higher layers of the protocol stack (*cross-layer optimization*) [6]. In designing cooperative cross-layer protocols, the system designer is faced by a fundamental dilemma: how to promote cooperation and, at the same time, avoid collisions by introducing the lowest amount of signaling overhead. As a starting point, one can look at existing medium access control (MAC) strategies, which do not enable cooperation at all. For instance, the IEEE 802.11 distributed coordination function (DCF) is a widely popular protocol for wireless local area networks [7]. Implementation of DCF in 802.11 employs a carrier sense multiple access with collision avoidance (CSMA/CA) mechanism, which is based on the exchange of two control messages: one from the source ($S$) to the destination ($D$), called request to send (RTS); one from $D$ to $S$, called clear to send (CTS). Such a functionality of 802.11 is aimed at avoiding collisions caused by hidden terminals[1] introducing a minimal amount of signaling overhead.

Noting that nodes overhearing both RTS and CTS messages are natural candidates to be recruited for cooperation, a *cooperative MAC (CoopMAC)* protocol has been recently developed in [6], [8], [9] to introduce cooperation in 802.11 networks. It is shown in [6], [8] that, with respect to the legacy 802.11 system, the CoopMAC protocol substantially improves network throughput and delay performance. The CoopMAC protocol uses the information collected by $S$ in a table of all the potential relays, called *CoopTable*, in order to recruit on the fly the *best* relay maximizing the end-to-end throughput of the $(S, D)$ pair exchanging the RTS/CTS frames. To facilitate cooperation, the RTS/CTS mode defined by 802.11 is extended in [8] to include an helper ready to send (HTS) control message, which is transmitted by the best relay to acknowledge its participation. Since the update of the CoopTable is done passively by listening to the radio activity in the neighborhood, there is no cost on the updating of such a table [8]. However, the price to pay for enabling cooperation is the HTS additional control message.

The CoopMAC protocol does not fully exploit the available

[1] Hidden terminals are nodes that are in the sensing range of the destination but not the source.

cooperative diversity since, even though there are several stations to provide help, only one of them can participate in the communication between $S$ and $D$. In this paper, we aim to answer the three following questions: (i) Can the RTS/CTS policy enable the recruitment of multiple relays with no additional signaling overhead to coordinate the relaying process? (ii) Is it possible to get even more cooperative gains while enjoying the low signaling overhead? (iii) Can cooperative gains be achieved even when the network state in not accurately known?

To answer to the first question, we propose a flexible MAC/PHY cross-layer design, whose MAC framework is inspired by [8] and whose enabling PHY technology is *randomized cooperative coding* [10], [11], a method that allows one to harvest cooperative diversity from multiple nodes in a decentralized manner. Specifically, we develop a simple strategy, referred to as *randomized cooperative MAC (RCoopMAC)*, which extends the MAC primitives proposed in [8] and enables, via the coding rule devised in [10], the recruitment of multiple relays in a randomized fashion, without introducing additional control signaling with respect to the RTS/CTS handshaking of 802.11. Contrary to the CoopMAC philosophy, which basically consists of harvesting the highest throughput gain through the recruitment of the best relay, the proposed randomized method is *opportunistic* in nature, in the sense that it allows multiple relays to simultaneously help, without sending any acknowledgement and without any coordination.

To answer to the second question, by considering a single source $S$, multiple relays and a single destination $D$, we derive an upper bound on the end-to-end average bit error probability (BEP) of the RCoopMAC scheme in the case of demodulate-and-forward relaying and complex Gaussian randomization. Using such an upper bound, we propose a rate-adaptive algorithm that, given the *mean* channel state information (CSI) regarding $S$ and the potential relays, allows $S$ and $D$ to jointly select the data rates ensuring the minimum transmission time with a quality of service (QoS) constraint in terms of average BEP (ABEP). The maximization of the end-to-end throughput for a single $(S, D)$ pair is a reasonable criterion to enable cooperation in all those environments where the aggregate network throughput is mainly dictated by the data rate over the individual source-destination links, e.g., small-scale wireless networks where only one source can transmit at a time or large-scale wireless systems with bursty traffic where multiple sources occasionally have to send large files to destinations.

To answer to the last question, capitalizing on our analytical framework that unveils the relationship between the achievable cooperative gain in terms of data rate and the amount/quality of mean CSI used in the selection process of the best relays, we highlight that a lower precision in the network state information results in easier MAC primitives at the expense of slightly lower PHY layer performance. Additionally, we show that implementing a decentralized selection of multiple nodes on the basis of imperfect knowledge of the mean CSI is preferable over an accurate selection of a single relay. Finally, it is worth noting that MAC implementation details regarding the proposed RCoopMAC approach can be found in [12], where a specific MAC sublayer is designed and a preliminary network analysis is provided.

The remainder of this paper is organized as follows. Section II introduces the considered PHY layer and presents a performance analysis of the randomized cooperative link in terms of ABEP, which allows us to derive the end-to-end ABEP of the proposed scheme. The RCoopMAC framework is described in Section III. In Section IV, we individuate the best setting of the PHY parameters when the relevant mean CSI is both updated and outdated. Section V provides numerical results, comparing different versions of RCoopMAC with the CoopMAC protocol in [8]. Section VI includes conclusions and suggestions for future work.

### A. Notation

Boldface upper [lower] case letters (e.g., $\mathbf{A}$ or $\mathbf{a}$) are matrices [vectors]; $\mathbb{C}^{m\times n}$ [$\mathbb{R}^{m\times n}$] is the field of $m \times n$ complex [real] matrices; $\mathbb{C}^{m}$ [$\mathbb{R}^{m}$] is a shorthand for $\mathbb{C}^{m\times 1}$ [$\mathbb{R}^{m\times 1}$]; $\{\mathbf{A}\}_{i_1,i_2}$ is the $(i_1, i_2)$th element of $\mathbf{A}$; $\mathrm{u}(x)$ denotes the unit step function, i.e., $\mathrm{u}(x) = 1$ if $x \geq 0$, zero otherwise, whereas $\lceil\cdot\rceil$ denotes ceiling-integer; $*$, $T$, $H$, $-1$ denote the conjugate, the transpose, the Hermitian and the inverse of a matrix; $\mathbf{0}_m \in \mathbb{R}^m$, is the null vector and $\mathbf{I}_m \in \mathbb{R}^{m\times m}$ is the identity matrix; $\odot$ denotes Hadamard product of two matrices and $\|\mathbf{a}\|$ is the Euclidean norm of $\mathbf{a}$; $\mathrm{rank}(\mathbf{A})$ is the rank of $\mathbf{A}$ and $\mathbf{A} = \mathrm{diag}(a_1, a_2, \ldots, a_n) \in \mathbb{C}^{n\times n}$ is a diagonal matrix whose $(i, i)$th entry is $a_i$; $\mathrm{E}[\cdot]$ stands for ensemble averaging; finally, a circular symmetric complex Gaussian random vector $\mathbf{x} \in \mathbb{C}^n$ with mean $\boldsymbol{\mu} \in \mathbb{C}^n$ and covariance matrix $\mathbf{K} \in \mathbb{C}^{n\times n}$ is denoted as $\mathbf{x} \sim \mathcal{CN}(\boldsymbol{\mu}, \mathbf{K})$.

## II. The PHY layer for RCoopMAC: problem statement and performance analysis

We consider a wireless network with (at most) $N+2$ nodes $\mathcal{N}_i$, with $i \in \{1, 2, \ldots, N+2\}$, deployed at random in a certain geographical region, where two of them are the source ($S$) and the destination ($D$), whereas all the remaining $N$ ones work as relays or helpers ($H_i$). The relays can cooperate with the $(S, D)$ pair to increase their data rate. The PHY layer selects the modulation level to meet a target average bit error probability $ABEP_{\text{target}}$ with uncoded transmission. Even though our approach can be generalized, for simplicity, the considered PHY layer is that of a single-carrier single-input single-output system, with symbol rate of $1/T_{\text{s}}$ symbols per second (s), designed to handle QAM square constellations. The PHY layer can support a set of $Q+1$ data rates $R_q = b_q/T_{\text{s}}$ (bits/s), where $b_q = \log_2(M_q)$ is the number of bits that are sent every symbol period, with $q \in \{0, 1, \ldots, Q\}$. Hence, $R_0 \leq R_1 \leq \cdots \leq R_Q$ form the *basic rate set* $\mathcal{J}$ and $R_0$ is the *base rate* at which the nodes exchange the RTS and CTS messages. The average transmitter energy is fixed for all the nodes and data rates.

We assume that $S$ wishes to send a packet of $B$ bits. There are two transmission modes in our network. In *direct mode*, the node $S$ transmits to $D$ a vector $\mathbf{a}^{(q)} \triangleq [a_1, a_2, \ldots, a_{K_q}]^T \in \mathbb{C}^{K_q}$ of $(M_q)$-QAM independent and identically distributed (i.i.d.) symbols at the data rate $R_q$, with $q \in \{1, 2, \ldots, Q\}$, where $M_q = 2^{R_q T_s}$ and $K_q \triangleq \lceil B/(R_q T_s) \rceil$. The transmission time for a direct communication is $B/R_q$ seconds. In *cooperative mode*, the transmission is divided into two phases: in *Phase I*, $S$ directly transmits to all its potential relays a vector $\mathbf{a}' \triangleq [a_{I,1}, a_{I,2}, \ldots, a_{I,K'}]^T \in \mathbb{C}^{K'}$ of $(M')$-QAM i.i.d. symbols at the data rate $R' \in \mathcal{J} - \{R_0\}$, with $M' = 2^{R' T_s}$ and $K' \triangleq \lceil B/(R' T_s) \rceil$; in *Phase II*, the relays demodulate the symbol block $\mathbf{a}'$ and, along with $S$ itself, they re-modulate the original source bits transmitting a vector $\mathbf{a}'' \triangleq [a_{II,1}, a_{II,2}, \ldots, a_{II,K''}]^T \in \mathbb{C}^{K''}$ of $(M'')$-QAM i.i.d. symbols at the data rate $R'' \in \mathcal{J} - \{R_0\}$ by using a randomized space-time block coding (STBC) rule [10] having a *code rate* $R_{\text{code}} \leq 1$, with $M'' = 2^{R'' T_s}$ and $K'' \triangleq \lceil B/(R'' T_s) \rceil$. The transmission time for such a two phase communication mode is $(B/R') + [B/(R'' R_{\text{code}})]$ seconds. We assume that $D$ processes only the signals received from Phase II. For simplicity, we consider the case of a frequency non-selective channel: let $\eta_{i,j}$ denote the distance between $\mathcal{N}_i$ and $\mathcal{N}_j$, for $i \neq j \in \{1, 2, \ldots, N+2\}$, the fading coefficients $g_{i,j}$ over the $(\mathcal{N}_i, \mathcal{N}_j)$ links are independent random variables modeled as $g_{i,j} \sim \mathcal{CN}(0, \eta_{i,j}^{-\alpha})$, where $\alpha$ is the path-loss exponent, independent of the transmitted symbols. Note that the channel is reciprocal, i.e., $g_{ij}$ and $g_{ji}$ are identically distributed. Finally, it is assumed that the underlying channels remain constant during (at least) the transmission of a data packet either in direct or cooperative mode.

### A. PHY performance for the direct mode

If $\mathcal{N}_i$ transmits in direct mode the block $\mathbf{a}^{(q)}$ to $\mathcal{N}_j$ at the data rate $R_q$, with $q \in \{0, 1, \ldots, Q\}$, a block of $K_q$ consecutive samples of the discrete-time baseband equivalent received signal can be expressed as

$$\mathbf{r}_{i,j}^{(q)} = g_{i,j}\, \mathbf{a}^{(q)} + \mathbf{n}_{i,j}^{(q)} , \tag{II.1}$$

where $\mathbf{n}_{i,j}^{(q)} \sim \mathcal{CN}(\mathbf{0}_{K_q}, N_0\, \mathbf{I}_{K_q})$ denotes additive white Gaussian noise (AWGN), which is independent of $g_{i,j}$ and $\mathbf{a}^{(q)}$. Let $P_{i,j}^{(q)}(e)$ denote the average (over the random gain $g_{i,j}$) BEP at the output of the maximum likelihood (ML) detector (assuming all the symbols within the block $\mathbf{a}^{(q)}$ experience the same fading), under the assumption that a Gray code is used to map the information bits into QAM symbols and the signal-to-noise ratio (SNR) is sufficiently high, one has [13]

$$P_{i,j}^{(q)}(e) \approx \frac{2}{\log_2(M_q)} \left(1 - \frac{1}{\sqrt{M_q}}\right) \cdot \left(1 - \sqrt{\frac{3\,\overline{\gamma}_{i,j}}{2(M_q - 1) + 3\,\overline{\gamma}_{i,j}}}\right) , \tag{II.2}$$

where $\overline{\gamma}_{i,j} \triangleq \gamma/\eta_{i,j}^{\alpha}$ represents the *average* SNR (ASNR) associated with the $(\mathcal{N}_i, \mathcal{N}_j)$ link and $\gamma$ is the ASNR per symbol expended by the transmitter (all nodes use the same transmission power), i.e., $\gamma \triangleq \mathrm{E}[|a_k|^2]/N_0 = (M_q - 1)\, \triangle_q^2/(6\, N_0)$, with $\Delta_q$ representing the minimum Euclidean distance of the symbol constellation. We say that the node $\mathcal{N}_i$ can *reliably* communicate (in direct mode) with the node $\mathcal{N}_j$ at the data rate $R_q$ if $P_{i,j}^{(q)}(e) \leq ABEP_{\text{target}}$, which, in light of (II.2), leads to the inequality

$$\overline{\gamma}_{i,j} \geq \overline{\gamma}_{\min,q} \triangleq [2(M_q - 1)\,(1 - \Omega_q)^2]/\{3\big[1 - (1 - \Omega_q)^2\,\big]\} , \tag{II.3}$$

with $\Omega_q \triangleq (ABEP_{\text{target}}/2)\log_2(M_q)\big(1 - 1/\sqrt{M_q}\big)^{-1}$ .

It is worth noting that $\overline{\gamma}_{\min,q}$ can be evaluated *off-line* for any $q \in \{0, 1, \ldots, Q\}$. A direct communication between $\mathcal{N}_i$ and $\mathcal{N}_j$ at a particular data rate $R_q$ can take place only if it is reliable, i.e., $\overline{\gamma}_{i,j} \geq \overline{\gamma}_{\min,q}$.

### B. PHY performance for randomized cooperation with demodulate-and-forward relays

Let us assume that there are $N_h \geq 1$ active relays in Phase II which have correctly demodulated the received signal transmitted from $S$. The $i$th relay transmits a linear combination $\mathbf{x}_i = \boldsymbol{\mathcal{C}}(\mathbf{a}'')\, \mathbf{r}_i \in \mathbb{C}^P$ of the columns of a code matrix $\boldsymbol{\mathcal{C}}(\mathbf{a}'') \in \mathbb{C}^{P \times L}$ associated with the $(K'')$-length vector $\mathbf{a}''$, where $P$ represents the block length and, hence, the code rate is defined as $R_{\text{code}} \triangleq K''/P \leq 1$, whereas $L$ denotes the number of *virtual* antennas and the choice of $\mathbf{r}_i \in \mathbb{C}^L$ leads to different randomization schemes. Additionally, the node $S$ participates to the transmission in Phase II sending its randomized code $\mathbf{x}_s = \boldsymbol{\mathcal{C}}(\mathbf{a}'')\, \mathbf{r}_s \in \mathbb{C}^P$, with $\mathbf{r}_s \in \mathbb{C}^L$. Our codes are based on orthogonal STBC (OSTBC) [14], for which $\boldsymbol{\mathcal{C}}^H(\mathbf{a}'')\, \boldsymbol{\mathcal{C}}(\mathbf{a}'') = \|\mathbf{a}''\|^2\, \mathbf{I}_L$. As a consequence of the orthogonal constraint, the code parameters $L$, $K''$ and $P$ cannot be chosen independently of each other. A systematic design method to generate high-rate complex orthogonal space-time block codes for any value of $L$ is given in [15].

If the relays and $S$ are time- and frequency-synchronized either by a centralized or distributed algorithm,[2] the signal received at $D$ is given by

$$\begin{aligned} \mathbf{r}_d &= g_{s,d}\, \mathbf{x}_s + \sum_{i=1}^{N_h} g_{h_i,d}\, \mathbf{x}_i + \mathbf{n}_d \\ &= \boldsymbol{\mathcal{C}}(\mathbf{a}'')\, \underbrace{[\mathbf{r}_s, \mathbf{r}_1, \ldots, \mathbf{r}_{N_h}]}_{\boldsymbol{\mathcal{R}} \in \mathbb{C}^{L \times (N_h+1)}} \underbrace{[g_{s,d}, g_{h_1,d}, \ldots, g_{h_{N_h},d}]^T}_{\mathbf{g}_d \in \mathbb{C}^{N_h+1}} + \mathbf{n}_d \\ &= \boldsymbol{\mathcal{C}}(\mathbf{a})\, \boldsymbol{\mathcal{R}}\, \mathbf{g}_d + \mathbf{n}_d , \end{aligned} \tag{II.4}$$

where $\mathbf{n}_d \sim \mathcal{CN}(\mathbf{0}_P, N_0\, \mathbf{I}_P)$ denotes AWGN independent of $\mathbf{a}''$ and $\mathbf{g}_d$, whereas $\mathbf{g}_d \sim \mathcal{CN}(\mathbf{0}_{N_h+1}, \boldsymbol{\Sigma}_{\mathbf{g}_d})$, with $\boldsymbol{\Sigma}_{\mathbf{g}_d} \triangleq \mathrm{diag}(\eta_{s,d}^{-\alpha}, \eta_{h_1,d}^{-\alpha}, \ldots, \eta_{h_{N_h},d}^{-\alpha}) \in \mathbb{R}^{(N_h+1)\times(N_h+1)}$.[3] The signal model (II.4) is quite general and subsumes different distributed STBC approaches. In a *centralized* scheme [1], each cooperating node transmits a *pre-assigned* column of the STBC matrix $\boldsymbol{\mathcal{C}}(\mathbf{a})$, i.e., $L = N_h + 1$ and $\boldsymbol{\mathcal{R}} = \mathbf{I}_{N_h+1}$. In a *decentralized deterministic* scheme [4], matrix $\boldsymbol{\mathcal{R}}$ collects node signature vectors to be properly optimized. In this paper, we are going to focus on the *decentralized randomized* scheme developed in [10], where the authors proposed that the vectors

[2] As recently shown in [11], randomized STBC can take care of (random) delays introduced by cooperating nodes and, hence, the assumption that relays and $S$ are time-synchronized can be relaxed in this case.

[3] The subscripts $s \in \{1, 2, \ldots, N+2\}$ and $d \in \{1, 2, \ldots, N+2\} - \{s\}$ denote the indices of $S$ and $D$, respectively. The $i$th relay or helper is referred to as $H_i$ and the subscript $h_i \in \{1, 2, \ldots, N+2\} - \{s, d\}$ denotes its index.

$\mathbf{r}_s$ and $\{\mathbf{r}_i\}_{i=1}^{N_h}$ are random and generated locally, with $L$ fixed irrespective of the number of relays $N_h$. Furthermore, as noted in [10], the knowledge of $\boldsymbol{\mathcal{R}}$ is not required at the destination in the randomized approach; indeed, the equivalent channel $\widetilde{\mathbf{g}}_d \triangleq \boldsymbol{\mathcal{R}}\,\mathbf{g}_d \in \mathbb{C}^L$ can be directly estimated via training (as customary, here we consider it perfectly known at the receiver) and $\mathbf{a}''$ can be demodulated according to the ML decision rule $\widehat{\mathbf{a}}'' = \arg\min_{\mathbf{a}''} \|\mathbf{r}_d - \boldsymbol{\mathcal{C}}(\mathbf{a}'')\,\widetilde{\mathbf{g}}_d\|^2$. For OSTBC schemes, the ML criterion is equivalent [14] to $K''$ independent scalar ML decision rules over the following $K''$ parallel and independent AWGN channels

$$y_k = \|\widetilde{\mathbf{g}}_d\|\, a_{II,k} + w_k \,, \tag{II.5}$$

for $k \in \{1, 2, \ldots, K''\}$, where $w_k \sim \mathcal{CN}(0, N_0)$ is the noise term, with $\mathrm{E}[w_{k_1} w_{k_2}^*] = 0$ for $k_1 \neq k_2 \in \{1, 2, \ldots, K''\}$. In order to streamline our analysis, we assume hereinafter that $\boldsymbol{\mathcal{R}} = \frac{1}{\sqrt{L}}\overline{\boldsymbol{\mathcal{R}}}$, where the entries of $\overline{\boldsymbol{\mathcal{R}}}$ are i.i.d. circular symmetric complex Gaussian random variables with zero mean and unit variance, independent of $\mathbf{a}''$, $\mathbf{g}_d$ and $\mathbf{n}_d$. Thus, we have $r = \mathrm{rank}(\boldsymbol{\mathcal{R}}) = \min(N_h + 1, L)$ with probability one.[4] Let $\zeta_1, \zeta_2, \ldots, \zeta_{N_h+1}$ be the diagonal entries of $\boldsymbol{\Sigma}_{\mathbf{g}_d}$ arranged in increasing order, the following result holds:

*Theorem 2.1:* Assume that $N_h \geq 1$ and $N_h + 1 \neq L$. Let $\mathcal{E}^c$ denote the event where all the $N_h$ relays demodulate $\mathbf{a}'$ correctly and let $P_d(e\,|\,\mathcal{E}^c, \boldsymbol{\mathcal{R}}, \mathbf{g}_d)$ denote the BEP at the output of the ML detector of $D$, conditioned on $\mathcal{E}^c$, $\boldsymbol{\mathcal{R}}$ and $\mathbf{g}_d$. The probability $P_d(e\,|\,\mathcal{E}^c)$, which is the expected value of $P_d(e\,|\,\mathcal{E}^c, \boldsymbol{\mathcal{R}}, \mathbf{g}_d)$ over the sample space of the pair $\{\boldsymbol{\mathcal{R}}, \mathbf{g}_d\}$, can be upper bounded as follows

$$P_d(e\,|\,\mathcal{E}^c) \leq P_d^{\mathrm{ub}}(e\,|\,\mathcal{E}^c) \triangleq \frac{4}{\log_2(M'')}\left(1 - \frac{1}{\sqrt{M''}}\right) \cdot \frac{\left[\frac{L\,(M''-1)}{3}\right]^{\min(N_h+1,L)}}{\prod_{i=1}^{\min(N_h+1,L)} \gamma\,\zeta_i}\,\frac{(|N_h - L + 1| - 1)!}{(\max(N_h+1, L) - 1)!}\,. \tag{II.6}$$

*Proof:* See Appendix A. ∎

We observe that, besides $N_h$ and $M''$ (i.e., $R''$), the upper bound (II.6) on $P_d(e\,|\,\mathcal{E}^c)$ also depends on the underlying space-time code by means of $L$ and, in particular, the *diversity gain* is $\min(N_h + 1, L)$ and saturates at $L$. Because $\zeta_1 \leq \zeta_2 \leq \cdots \leq \zeta_{N_h+1}$, the derived upper bound is pessimistic since it depends only on the ASNRs over the $\min(N_h + 1, L)$ most attenuated links and it turns out to be useful if the ASNR $\gamma$ is sufficiently high such that $P_d^{\mathrm{ub}}(e\,|\,\mathcal{E}^c) < 1$.

The key point of using a randomized STBC (RSTBC) scheme is that the selection of the relays can be completely decentralized. As we will show next, in our framework, such a selection is done based on a precomputed threshold $\overline{\gamma}_{\mathrm{coop}} = \gamma/\eta_{\mathrm{coop}}^{\alpha}$ for the broadcast links between $S$ and the relays, i.e., the nodes that can cooperate in Phase II are only those whose ASNR over the link towards $S$ is greater than or equal to $\overline{\gamma}_{\mathrm{coop}}$ or, equivalently, whose distance from $S$ is smaller than or equal to $\eta_{\mathrm{coop}}$. However, along with the problem of selecting the threshold $\overline{\gamma}_{\mathrm{coop}}$, the problem of choosing the best data rates $R'$ and $R''$, and the code order $L$ to maximize the end-to-end throughput and, at the same time, meet the ABEP constraint needs to be solved as well. Prior to delving into the details on how these parameters are selected, we have to evaluate the end-to-end ABEP of the overall cooperative communication protocol.

[4] As discussed in [10], other randomization rules can be used, such as real Gaussian, uniform phase, real/complex spherical distribution, random antenna selection, all leading to slightly different diversity orders and implementation features.

### C. PHY performance for the cooperative mode

Let $P_{\mathrm{coop}}(e)$ be the end-to-end ABEP of the two-phase cooperative transmission, for a given network configuration, the probability $P_{\mathrm{coop}}(e)$ also depends on $R'$, $R''$, $L$ and $N_h$, i.e., $P_{\mathrm{coop}}(e) = f(R', R'', L, N_h)$. We will characterize such a function in this subsection. Let $\mathcal{E}$ denote the event that at least one of the $N_h$ active relays makes an error in demodulating the $k$th symbol $a_{I,k}$ belonging to $\mathbf{a}'$, for $k \in \{1, 2, \ldots, K'\}$. Observe that $\mathcal{E}^c$, already defined in Subsection II-B, is the complement event of $\mathcal{E}$. Under our hypotheses the probability $P_{\mathrm{coop}}(e)$ is independent of $k$ and can be written as

$$P_{\mathrm{coop}}(e) = P_{\mathrm{coop}}(e\,|\,\mathcal{E})\,[1 - P(\mathcal{E}^c)] + P_{\mathrm{coop}}(e\,|\,\mathcal{E}^c)\,P(\mathcal{E}^c)\,, \tag{II.7}$$

where $P(\mathcal{E}^c)$ is the probability of $\mathcal{E}^c$, whereas $P_{\mathrm{coop}}(e\,|\,\mathcal{E})$ and $P_{\mathrm{coop}}(e\,|\,\mathcal{E}^c)$ denote the end-to-end ABEPs given that $\mathcal{E}$ and $\mathcal{E}^c$ occurred, respectively. Let us assume that $R' = R_{q'}$, with $q' \in \{1, 2, \ldots, Q\}$, since the channels are statistically independent by assumption and the nodes make decisions independently of each other, one has

$$P(\mathcal{E}^c) = \prod_{i=1}^{N_h}\left[1 - P_{s,h_i}^{(q')}(e)\right]\,, \tag{II.8}$$

where $P_{s,h_i}^{(q')}(e)$ can be obtained from (II.2). Furthermore, we assume that, if at least one relay demodulates incorrectly, then $D$ makes an error in demodulating, too, i.e., $P_{\mathrm{coop}}(e\,|\,\mathcal{E}) = 1$.[5] Thus, it follows that

$$P_{\mathrm{coop}}(e) = 1 - [1 - P_{\mathrm{coop}}(e\,|\,\mathcal{E}^c)]\prod_{i=1}^{N_h}\left[1 - P_{s,h_i}^{(q')}(e)\right]\,, \tag{II.9}$$

which, if $P_{\mathrm{coop}}(e\,|\,\mathcal{E}^c), P_{s,h_1}^{(q')}(e), \ldots, P_{s,h_{N_h}}^{(q')}(e) \ll 1$, can be approximated as

$$P_{\mathrm{coop}}(e) = f(R', R'', L, N_h) \approx P_{\mathrm{coop}}(e\,|\,\mathcal{E}^c) + \sum_{i=1}^{N_h} P_{s,h_i}^{(q')}(e)\,, \tag{II.10}$$

where, since $P_{\mathrm{coop}}(e\,|\,\mathcal{E}^c)$ is the probability that $D$ makes an error when all the $N_h$ relays demodulated correctly, it is equal to $P_d(e\,|\,\mathcal{E}^c)$, which is defined and upper bounded in Theorem 2.1. It is noteworthy that, of the two terms in (II.10), according to (II.6), the first one decreases rapidly as $N_h$ increases, while, by virtue of (II.2), the second one roughly

[5] Since the relays demodulate each symbol separately, they may end up forwarding wrong estimates to $D$. In general, a demodulation error made by some relays does not necessarily lead to an error in the end-to-end link. However, even though more detailed (and, inevitably, more complicated) analyses might be developed, such an assumption has the advantage of leading to a simplified upper bound on the ABEP, which is well-suited for setting the cooperative PHY parameters in a tractable way.

increases proportionally to $N_h$. Furthermore, the evaluation of the former term in (II.10) requires the knowledge of the ASNRs over the $(S, D)$ and $(H_i, D)$ links, for $i \in \{1, 2, \ldots, \min(N_h + 1, L)\} - \{s, d\}$, whereas the calculation of the latter one involves the knowledge of the ASNRs over the $(S, H_i)$ links, for $i \in \{1, 2, \ldots, N_h\} - \{s\}$.

## III. THE PROPOSED RCOOPMAC FRAMEWORK

In this section, by keeping the definition of the MAC sublayer as generic as we can, we describe the proposed cross-layer optimization framework that allows for an efficient and robust design of a cooperative MAC scheme supporting at the PHY layer the RSTBC rule analyzed in Subsection II-B. Similarly to [8], we rely on the CSMA/CA mechanism to manage cooperation, however without requiring transmission of additional control frames to coordinate the cooperative nodes. In Subsection III-A, we point out what is the amount of network state information that each node can learn by passive listening. Subsequently, we indicate in Subsection III-B how this wealth of information, that is naturally accessible by the nodes in an IEEE 802.11 network, can be utilized to recruit multiple cooperative relays.

### A. Learning process at each node by passive listening

Each node is required to measure the *mean* channel conditions with its neighboring nodes and, moreover, maintain information about which stations can communicate with its neighbors. More precisely, for $i \in \{1, 2, \ldots, N + 2\}$, the generic node $\mathcal{N}_i$ creates and updates a particular matrix $\mathbf{A}_i \in \mathbb{R}^{(N+2)\times(N+2)}$, called the *CoopMatrix*, whose $(p_1, p_2)$th entry, for $p_1, p_2 \in \{1, 2, \ldots, N + 2\}$, is defined as follows

$$\{\mathbf{A}_i\}_{p_1,p_2} = \begin{cases} \overline{\gamma}_{i,p_2}, & \text{if } i \neq p_2 \text{ and } \mathcal{N}_i \text{ can directly communicate with } \mathcal{N}_{p_2}; \\ \overline{\gamma}_{p_1,i}, & \text{if } p_1 \neq i \text{ and } \mathcal{N}_{p_1} \text{ can directly communicate with } \mathcal{N}_i; \\ 1, & \text{if } p_1 \neq p_2 \neq i \text{ and } \mathcal{N}_{p_1} \text{ can directly communicate with } \mathcal{N}_{p_2}; \\ 0, & \text{else.} \end{cases} \quad \text{(III.1)}$$

Due to the channel reciprocity $\mathbf{A}_i$ turns out to be symmetric, i.e., $\{\mathbf{A}_i\}_{p_1,p_2} = \{\mathbf{A}_i\}_{p_2,p_1}$. The CoopMatrix can be created and updated by passively listening to all ongoing communications occurring between nodes [6], [8], without the exchange of dedicated control frames between them. In particular, passive listing allows $\mathcal{N}_i$ to directly estimate the ASNRs over the $(\mathcal{N}_p, \mathcal{N}_i)$ links, for $p \in \{1, 2, \ldots, N + 2\} - \{i\}$, by using non-data-aided estimators (see, e.g., [16]), thus filling in the corresponding entries $\{\mathbf{A}_i\}_{p,i}$ and $\{\mathbf{A}_i\}_{i,p}$ of the CoopMatrix. On the other hand, the ASNRs over the $(\mathcal{N}_{p_1}, \mathcal{N}_{p_2})$ links, with $p_1 \neq i$ and $p_2 \neq i$, cannot be estimated at $\mathcal{N}_i$ by only overhearing the signals that $\mathcal{N}_{p_1}$ and $\mathcal{N}_{p_2}$ exchange each other.[6] However, in an IEEE 802.11 network, the node $\mathcal{N}_i$ can discover which relays can directly communicate with its neighbors [6], [8], by decoding the source and destination MAC addresses contained in the header of all the packets it receives. We assume that such MAC addresses are stored in a table according to a pre-established order criterion which is adopted by all the stations.

### B. Transmission algorithm

The considered scheme is distributed, thus both $S$ and $D$ are responsible for setting the PHY parameters $R'$, $R''$, $L$ and $\overline{\gamma}_{\text{coop}}$. Let us consider that $S$ has to transmit to $D$ a block of $B$ bits at the target data rate $R_{\text{target}} = R_{\overline{q}}$, for $\overline{q} \in \{1, 2, \ldots, Q - 1\}$, with the ABEP being smaller than or equal to $ABEP_{\text{target}}$. Both $S$ and $D$ are in charge of jointly choosing between direct or cooperative mode. To do that, by jointly exploiting the information stored in their CoopMatrices, the nodes $S$ and $D$ compare estimates of the performances achievable by both the direct and cooperative transmission modes. In particular, the source $S$ uses the mean CSI stored in its CoopMatrix to individuate a "qualified" subset $\mathcal{H} \subseteq \{H_1, H_2, \ldots, H_N\}$ of $N_h$ *potential* relays (see Section IV for further details), where $N_h$ is a variable to be optimized in its turn. A node is a potential relay if the data rate over the link between $S$ and itself can be strictly grater than $R_{\text{target}}$. The transmission mode selection (direct or cooperative) is based on the *minimum transmission time criterion*: if the cooperative transmission is more time efficient than the direct communication at the data rate $R_{\text{target}}$ and the target requirement in terms of ABEP is fulfilled, then $S$ and $D$ will jointly decide in favour of the cooperative mode. Therefore, for a given value of $L$, let $R'_{\text{coop}}$, $R''_{\text{coop}}$ and $N_{\text{coop}}$ denote the largest value of $R'$, $R''$ and $N_h$ satisfying the QoS constraint

$$P_{\text{coop}}(e) = f(R', R'', L, N_h) \leq ABEP_{\text{target}}\,, \quad \text{(III.2)}$$

cooperative transmission between $S$ and $D$ can be sustained if $N_{\text{coop}} \geq 1$ and

$$(R_{\text{coop}})^{-1} \triangleq (R'_{\text{coop}})^{-1} + (R''_{\text{coop}}\, R_{\text{code}})^{-1} < (R_{\text{target}})^{-1}\,. \quad \text{(III.3)}$$

The subset collecting the best $N_{\text{coop}}$ potential relays is referred to as $\mathcal{H}_{\text{coop}}$ and the ASNR threshold $\overline{\gamma}_{\text{coop}}$ used for the selection of the relays turns out to be

$$\overline{\gamma}_{\text{coop}} = \frac{\gamma}{\eta^{\alpha}_{\text{coop}}} = \min_{H_i \in \mathcal{H}_{\text{coop}}} \overline{\gamma}_{s,h_i}\,. \quad \text{(III.4)}$$

Note that fulfillment of (III.3) necessarily requires that $R'_{\text{coop}} > R_{\text{target}}$ and $R''_{\text{coop}}\, R_{\text{code}} > R_{\text{target}}$. In the sequel, we refer to $R_{\text{coop}}$ as the *cooperative data rate* of the two-phase communication; its inverse $R^{-1}_{\text{coop}}$ represents the time (in s) required to transmit one information bit over the cooperative link.

[6]The node $\mathcal{N}_i$ can acquire estimates of the ASNRs over the $(\mathcal{N}_{p_1}, \mathcal{N}_{p_2})$ links, with $p_1 \neq i$ and $p_2 \neq i$, by means of an explicit exchange of feedback information with other stations. In such a case, there is a crucial tradeoff between timeliness of the acquired information and waste of communication resources: the higher the frequency of information exchange, the more accurate the ASNR estimates are and, at the same time, the more one has to pay in terms of signaling overhead, which might be too costly in dense networks.

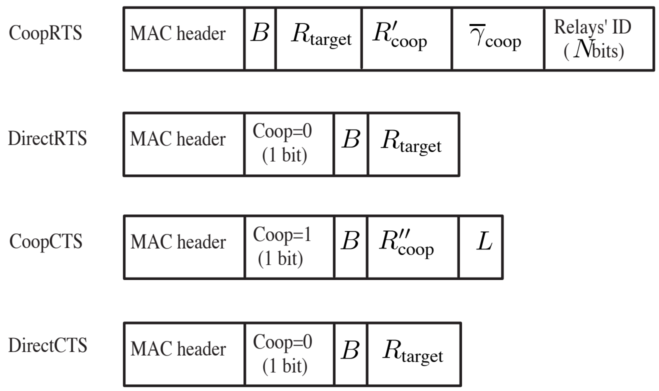


Fig. 1. Frame formats for CoopRTS, DirectRTS, CoopCTS and DirectCTS. The variable Coop is boolean: if Coop $= 1$, then cooperation is requested; if Coop $= 0$, then cooperation is forbidden.

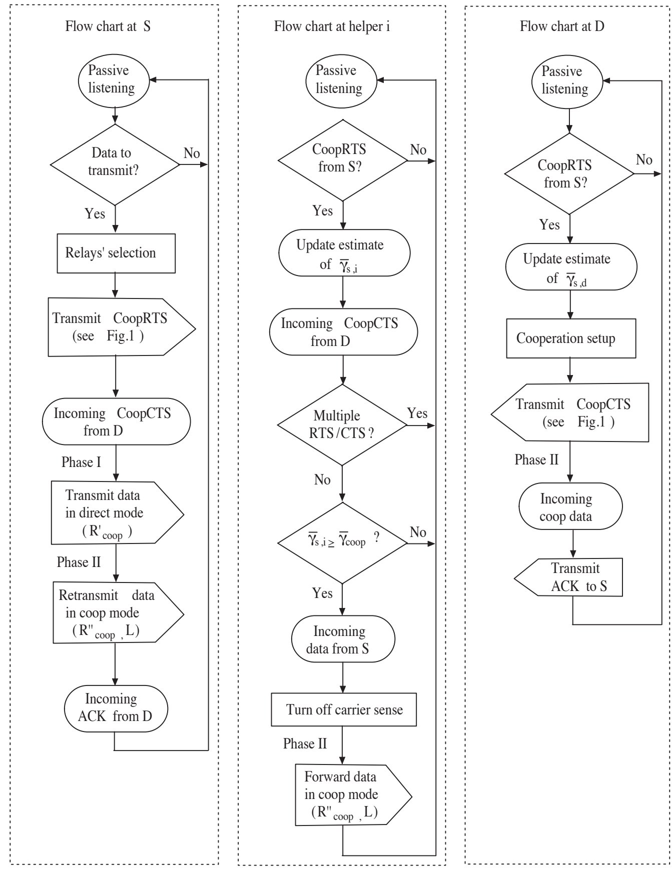


Fig. 2. Flow chart of the RCoopMAC framework. For the sake of simplicity, the most interesting case when at least one out of the potential relays is able to cooperate is considered in the figure.

It is crucial to note that, by virtue of its CoopMatrix $\mathbf{A}_s$, the node $S$ knows enough to compute the second summand in $P_{\text{coop}}(e)$ given by (II.10), i.e., it has a reliable estimate of the ASNRs over the $(S, H_i)$ links, for $i \in \{1, 2, \ldots, N_h\}$, but it does not know the average channel quality of relays' links towards $D$, since it can only discover which stations can directly communicate with $D$. Vice versa, by exploiting its CoopMatrix $\mathbf{A}_d$, the node $D$ is able to evaluate the first summand in (II.10), i.e., it has a reliable estimate of the ASNRs over the $(S, D)$ and $(H_i, D)$ links, for $i \in \{1, 2, \ldots, \min(N_h+1, L)\} - \{s, d\}$, but it does not know the average channel quality of relays' links towards $S$, since it can only learn which stations can directly communicate with $S$. Indeed, the task of determining the best values of $R'$ and $\overline{\gamma}_{\text{coop}}$ is demanded to $S$, whereas the best values of the remaining parameters $R''$ and $L$ are computed by $D$. To transmit such PHY parameters along with the request for help, the nodes $S$ and $D$ use the *cooperative RTS (CoopRTS)* and *cooperative CTS (CoopCTS)* frames shown in Fig. 1, replacing the regular RTS and CTS packets of 802.11. More precisely, the proposed RTS/CTS handshaking follows the flow chart reported in Fig. 2. Specifically, the participants in the communication undertake the following actions.

*1) First action:* Let $ABEP'_{\text{target}} \triangleq \varrho\, ABEP_{\text{target}}$ be a (large) percentage of the total error rate budget, with $0 < \varrho < 1$ known at both $S$ and $D$. By acceding to the information contained in $\mathbf{A}_s$, as a first step $S$ searches for the largest values $R'_{\text{coop}}$ and $N_{\text{coop}}$ of $R' = R_{q'}$ and $N_h$, respectively, fulfilling the constraint

$$\sum_{H_i \in \mathcal{H}} P_{s,h_i}^{(q')}(e) \leq ABEP'_{\text{target}} \, . \tag{III.5}$$

If $N_{\text{coop}} \geq 1$, then $S$ will communicate through the CoopRTS frame (see Fig. 1) the packet length $B$, the target data rate $R_{\text{target}}$, the computed data rate $R'_{\text{coop}}$ for data transmission in Phase I, the threshold $\overline{\gamma}_{\text{coop}}$ evaluated as in (III.4) and, finally, the identities of the potential relays belonging to $\mathcal{H}_{\text{coop}}$. It is interesting to underline that, since $D$ knows which ones of the $N$ nodes can directly communicate with $S$,[7] to inform $D$ about which are the $N_{\text{coop}}$ relays selected by $S$, it is sufficient to piggyback in the CoopRTS packet a string of only $N$ bits: if the node $H_i$ has been selected as a relay, for $i \in \{1, 2, \ldots, N\} - \{s, d\}$, then the $i$th bit of the string takes on the value $1$, otherwise, it is set equal to $0$. On the other hand, if $N_{\text{coop}} = 0$, then $S$ will conclude that cooperation is useless; in this case, provided that $S$ can reliably communicate in direct mode with $D$ at the data rate $R_{\text{target}}$, i.e., $\overline{\gamma}_{s,d} \geq \overline{\gamma}_{\min,\overline{q}}$, then $S$ will send a *direct RTS (DirectRTS)* frame announcing that the transmission in direct mode can support $R_{\text{target}}$, forbidding any cooperative transmission. The DirectRTS frame also specifies the packet length $B$ and the target data rate $R_{\text{target}}$ for data transmission (see Fig. 1). Finally, failure to find at least one potential relay and meet $\overline{\gamma}_{s,d} \geq \overline{\gamma}_{\min,\overline{q}}$ implies that $R_{\text{target}}$ cannot be met and, thus, $S$ will reduce $R_{\text{target}}$ and/or increase $ABEP_{\text{target}}$.

[7]This information is stored in the CoopMatrix $\mathbf{A}_d$ and in the table of the MAC addresses.

*2) Second action:* The RTS message (Direct or Coop) is overheard by $D$, as well as by all the other nodes within the transmission range of $S$. For $i \in \{1, 2, \ldots, N+2\} - \{s\}$, by using non-data-aided estimation techniques [16], the node $\mathcal{N}_i$ can thereby update the estimate of the ASNRs $\overline{\gamma}_{s,i} = \gamma / \eta_{s,i}^{\alpha}$ already available from prior transmissions; consequently, from the knowledge of $\overline{\gamma}_{s,i}$,[8] the $i$th node can learn its distance $\eta_{s,i} = (\gamma/\overline{\gamma}_{s,i})^{1/\alpha}$ from $S$. If a DirectRTS packet is received, all the nodes within the transmission range of $S$ can learn $B$ and $R_{\text{target}}$ from it. In such a case, if $D$ is capable to receive the packet, it replies with a *direct CTS (DirectCTS)* control message containing $B$ and $R_{\text{target}}$ (see Fig. 1). All the nodes overhearing the DirectRTS packet and/or the DirectCTS one will defer its own transmissions for the duration of the direct transmission between $S$ and $D$. On the other hand, if a CoopRTS frame is transmitted by $S$, then the definitive choice between direct or cooperative mode is assigned to $D$. Let $ABEP''_{\text{target}} \triangleq (1-\varrho)\, ABEP_{\text{target}}$ be a (small) percentage of the total error rate budget, by using the information contained in $\mathbf{A}_d$, for a given value of $L$, first $D$ searches for the largest values $R''_{\text{coop}}$ of $R''$ meeting the constraints $R''\, R_{\text{code}} > R_{\text{target}}$ and

$$P_{\text{coop}}(e \,|\, \mathcal{E}^c) \leq ABEP''_{\text{target}} \,. \tag{III.6}$$

To solve such an optimization procedure, $D$ will use the information contained in the received CoopRTS packet. Since $ABEP'_{\text{target}} + ABEP''_{\text{target}} = ABEP_{\text{target}}$, it should be observed that conditions (III.5) and (III.6) automatically ensure the fulfillment of the constraint (III.2). The cooperative mode is feasible if (III.3) is satisfied. In this case, $D$ will issue a CoopCTS frame (see Fig. 1) requesting cooperation and piggybacking the parameters $B$, $R''_{\text{coop}}$ and $L$. On the other hand, if there exist any values of $L$ and $R''\, R_{\text{code}} > R_{\text{target}}$ satisfying (III.6), or (III.3) is not fulfilled, then $D$ will conclude that cooperation does not offer advantages; in this case, provided that $\overline{\gamma}_{s,d} \geq \overline{\gamma}_{\min,\overline{q}}$, the node $D$ will transmit the DirectCTS frame starting up a direct communication at the data rate $R_{\text{target}}$ and excluding any cooperative transmission. Failure to meet (III.2)–(III.3) and $\overline{\gamma}_{s,d} \geq \overline{\gamma}_{\min,\overline{q}}$ implies that $R_{\text{target}}$ cannot be met and, thus, $D$ will transmit a *negative CTS (NegCTS)* packet, mandating a reduction of $R_{\text{target}}$ and/or an increase of $ABEP_{\text{target}}$.

*3) Third action:* If a CoopCTS message is received, during Phase I $S$ transmits $\mathbf{a}'$ at a starting time $T'$ in direct mode using the data rate $R'_{\text{coop}}$. From the knowledge of $\overline{\gamma}_{\text{coop}}$, the node $S$ is able to recover the distance $\eta_{\text{coop}} = (\gamma/\overline{\gamma}_{\text{coop}})^{1/\alpha}$. We remember that $\eta_{\text{coop}}$ is the distance of the farthest recruited relay from $S$. To ensure a synchronous cooperative transmission, the node $S$ starts Phase II at the extended time $T'' = T' + (\eta_{\text{coop}}/c)$, with $c$ denoting the speed of light. Moreover, starting from the value of $R''_{\text{coop}}$, the node $S$ can determine the number of symbols $K''_{\text{coop}} = \lceil B/(R''_{\text{coop}}\, T_s)\rceil$ to be transmitted in Phase II; from the knowledge of both $K''_{\text{coop}}$ and the code length $L$, the node $S$ can choose the row dimension $P$ of the OSTBC matrix $\boldsymbol{\mathcal{C}}(\mathbf{a}'')$. Consequently, during Phase II $S$ transmits $\mathbf{a}''$ in cooperative mode at the data rate $R''_{\text{coop}}$, with a randomized OSTBC (ROSTBC) of order $L$. The transmission ends with an ACK/NACK control packet sent by $D$ in order to acknowledge a correct/incorrect reception of the data packet. On the other hand, if a DirectCTS frame is overheard, the node $S$ transmits $\mathbf{a}^{(\overline{q})}$ in direct mode at the rate $R_{\text{target}}$. Finally, when $S$ receives a NegCTS packet, no transmission will be started.

*4) Fourth action:* As a basic rule, we stipulate that, if a relay $H_i$ receives multiple (Direct or Coop) RTS and CTS messages, it will not cooperate to reduce the amount of interference introduced to the network. Clearly, the node $H_i$ will stay inactive throughout the subsequent session also when either a DirectRTS/DirectCTS messages or a NegCTS frame is received. On the other hand, if the relay $H_i$, that received the CoopRTS message, overhears the corresponding CoopCTS frame as well, then it decides whether or not it can join in Phase II: if $\overline{\gamma}_{s,h_i} \geq \overline{\gamma}_{\text{coop}}$, then $H_i$ will take part in the cooperative transmission by carrying out the following actions. The node $H_i$ receives the data transmitted by $S$ in Phase I at the time epoch $T'_i = T' + (\eta_{s,i}/c)$. From that time on, carrier sense at the relay is turned off for all the duration of the cooperative transmission since, during Phase II, the concurrent transmissions from the cooperating relays are not collisions. Similarly to $S$, from the knowledge of $\overline{\gamma}_{\text{coop}}$, the relay node can obtain the distance $\eta_{\text{coop}}$, and, starting from the values of $R''_{\text{coop}}$ and $L$, it can set the appropriate value of $P$. In Phase II, after demodulating the received packet, at the deferred time $T''_i = T'_i + [(\eta_{\text{coop}} - \eta_{s,i})/c]$, the node $H_i$ transmits $\mathbf{a}''$ in cooperative mode at the data rate $R''_{\text{coop}}$, with a $L$-order ROSTBC. It is readily seen that $T''_i = T''$ for each cooperating relay, which means that the transmissions from $S$ and the recruited relays towards $D$ are synchronized. Moreover, it is worth observing that only the nodes that overhear both the CoopRTS and CoopCTS control frames may cooperate subsequently and that the number of relays actually involved in Phase II depends on $\overline{\gamma}_{\text{coop}}$.

*Remark 1.* A first distinguishing feature of our approach is that no exchange of control information (e.g., an HTS message) at the MAC sublayer is required between the relays and the $(S, D)$ pair. Indeed, piggybacking the ASNR threshold $\overline{\gamma}_{\text{coop}}$ in the CoopRTS packet is the mechanism used indirectly to recruit nodes. With respect to existing CoopMAC protocols [6], [8], this allows one to reduce the signaling overhead, especially in heavily loaded networks.

*Remark 2.* Although all the participants in Phase II are synchronous, their transmitted signals might be received at $D$ with different delays since the cooperating transmitters are not co-located. In a small size network, the relative delays between such signals are fairly small [17]. Even if the signals transmitted from $S$ and the relays do not arrive at $D$ within a very small gap (compared to $T_s$), the effect is that of a frequency-selective channel [11] and one can use time-reverse space-time codes or space-time orthogonal frequency-division multiplexing transmission techniques to harvest cooperative gains.

## IV. BEST SETTING OF THE PHY PARAMETERS

In order to determine the data rates $R'_{\text{coop}}$ and $R''_{\text{coop}}$ satisfying (III.2) and (III.3), as well as the threshold $\overline{\gamma}_{\text{coop}}$ and the code length $L$, the nodes $S$ and $D$ need to jointly compute the function $f(R', R'', L, N_h)$ representing the dependence of $P_{\text{coop}}(e)$ on these parameters. To be specific, we recall that

[8]Hereinafter, we assume that the path-loss exponent $\alpha$ and the ASNR per symbol $\gamma$ are known at all the nodes, which are reasonable assumptions if each node expends the same transmission energy.

$R_{\text{target}} = R_{\overline{q}}$, with $\overline{q} \in \{1, 2, \ldots, Q-1\}$, and let us set $R' = R_{\overline{q}+m} > R_{\overline{q}}$, with $m \in \{1, 2, \ldots, Q-\overline{q}\}$ being a parameter to be optimized.[9] The information contained in $\mathbf{A}_s$ and $\mathbf{A}_d$ randomly changes as a consequence of fading, station mobility, node failure or expired battery life. Since having up to date the CoopMatrices is a noticeable problem in practice, we will propose different settings of the PHY parameters $R'_{\text{coop}}$, $R''_{\text{coop}}$, $L$ and $\overline{\gamma}_{\text{coop}}$, which account for different degrees of accuracy of the information stored in $\mathbf{A}_s$ and $\mathbf{A}_d$. Specifically, in Subsection IV-A, we first consider the case when $S$ and $D$ have perfect knowledge of the average network status and, then, in Subsection IV-B, we focus attention on the more realistic scenario wherein $S$ and $D$ have a *partial* information about the average channel quality of the relay links. While the former case is an ideal situation, the algorithm developed in Subsection IV-A provides a useful benchmark for comparing the methods proposed in Subsection IV-B. Furthermore, the treatment of Subsection IV-A is an intermediate step which will prove instrumental in Subsection IV-B.

### *A. Setting of the PHY parameters when the CoopMatrices are perfectly updated*

Suppose that the CoopMatrices at $S$ and $D$ disposal exactly reflect the current average network status. In such a case, for a given data rate $R' = R_{\overline{q}+m}$, with $m \in \{1, 2, \ldots, Q-\overline{q}-1\}$, the $i$th node $H_i$, for $i \in \{1, 2, \ldots, N+2\} - \{s, d\}$, is a *potential* relay if $\{\mathbf{A}_s\}_{s,h_i} \geq \overline{\gamma}_{\min,\overline{q}+m}$, i.e., a direct transmission between $S$ and $H_i$ can be reliably sustained at a data rate greater than or equal to $R_{\overline{q}+m}$. This is a necessary (but not sufficient) condition for selecting the relays, because the selection of the best number of active relays comes from reaching the best tradeoff among the two terms in (II.10). Therefore, the number $0 \leq N_{s,d}(m) \leq N$ of potential relays is evaluated as

$$N_{s,d}(m) = \sum_{\substack{p=1 \\ p \neq s,d}}^{N+2} \mathrm{u}(\{\mathbf{A}_s\}_{s,p} - \overline{\gamma}_{\min,\overline{q}+m}) \,. \tag{IV.1}$$

In order to avoid the saturation effect of the diversity order in Phase II,[10] we assume that $L-1 > N$. In this case, let $N_{s,d}(m) \geq 1$ and consider the variable $0 \leq N_h \leq N_{s,d}(m)$, from (II.2), (II.6) and (II.10), the end-to-end ABEP of the proposed scheme is upper bounded by (IV.2) reported at the top of the next page, where $\mathcal{I}_h \subseteq \{1, 2, \ldots, N_{s,d}(m)\}$ is a set of cardinality $N_h$. To meet (III.2) in a distributed fashion, the evaluation of $P_{\text{coop}}(e)$ is split according to (III.5) and (III.6). Henceforth, the PHY parameters $R'_{\text{coop}}$, $N_{\text{coop}}$ and $\overline{\gamma}_{\text{coop}}$ are determined at $S$ by searching for the maximum values of $m$ and $N_h$ satisfying

$$f_2(m, N_h) \leq ABEP'_{\text{target}} \,, \tag{IV.3}$$

whereas, for a given value of $L > N-1$, the remaining PHY parameter $R''_{\text{coop}}$ is determined at $D$ by searching for the maximum value of $M''$ obeying

$$f_1(M'', L, N_{\text{coop}}) \leq ABEP''_{\text{target}} \,. \tag{IV.4}$$

In general, for a given value of $m$, the solution $N_{\text{coop}} = N_{s,d}(m)$ is not the one found to satisfy the target ABEP requirement. This is because, when the sum of the ABEPs $\{P_{s,h_i}^{(\overline{q}+m)}(e)\}_{i \in \mathcal{I}_h}$ over the broadcast link from $S$ to the relays is equal to or greater than $ABEP'_{\text{target}}$, one has $f_2(m, N_h) > ABEP'_{\text{target}}$, which leaves no possible choice available. This situation may occur even when the ABEPs over the $(S, H_i)$ links are individually much smaller than $ABEP'_{\text{target}}$ and confirms that the bottleneck of our cooperative strategy is the broadcast link from $S$ to the relays. To overcome this problem, we use only a subset $\mathcal{H}_{\text{coop}}$ of the potential relays belonging to $\mathcal{H} = \{H_1, H_2, \ldots, H_{N_{s,d}(m)}\}$, by successively discarding those relays that are farthest from $S$ or, equivalently, those relays whose $(S, H_i)$ links exhibit the smallest ASNRs. To this aim, we propose the procedure summarized in Table I (see the next page) to solve (IV.3) and (IV.4).

*Remark 3*. When the CoopMatrix is perfectly updated, all the participants in the communication know not only the number $N_{\text{coop}}$ of cooperating relays (i.e., those nodes $H_i$ for which $\overline{\gamma}_{s,h_i} \geq \overline{\gamma}_{\text{coop}}$), but also which ones will cooperate, that is, the subset $\mathcal{H}_{\text{coop}}$. Hence, at the expense of explicitly assigning the codes, an algorithm similar to that of Table I can be also developed for both centralized [1] and decentralized deterministic [4] schemes. However, if one resorts to [4], the matter of evaluating the PHY parameters $R'_{\text{coop}}$, $R''_{\text{coop}}$, $L$ and $\overline{\gamma}_{\text{coop}}$ is complicated by the fact that, in its turn, matrix $\boldsymbol{\mathcal{R}}$ has to be optimized to reduce the performance loss entailed by the distributed implementation.

*Remark 4*. Some of the relays belonging to $\mathcal{H}_{\text{coop}}$ could be unwilling to cooperate. These nodes will be referred to as "dissident" relays. In such a case, our method still works[11] and the cooperative transmission at the data rates $R'_{\text{coop}}$ and $R''_{\text{coop}}$ can take place as well. If the number of relays actually joining in Phase II is smaller than $N_{\text{coop}}$, the only problem which possibly arises is that inequality $P_{\text{coop}}(e) \leq ABEP_{\text{target}}$ could not be satisfied. However, as shown in Subsection V-A, the degradation is graceful since our algorithm includes natural margins due to the use of an upper bound on $P_{\text{coop}}(e)$. Thus, successful decoding occurs even if $N_{\text{coop}}$ is overestimated by a small difference.

### *B. Setting of the PHY parameters when the CoopMatrices are partially outdated*

It is difficult for each node to frequently update its CoopMatrix, especially in rapidly time-varying channels and, thus, the information at both $S$ and $D$ disposal might not reflect exactly the current status of the network. However, each node $\mathcal{N}_i$, for $i \in \{1, 2, \ldots, N+2\}$, is able to judge whether or not its CoopMatrix is updated [6], by additionally storing for each

[9] $R'_{\text{coop}}$ and, thus, $R'$, must be necessarily greater than $R_{\text{target}}$.

[10] In practice, when $N$ takes on large values, it might be unsustainable using OSTBC with large enough values of $L$. In such a case, setting $L-1 < N$ might be preferred for complexity savings. Anyway, the proposed method can be straightforwardly modified to account for the case when $L-1 < N$.

[11] Remember that, thanks to the randomized coding rule, the knowledge of actual number of cooperating nodes is not required.

$$P_{\text{coop}}(e) = f(R', R'', N_h) \leq \overbrace{\frac{4}{\log_2(M'')}\left(1 - \frac{1}{\sqrt{M''}}\right) \frac{\left[\frac{L\,(M''-1)}{3}\right]^{N_h+1}}{\{\mathbf{A}_d\}_{s,d} \prod_{i\in\mathcal{I}_h}\{\mathbf{A}_d\}_{h_i,d}} \frac{(L-N_h-2)!}{(L-1)!}}^{f_1(M'',L,N_h)} + \underbrace{\frac{2}{\log_2(M_{\overline{q}+m})}\left(1 - \frac{1}{\sqrt{M_{\overline{q}+m}}}\right)\sum_{i\in\mathcal{I}_h}\left(1 - \sqrt{\frac{3\,\{\mathbf{A}_s\}_{s,h_i}}{2(M_{\overline{q}+m}-1) + 3\,\{\mathbf{A}_s\}_{s,h_i}}}\right)}_{f_2(m,N_h)} = f_1(M'', L, N_h) + f_2(m, N_h)\,, \quad \text{(IV.2)}$$

---

Given $R_{\text{target}} = R_{\overline{q}}$, with $\overline{q} \in \{1, 2, \ldots, Q-1\}$, first set $m = Q - \overline{q} - 1$, i.e., the node $S$ starts by considering the highest value of the data rate $R' = R_{\overline{q}+m}$ in Phase I. Subsequently $S$ performs the first four steps of the procedure:

- *Step* 1: calculate $N_{s,d}(m)$ according to (IV.1).
- *Step* 2: if $N_{s,d}(m) = 0$, then decrement $m$ by one unit and, if $m \geq 1$, repeat Step 1 again, otherwise ($m = 0$), conclude that cooperation does not offer advantages; else [$N_{s,d}(m) \geq 1$], initially set $\mathcal{I}_h = \{1, 2, \ldots, N_{s,d}(m)\}$, i.e., the node $S$ starts by involving all the potential relays and hence $N_h = N_{s,d}(m)$.
- *Step* 3: evaluate $\tau(m, N_h) \triangleq ABEP'_{\text{target}} - f_2(m, N_h)$ [see (IV.3)].
- *Step* 4: if $\tau(m, N_h) \geq 0$, then set $\overline{\gamma}_{\text{coop}} = \min_{i\in\mathcal{I}_h}\{\mathbf{A}_s\}_{s,h_i}$, $N_{\text{coop}} = N_h$ and $R'_{\text{coop}} = R_{\overline{q}+m}$; otherwise [$\tau(m, N_h) < 0$],
  - (i) set $i_{\min} = \arg\min_{i\in\mathcal{I}_h}\{\mathbf{A}_s\}_{s,h_i}$ and reduce $N_h$ by one unit by setting $\mathcal{I}_h = \mathcal{I}_h - \{i_{\min}\}$;
  - (ii) if $N_h \geq 1$, repeat Steps 3 and 4 again, otherwise ($N_h = 0$), decrement $m$ by one unit and, if $m \geq 1$, repeat Steps 1, 2, 3 and 4 again; else ($m = 0$), conclude that cooperation does not offer advantages.

The last two steps of the procedure are carried out by $D$:

- *Step* 5: denote with $M''_{\text{coop}} = 2^{b''_{\text{coop}}}$ the largest value of $M''$ satisfying (IV.4), with $b''_{\text{coop}}$ even, if $R''_{\text{coop}}\, R_{\text{code}} \leq R_{\text{target}}$, with $R''_{\text{coop}} = b''_{\text{coop}}/T_s$, then conclude that cooperation does not offer advantages.
- *Step* 6: if $R'_{\text{coop}}$ and $R''_{\text{coop}}\, R_{\text{code}} > R_{\text{target}}$ satisfy condition (III.3), the cooperative communication can be carried out along the framework described in Section III; otherwise [(III.3) is violated], conclude that cooperation does not offer advantages.

TABLE I

THE COOPMATRICES ARE PERFECTLY UPDATED: SUMMARY OF THE PROCEDURE PROPOSED TO SOLVE (IV.3)–(IV.4).

neighboring node $\mathcal{N}_j$, with $j \neq i \in \{1, 2, \ldots, N+2\}$, the time of the last transmission received from it. If $\mathcal{N}_i$ does not hear a transmission from $\mathcal{N}_j$ for a certain time exceeding a predefined threshold value, the corresponding entries of the CoopMatrix $\mathbf{A}_i$ are considered outdated. In our mathematical framework, this fact can be accounted for by assuming that $\mathbf{A}_i$ contains *exact* information about the actual channel conditions and status, and introducing the additional matrix $\mathbf{B}_i \in \mathbb{R}^{(N+2)\times(N+2)}$, whose $(p_1, p_2)$th entry, for $p_1, p_2 \in \{1, 2, \ldots, N+2\}$, is equal to $\{\mathbf{B}_i\}_{p_1,p_2} = 1$ if $p_1 \neq p_2$ and the information over the $(N_{p_1}, N_{p_2})$ link is updated, $\{\mathbf{B}_i\}_{p_1,p_2} = 0$ else. Since $\mathbf{A}_i$ is symmetric, the matrix $\mathbf{B}_i$ turns out to be symmetric, too. In general, the matrix $\mathbf{B}_i$ depends on the node index $i \in \{1, 2, \ldots, N\}$. However, since the potential relays must be located within the overlapping area of the transmission ranges of $S$ and $D$, we assume that, if $S$ has updated information over the $(S, H_p)$ link, for $p \in \{1, 2, \ldots, N\} - \{s, d\}$, then the information over the $(H_p, D)$ link at $D$ disposal is also updated, i.e, $\{\mathbf{B}_s\}_{s,p} = \{\mathbf{B}_d\}_{p,d}$. Thus, the CoopMatrices at disposal of $S$ and $D$ containing *partial* network state information are $\widetilde{\mathbf{A}}_s \triangleq \mathbf{A}_s \odot \mathbf{B}_s$ and $\widetilde{\mathbf{A}}_d \triangleq \mathbf{A}_d \odot \mathbf{B}_d$. In this case, for a given $R' = R_{\overline{q}+m}$, with $m \in \{1, 2, \ldots, Q-\overline{q}-1\}$, the number $0 \leq \widetilde{N}_{s,d}(m) \leq N$ of potential relays is calculated as follows

$$\widetilde{N}_{s,d}(m) = \sum_{\substack{p=1\\ p\neq s,d}}^{N+2} \mathrm{u}(\{\widetilde{\mathbf{A}}_s\}_{s,p} - \overline{\gamma}_{\min,\overline{q}+m})\,. \quad \text{(IV.5)}$$

In this case, the optimization variable $N_h$ representing the *actual* number of active relays can be expressed as $N_h = \widetilde{N}_h + \Delta N_h \leq N$, where $0 \leq \widetilde{N}_h \leq \widetilde{N}_{s,d}(m)$ denotes the number of potential relays whose corresponding ASNRs are *known* to $S$, whereas $0 \leq \Delta N_h \leq \widetilde{N}_{\text{unknown}}$ represents the number of potential relays whose average channel quality is *unknown* to both $S$ and $D$, with

$$\widetilde{N}_{\text{unknown}} \triangleq \sum_{\substack{p=1\\ p\neq s,d}}^{N+2} (1 - \{\mathbf{B}_s\}_{s,p}) = \sum_{\substack{p=1\\ p\neq s,d}}^{N+2} (1 - \{\mathbf{B}_d\}_{p,d})\,. \quad \text{(IV.6)}$$

In the sequel, without loss of generality, we assume that the relays with "known" mean CSI are the first $\widetilde{N}_h$ nodes of the set $\mathcal{H} = \{H_1, H_2, \ldots, H_{N_h}\}$, while the last $\Delta N_h$ ones are those with "unknown" mean CSI. Starting from the ASNRs of $H_1, H_2, \ldots, H_{\widetilde{N}_h}$, if the presence of the other relays $H_{\widetilde{N}_h+1}, H_{\widetilde{N}_h+2}, \ldots, H_{N_h}$ is completely ignored and the procedure of Subsection IV-B is carried out by replacing $N_{s,d}(m)$ with $\widetilde{N}_{s,d}(m)$, then $S$ and $D$ will calculate cer-

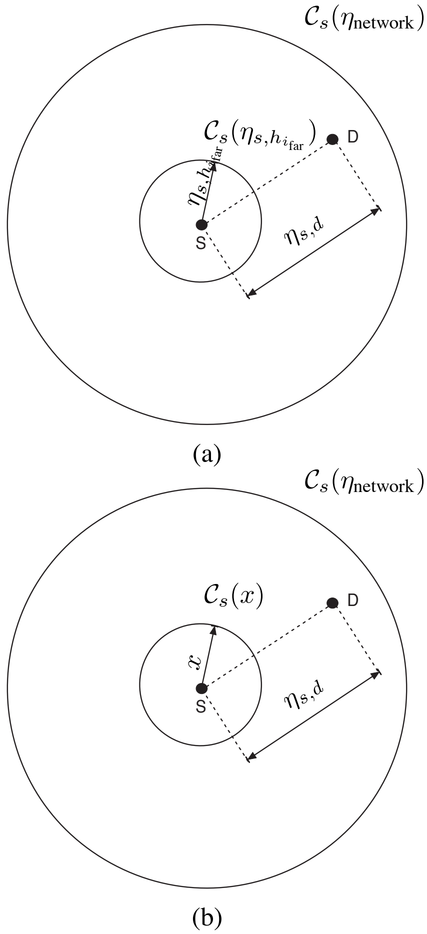


Fig. 3. (a) When $\widetilde{N}_{s,d}(m) > 0$, all the active relays (with either known or unknown mean CSI) will be located within the overlap area of the two circles $\mathcal{C}_s(\eta_{s,h_{i_{\text{far}}}})$ and $\mathcal{C}_s(\eta_{\text{network}})$. (b) When $\widetilde{N}_{s,d}(m) = 0$, $S$ and $D$ are completely in the dark about the availability of potential relays and, thus, they adaptively search for those that will be located within the overlap area of the two circles $\mathcal{C}_s(x)$ and $\mathcal{C}_s(\eta_{\text{network}})$, where in its turn $x$ is a parameter to be optimized.

tain ASNR threshold $\overline{\gamma}_{\text{coop}}$ and data rates $R'_{\text{coop}}$ and $R''_{\text{coop}}$. However, by passing these parameters via the CoopRTS and CoopCTS frames, $S$ and $D$ may recruit not only a subset of the relays $H_1, H_2, \ldots, H_{\widetilde{N}_h}$ with known mean CSI, but also a subset of the relays $H_{\widetilde{N}_h+1}, H_{\widetilde{N}_h+2}, \ldots, H_{N_h}$ with unknown mean CSI. Consequently, the computed $R'_{\text{coop}}$ and $R''_{\text{coop}}$ do not fully account for the available spatial diversity and, even worse, they could not satisfy the target ABEP requirement. To avoid such a shortcoming, we derive a looser upper bound on $P_{\text{coop}}(e)$ than (IV.2), by modeling $\Delta N_h$ as a random variable. Besides $L-1 > N$, we assume that all the nodes are uniformly and independently distributed over a circle $\mathcal{C}_s(\eta_{\text{network}})$ of radius $\eta_{\text{network}}$ centered around $S$. Henceforth, we distinguish the following two cases:

*1) Case A:* when $\widetilde{N}_{s,d}(m) > 0$, let $\widetilde{\mathcal{I}}_h \subseteq \{1, 2, \ldots, \widetilde{N}_{s,d}(m)\}$ be a set of cardinality $\widetilde{N}_h$ collecting a subset of the indices of all the potential relays whose mean CSI is known to $S$ and, moreover, let $i_{\text{far}} \triangleq \arg\min_{i \in \widetilde{\mathcal{I}}_h} \{\widetilde{\mathbf{A}}_s\}_{s,h_i}$ be the index of the relay $H_{i_{\text{far}}}$ that is farthest from $S$; the distance of $H_{i_{\text{far}}}$ from $S$ is $\eta_{s,h_{i_{\text{far}}}} = (\gamma/\widetilde{\gamma}_{s,h_{i_{\text{far}}}})^{1/\alpha}$, where $\widetilde{\gamma}_{s,h_{i_{\text{far}}}} = \{\widetilde{\mathbf{A}}_s\}_{s,h_{i_{\text{far}}}}$. Since $\overline{\gamma}_{\text{coop}}$ is computed by using the available ASNRs $\{\widetilde{\mathbf{A}}_s\}_{s,h_i}$, with $i \in \widetilde{\mathcal{I}}_h$, all the $\Delta N_h$ potential relays whose mean CSI is unknown must be located in the circle $\mathcal{C}_s(\eta_{s,h_{i_{\text{far}}}})$ of radius $\eta_{s,h_{i_{\text{far}}}}$ centered around $S$ [see Fig. 3(a)].

*2) Case B:* when $\widetilde{N}_{s,d}(m) = 0$, the node $S$ does not have any *a priori* information about the availability of potential relays, that is, $\widetilde{\mathcal{I}}_h = \varnothing$, $\widetilde{N}_h = 0$ and $\Delta N_h = N_h$, and, thus, it has to search for them adaptively. To do this, starting from the ASNR over the $(S, D)$ link, the distance between $S$ and $D$ can be computed as $\eta_{s,d} = (\gamma/\{\widetilde{\mathbf{A}}_s\}_{s,d})^{1/\alpha}$. Let $\mathcal{C}_s(x)$ denote the circle of radius $x \in (0, \eta_{s,d}]$ (to be adaptively optimized) centered about $S$, then $S$ aims at recruiting all the nodes that are included in $\mathcal{C}_s(x)$ [see Fig. 3(b)]. In such a case, let $H_{i_{\text{far}}} \in \mathcal{C}_s(x)$ denote the potential relay that is farthest from $S$, the ASNR over the $(S, H_{i_{\text{far}}})$ link is $\widetilde{\gamma}_{s,h_{i_{\text{far}}}} = \gamma/\eta^{\alpha}_{s,h_{i_{\text{far}}}}$, with $\eta_{s,h_{i_{\text{far}}}} = x$.

In either case, all the ABEPs over the $(S, H_i)$ links with unknown mean CSI are upper bounded by the ABEP over the $(S, H_{i_{\text{far}}})$ link, i.e.,

$$P^{(\overline{q}+m)}_{s,h_i}(e) \leq P^{(\overline{q}+m)}_{s,h_{i_{\text{far}}}}(e) \,, \tag{IV.7}$$

$\forall i \in \{\widetilde{N}_h + 1, \widetilde{N}_h + 2, \ldots, N_h\}$. Moreover, for a given value of $\Delta N_h$, on the basis of (II.6), we can obtain an upper bound on the first term in (II.10). Let us assume without loss of generality that the ASNRs over the cooperative link from the active relays to $D$ are in increasing order, i.e., $\overline{\gamma}_{h_1,d} \leq \overline{\gamma}_{h_2,d} \leq \cdots \leq \overline{\gamma}_{h_{N_h},d}$, it holds that[12]

$$\prod_{i=\widetilde{N}_h+1}^{N_h} \overline{\gamma}_{h_i,d} \geq [(\widetilde{\gamma}_{h_{i_{\text{far}}},d})_{\max}]^{\Delta N_h} \,, \tag{IV.8}$$

with $(\widetilde{\gamma}_{h_{i_{\text{far}}},d})_{\max} \triangleq \gamma/(\eta_{s,d} + \eta_{s,h_{i_{\text{far}}}})^{\alpha}$. Therefore, accounting for (II.6) and (II.10), the end-to-end ABEP of the two-phase transmission can be upper bounded from a unified perspective as in (IV.9) at the top of the next page, which is obtained by further averaging over the random variable $\Delta N_h$. Clearly, the upper bound in (IV.9) is worse than that in (IV.2) since, for a given value of $\Delta N_h$, the two terms in (II.10) are upper bounded by assuming that all the relays with unknown mean CSI are located on the edge of the circle $\mathcal{C}_s(\eta_{s,h_{i_{\text{far}}}})$ in Case A or the circle $\mathcal{C}_s(x)$ in Case B, at the maximum distance from $D$. To evaluate the expected values in (IV.9), we observe that under our hypotheses $\Delta N_h$ is a binomial$(\widetilde{N}_{\text{unknown}}, p)$ random variable, whose probability mass function is

$$d(n) \triangleq P(\Delta N_h = n) \,, \tag{IV.10}$$

where $p \triangleq \chi^2/\eta^2_{\text{network}}$ is the probability of finding a node within the overlap area of the two circles $\mathcal{C}_s(\cdot)$ and $\mathcal{C}_s(\eta_{\text{network}})$, with $\mathcal{C}_s(\cdot) \equiv \mathcal{C}_s(\eta_{s,h_{i_{\text{far}}}})$ and $\chi = \eta_{s,h_{i_{\text{far}}}}$ in Case A, or $\mathcal{C}_s(\cdot) \equiv \mathcal{C}_s(x)$ and $\chi = x$ in Case B. Consequently, one obtains that

[12]If the relay node $H_{i_{\text{far}}}$ is at distance $\eta_{s,h_{i_{\text{far}}}}$ from $S$ at an angle $\beta$ with respect to the line joining $S$ and $D$, its distance to $D$ is given by $\eta_{h_{i_{\text{far}}},d} = \sqrt{\eta^2_{s,d} + \eta^2_{s,h_{i_{\text{far}}}} - 2\,\eta_{s,d}\,\eta_{s,h_{i_{\text{far}}}}\cos(\beta)}$, which assumes the maximum value $(\eta_{h_{i_{\text{far}}},d})_{\max} \triangleq \eta_{s,d} + \eta_{s,h_{i_{\text{far}}}}$ for $\beta = \pi$.

$$
\begin{aligned}
P_{\text{coop}}(e) \leq & \overbrace{\frac{4\left(1-\frac{1}{\sqrt{M''}}\right)}{\log_2(M'')(L-1)!} \frac{\left[\frac{L\,(M''-1)}{3}\right]^{\widetilde{N}_h+1}}{\{\widetilde{\mathbf{A}}_d\}_{s,d}\prod_{i\in\widetilde{\mathcal{I}}_h}\{\widetilde{\mathbf{A}}_d\}_{h_i,d}} \mathrm{E}\left\{\left[\frac{L\,(M''-1)}{3\,(\widetilde{\gamma}_{h_{i_{\text{far}}},d})_{\max}}\right]^{\Delta N_h}(L-\widetilde{N}_h-\Delta N_h-2)!\right\}}^{\widetilde{f}_1(M'',L,\widetilde{N}_h,x)} \\
& + \frac{2}{\log_2(M_{\overline{q}+m})}\left(1-\frac{1}{\sqrt{M_{\overline{q}+m}}}\right)\left\{\sum_{i\in\widetilde{\mathcal{I}}_h}\left(1-\sqrt{\frac{3\,\{\widetilde{\mathbf{A}}_s\}_{s,h_i}}{2(M_{\overline{q}+m}-1)+3\,\{\widetilde{\mathbf{A}}_s\}_{s,h_i}}}\right)\right. \\
& \left. +\mathrm{E}[\Delta N_h]\left(1-\sqrt{\frac{3\,\widetilde{\gamma}_{s,h_{i_{\text{far}}}}}{2(M_{\overline{q}+m}-1)+3\,\widetilde{\gamma}_{s,h_{i_{\text{far}}}}}}\right)\right\}\,, \qquad \text{(IV.9)}
\end{aligned}
$$

$\mathrm{E}[\Delta N_h] = p\,\widetilde{N}_{\text{unknown}}$ and additionally has

$$
\begin{aligned}
\mathrm{E}&\left\{\left[\frac{L\,(M''-1)}{3\,(\widetilde{\gamma}_{h_{i_{\text{far}}},d})_{\max}}\right]^{\Delta N_h}(L-\widetilde{N}_h-\Delta N_h-2)!\right\} \\
&= \sum_{n=0}^{\widetilde{N}_{\text{unknown}}} d(n)\,\left[\frac{L\,(M''-1)}{3\,(\widetilde{\gamma}_{h_{i_{\text{far}}},d})_{\max}}\right]^{n}(L-\widetilde{N}_h-n-2)!\,. \qquad \text{(IV.11)}
\end{aligned}
$$

To fulfill $P_{\text{coop}}(e) = \widetilde{f}(R', R'', L, \widetilde{N}_h, x) \leq ABEP_{\text{target}}$, the PHY parameters $R'_{\text{coop}}$, $N_{\text{coop}}$ and $\overline{\gamma}_{\text{coop}}$ are computed at $S$ by searching for the maximum values of $m$ and $\widetilde{N}_h$ in Case A or $x$ in Case B satisfying

$$
\widetilde{f}_2(m, \widetilde{N}_h, x) \leq ABEP'_{\text{target}}\,, \qquad \text{(IV.12)}
$$

with

$$
\begin{aligned}
\widetilde{f}_2(m,\widetilde{N}_h,x) &\triangleq \frac{2}{\log_2(M_{\overline{q}+m})}\left(1-\frac{1}{\sqrt{M_{\overline{q}+m}}}\right) \\
&\cdot\left\{\sum_{i\in\widetilde{\mathcal{I}}_h}\left(1-\sqrt{\frac{3\,\{\widetilde{\mathbf{A}}_s\}_{s,h_i}}{2(M_{\overline{q}+m}-1)+3\,\{\widetilde{\mathbf{A}}_s\}_{s,h_i}}}\right)\right. \\
&\left. +p\,\widetilde{N}_{\text{unknown}}\left(1-\sqrt{\frac{3\,\widetilde{\gamma}_{s,h_{i_{\text{far}}}}}{2(M_{\overline{q}+m}-1)+3\,\widetilde{\gamma}_{s,h_{i_{\text{far}}}}}}\right)\right\}\,, \qquad \text{(IV.13)}
\end{aligned}
$$

whereas, for a given value of $L > N-1$, the remaining PHY parameter $R''_{\text{coop}}$ is determined at $D$ by searching for the maximum value of $M''$ obeying

$$
\widetilde{f}_1(M'', L, \widetilde{N}_h, x) \leq ABEP''_{\text{target}}\,. \qquad \text{(IV.14)}
$$

The procedure in Table II (see the next page) is proposed to solve (IV.12) and (IV.14), where "ineffective" relays are successively discarded to meet the target requirements.

*Remark 5*. For the realistic scenario at hand, a procedure similar to that reported in Table II cannot be developed for neither centralized [1] nor decentralized deterministic [4] schemes. Indeed, in this case, starting from the information at its disposal, $S$ and $D$ can predict (in a probabilistic sense) the number of cooperating nodes, but it is unable to "identify" exactly which ones will be recruited. The robustness of the RCoopMAC framework against average network status misinformation stems from the fact that a code allocation (and thus a relay identification) is unnecessary, since coding is based on independent randomization at each cooperating node.

## V. NUMERICAL PHY PERFORMANCE

In this section, we present a comparative performance study of a generic version of the CoopMAC protocol and the RCoopMAC approach, either when the CoopMatrices $\mathbf{A}_s$ and $\mathbf{A}_d$ are perfectly updated (referred to as "CoopMAC-up" and "RCoopMAC-up") or when these CoopMatrices are partially outdated (referred to as "CoopMAC-out" and "RCoopMAC-out").[13] Both CoopMAC and RCoopMAC schemes rely on CSMA/CA as medium access protocol and, therefore, performance comparison between CoopMAC and RCoopMAC is fair. Indeed, although there are several stations in the region of interest, which is given by the union of the RTS range of $S$ and the CTS range of $D$, when the CoopMAC protocol is active, only three of them ($S$, $D$ and the best relay) can participate in the communication due to the CA mechanism. Hence, the RCoopMAC method uses relays that are silenced by the CoopMAC protocol. We also report the performance of the transmission in direct mode from $S$ to $D$ at the data rate $R_{\text{target}}$ (referred to as "DM"). Herein, we focus on the PHY performances of the methods under comparison. An initial study on the MAC layer performance of such approaches in terms of network throughput and service delay can be found in [12].

In all the experiments, the following simulation setting is adopted. We assume that $S$ and $D$ can reliably communicate at the target data rate. According to (II.3), a direct communication between $S$ and $D$ at the data rate $R_{\text{target}}$ can take place only if the ASNR $\overline{\gamma}_{s,d} = \gamma/\eta^{\alpha}_{s,d}$ at $D$ is greater than $\overline{\gamma}_{\min,\overline{q}}$: hence, the maximum separation distance within which a packet is successfully received is $(\eta_{s,d})_{\max} \triangleq \left(\gamma/\overline{\gamma}_{\min,\overline{q}}\right)^{1/\alpha}$. The

[13]The generic version of the CoopMAC protocol is implemented as follows. Let $R_{s,h_i} \in \{R_1, R_2, \ldots, R_{Q-1}\}$ and $R_{h_i,d} \in \{R_1, R_2, \ldots, R_{Q-1}\}$ denote the data rates over the $(S, H_i)$ and $(H_i, D)$ links, respectively, for $i \in \{1, 2, \ldots, N+2\} - \{s, d\}$. Relying on the matrix $\mathbf{A}_s$, the node $S$ individuates the best relay $H_{i_{\text{best}}}$ that minimizes the average bit period $(R_{i,\text{coop}})^{-1} \triangleq (R_{s,h_i})^{-1} + (R_{h_i,d})^{-1}$: if $(R_{i_{\text{best}},\text{coop}})^{-1} < (R_{\text{target}})^{-1}$, then $H_{i_{\text{best}}}$ is recruited; otherwise, the node $S$ will transmit to $D$ in direct mode. The "CoopMAC-out" method works similarly, with the only difference that, in this case, the outdated CoopMatrix $\widetilde{\mathbf{A}}_s$ is used for recruiting the best relay.

Let $\Delta_x$ be a small positive step-size. Given $R_{\text{target}} = R_{\overline{q}}$, with $\overline{q} \in \{1, 2, \ldots, Q-1\}$, first set $m = Q - \overline{q} - 1$, i.e., the node $S$ starts by considering the highest value of the data rate $R = R_{\overline{q}+m}$ in Phase I. Subsequently $S$ performs the first four steps of the procedure:

- *Step* 1: compute $\widetilde{N}_{s,d}(m)$ according to (IV.5).
- *Step* 2: if $\widetilde{N}_{s,d}(m) > 0$, set $\widetilde{\mathcal{I}}_h = \{1, 2, \ldots, \widetilde{N}_{s,d}(m)\}$ and refer to Case A, otherwise $[\widetilde{N}_{s,d}(m) = 0]$, set $x = \eta_{s,d}$ and refer to Case B.
- *Step* 3: evaluate $\widetilde{\tau}(m, \widetilde{N}_h, x) \triangleq ABEP'_{\text{target}} - \widetilde{f}_2(m, \widetilde{N}_h, x)$ [see (IV.12)].
- *Step* 4: if $\widetilde{\tau}(m, \widetilde{N}_h, x) \geq 0$, then set $\overline{\gamma}_{\text{coop}} = \min_{i \in \widetilde{\mathcal{I}}_h} \{\widetilde{\mathbf{A}}_s\}_{s,h_i}$ in Case A or $\overline{\gamma}_{\text{coop}} = \gamma / x^{\alpha}$ in Case B, and $R'_{\text{coop}} = R_{\overline{q}+m}$; otherwise $[\widetilde{\tau}(m, \widetilde{N}_h, x) < 0]$,
  - (i) in Case A, set $\widetilde{\imath}_{\min} = \arg\min_{i \in \widetilde{\mathcal{I}}_h} \{\widetilde{\mathbf{A}}_s\}_{s,h_i}$ and reduce $\widetilde{N}_h$ by one unit by setting $\widetilde{\mathcal{I}}_h = \widetilde{\mathcal{I}}_h - \{\widetilde{\imath}_{\min}\}$ or, in Case B, reduce the radius of the circle $\mathcal{C}_s(x)$ by setting $x = x - \Delta_x$;
  - (ii) if $\widetilde{N}_h \geq 1$ in Case A or $x > 0$ in Case B, repeat Steps 3 and 4 again, otherwise ($\widetilde{N}_h = 0$ in Case A or $x \leq 0$ in Case B), decrement $m$ by one unit and, if $m \geq 1$, repeat Steps 1, 2, 3 and 4 again; else ($m = 0$), conclude that cooperation does not offer advantages.

The last two steps of the procedure are carried out by $D$:

- *Step* 5: denote with $M''_{\text{coop}} = 2^{b''_{\text{coop}}}$ the largest value of $M''$ satisfying (IV.14), with $b''_{\text{coop}}$ even, if $R''_{\text{coop}} R_{\text{code}} \leq R_{\text{target}}$, with $R''_{\text{coop}} = b''_{\text{coop}}/T_s$, then conclude that cooperation does not offer advantages.
- *Step* 6: if $R'_{\text{coop}}$ and $R''_{\text{coop}} R_{\text{code}} > R_{\text{target}}$ satisfy condition (III.3), the cooperative communication can be carried out along the framework described in Section III; otherwise [(III.3) is violated], conclude that cooperation does not offer advantages.

TABLE II
THE COOPMATRICES ARE PARTIALLY OUTDATED: SUMMARY OF THE PROCEDURE PROPOSED TO SOLVE (IV.12)–(IV.14).

position of $S$ is kept fixed (see also Fig. 3), while the position of $D$ changes randomly from run to run, with the distance between $S$ and $D$ set to $\eta_{s,d} = \eta_{\text{network}} = 0.8\,(\eta_{s,d})_{\max}$ meters.[14] All the remaining $N$ nodes are uniformly and independently distributed in a circle of radius $\eta_{\text{relay}} \leq \eta_{\text{network}}$ meters centered around $S$. It is interesting to note that $\eta_{\text{relay}}$ is inversely related to the relay density, i.e., the number of potential relays per unit area, which is equal to $N/(\pi\,\eta_{\text{relay}}^2)$. The symbol period is equal to $T_s = 10^{-6}$ seconds, with $ABEP_{\text{target}} = 10^{-5}$ and $R_{\text{target}} = 4$ Mbps. The PHY can support $Q = 10$ data rates $R_q = 2\,(q+1)$ Mbps, for $q \in \{0, 1, \ldots, 9\}$. We set the path-loss exponent $\alpha = 4$, which is commonly used as a baseline value in wireless channel [18], and, for $p \in \{1, 2, \ldots, N\} - \{s, d\}$, the entries $\{\mathbf{B}_s\}_{s,p}$ of the matrix $\mathbf{B}_s$ are i.i.d. random variables assuming equiprobable values in $\{0, 1\}$, randomly generated from run to run. For the "RCoopMAC-out" algorithm, the step-size $\Delta_x$ is fixed to $\eta_{s,d}/100$ and, for both the "RCoopMAC-up" and "RCoopMAC-out" approaches, we set $\varrho = 0.9$. The ASNR per symbol is $\gamma = 25$ dB. As performance measures, we considered the end-to-end ABEP and the average cooperative data rate $R_{\text{coop}}$. All the results are obtained by carrying out $10^5$ independent Monte Carlo trials, with each run using a different set of node configurations and channel realizations.

### A. ABEP versus the number of dissident nodes $N_{diss}$

In this example, we test the robustness of the proposed RCoopMAC protocol against possible drops in the number $N_{\text{coop}}$ of the recruited nodes (see Remark 4). To separate the different effects, we only consider the case when the CoopMatrices $\mathbf{A}_s$ and $\mathbf{A}_d$ are perfectly updated, and we set $\eta_{\text{relay}} = 0.2\,\eta_{s,d}$, $N_{\text{coop}} = 8$ and $L = 10$. We report in Fig. 4 the ABEP of the "RCoopMAC-up" method as a function of the number of dissident nodes $N_{\text{diss}}$. The best situation in terms of average cooperative data rate corresponds to the case when all the eight recruited relays are willing to cooperate, i.e., when $N_{\text{diss}} = 0$, for which the average data rates in Phase I and II turns out to be $R'_{\text{coop}} = 14.8884$ Mbps and $R''_{\text{coop}} = 16$ Mbps, respectively. For each value of $N_{\text{diss}} \geq 1$, the dissident nodes are randomly selected from run to run among the $N_{\text{coop}}$ recruited nodes. We recall that, if the number of relays actually joining in Phase II is smaller than $N_{\text{coop}}$, the inequality $P_{\text{coop}}(e) \leq ABEP_{\text{target}}$ might not be satisfied: indeed, in this case, the former term in (II.10) increases, while the latter one decreases. Results of Fig. 4 show that, compared to the case when all the recruited nodes cooperate in Phase II, the ABEP slightly decreases if the number of dissident nodes ranges from 1 to 4, i.e., the decrease of the second term in (II.10) predominates over the increase of the first one. On the contrary, the ABEP increases for $N_{\text{diss}} > 4$, i.e., the increase of the first term in (II.10) predominates over the decrease of the second one, and the QoS threshold $ABEP_{\text{target}} = 10^{-5}$ is exceeded only when six out of the eight recruited relays will not cooperate deliberately.

### B. ABEP, average data rate and average number of recruited relays versus $\eta_{relay}$

In Figs. 5, 6 and 7, we evaluate the performances of the considered approaches as a function of $\eta_{\text{relay}}$. In this experiment, the number of potential relays is $N = 8$; we set $L = 10$ for which, according to [15, Table I], it results that $R_{\text{code}} = 3/5$. Results of Figs. 5 and 6 show that the proposed "RCoopMAC-up" and "RCoopMAC-out" strategies significantly outperform the "CoopMAC-up" and "CoopMAC-out" ones in terms of

[14] The RCoopMAC approach offers advantages over the DM and CoopMAC alternatives even when $S$ and $D$ cannot reliably communicate at the target data rate, i.e., $\eta_{s,d} > (\eta_{s,d})_{\max}$. However, due to the lack of space, results in this environment are not reported here.

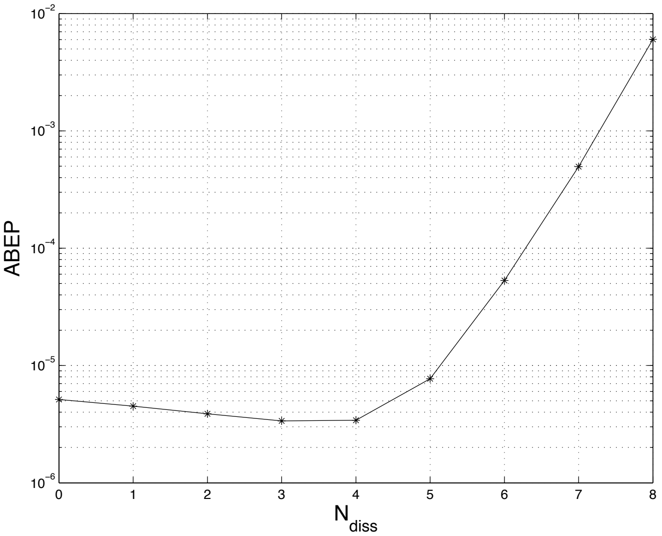


Fig. 4. ABEP versus number of dissident relays $N_{\text{diss}}$ for the proposed algorithm detailed in Table I ($\eta_{\text{relay}} = 0.2\,\eta_{s,d}$, $N_{\text{coop}} = 8$ and $L = 10$). The average data rates in Phase I and II turns out to be $R'_{\text{coop}} = 14.8884$ Mbps and $R''_{\text{coop}} = 16$ Mbps, respectively.

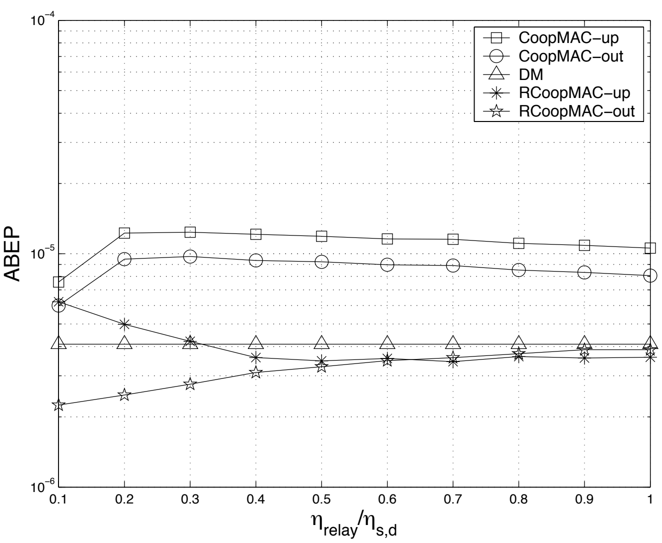


Fig. 5. ABEP versus $\eta_{\text{relay}}/\eta_{s,d}$ ($N = 8$, $L = 10$ and $R_{\text{code}} = 3/5$). CoopMAC-up: the CoopMAC method based on the perfectly updated matrix $\mathbf{A}_s$. CoopMAC-out: the CoopMAC method based on the partially outdated matrix $\widetilde{\mathbf{A}}_s$. DM: direct transmission from $S$ to $D$. RCoopMAC-up: the proposed method detailed in Table I. RCoopMAC-out: the proposed method detailed in Table II.

average $R_{\text{coop}}$, especially for high values of the relay density, i.e., small values of $\eta_{\text{relay}}$, while keeping the ABEP less than the assigned threshold $ABEP_{\text{target}}$. In particular, compared with the "RCoopMAC-up" one, the penalty paid by the "RCoopMAC-out" method in Fig. 6 is approximatively equal to $0.5$ Mbps for all the considered values of the node density. It is noteworthy from Fig. 5 that, since the CoopMAC protocol chooses the best relay ensuring the highest $R_{\text{coop}}$ without imposing any constraint on the resulting end-to-end ABEP, the curves of the "CoopMAC-up" and "CoopMAC-out" methods may not meet the target requirement $ABEP \leq ABEP_{\text{target}}$. Moreover, comparing the curves of the "RCoopMAC-up" and

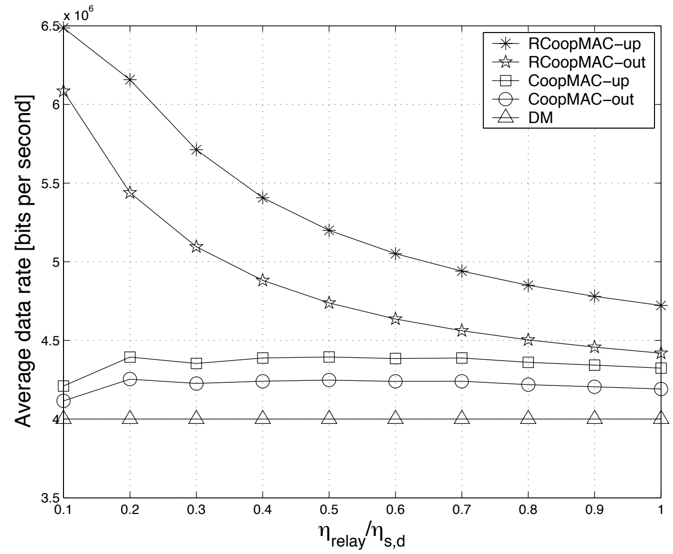


Fig. 6. Average cooperative data rate versus $\eta_{\text{relay}}/\eta_{s,d}$ ($N = 8$, $L = 10$ and $R_{\text{code}} = 3/5$). RCoopMAC-up: the proposed method detailed in Table I. RCoopMAC-out: the proposed method detailed in Table II. CoopMAC-up: the CoopMAC method based on the perfectly updated matrix $\mathbf{A}_s$. CoopMAC-out: the CoopMAC method based on the partially outdated matrix $\widetilde{\mathbf{A}}_s$. DM: direct transmission from $S$ to $D$.

"RCoopMAC-out" methods, it is apparent from Figs. 5 and 6 that recruiting more relays does not necessarily lead to higher values of $R_{\text{coop}}$. Indeed, remember that, in Phase II, the diversity gain provided by the recruitment of multiple relays may allow the data rate $R''_{\text{coop}}$ to take on values greater than $R'_{\text{coop}}$ even when the number of relays is not so high. This means that the cooperative data rate $R_{\text{coop}}$ is essentially limited by $R'_{\text{coop}}$ which, in its turn, is determined by the average channel quality between $S$ and the relays. Therefore, due to the lack of network state information, the "RCoopMAC-out" algorithm might determine a lower ASNR threshold $\overline{\gamma}_{\text{coop}}$ with respect to the "RCoopMAC-up" one and, consequently, it may engage not only all the relays recruited by the "RCoopMAC-up" method, but also additional relays with $(S, H_i)$ links exhibiting smaller ASNRs, which have the net effect of further increasing $R''_{\text{coop}}$ at the expense of significantly reducing $R'_{\text{coop}}$ and, consequently, $R_{\text{coop}}$.

### *C. ABEP, average data rate and average number of recruited relays versus $N$*

In Figs. 8, 9 and 10, the performances of the considered methods are studied as a function of the maximum number of potential relays $N$, by setting $\eta_{\text{relay}} = 0.2\,\eta_{s,d}$. The number of virtual antennas is fixed to $L = 18$ for which, according to [15, Table I], it results that $R_{\text{code}} = 5/9$. It can be observed from Fig. 9 that, contrary to the "CoopMAC-up" and "CoopMAC-out" strategies whose average cooperative data rates quickly saturate at the values $4.5$ and $4.4$ Mbps, respectively, the $R_{\text{coop}}$ values of the proposed "RCoopMAC-up" and "RCoopMAC-out" approaches improve as the number of nodes increases. As also evident from Fig. 10, this is the natural consequence of the fact that our approaches ensure a much better exploitation of the spatial diversity offered by the presence of multiple relays,

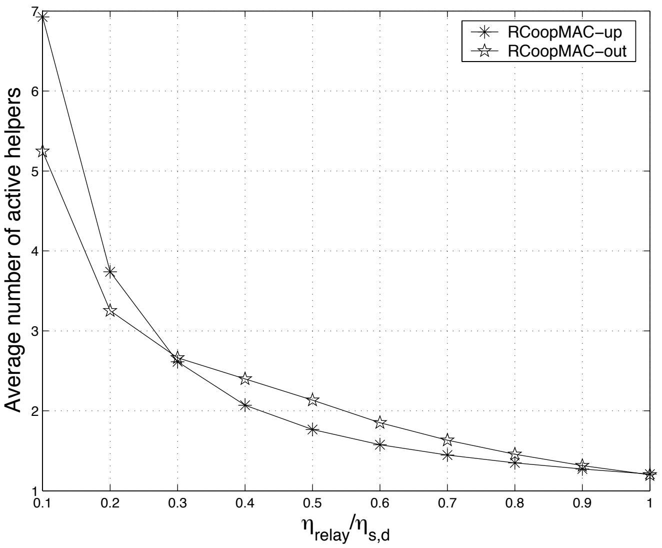


Fig. 7. Average number of active relays versus $\eta_{\text{relay}}/\eta_{s,d}$ ($N = 8$, $L = 10$ and $R_{\text{code}} = 3/5$). RCoopMAC-up: the proposed method detailed in Table I. RCoopMAC-out: the proposed method detailed in Table II.

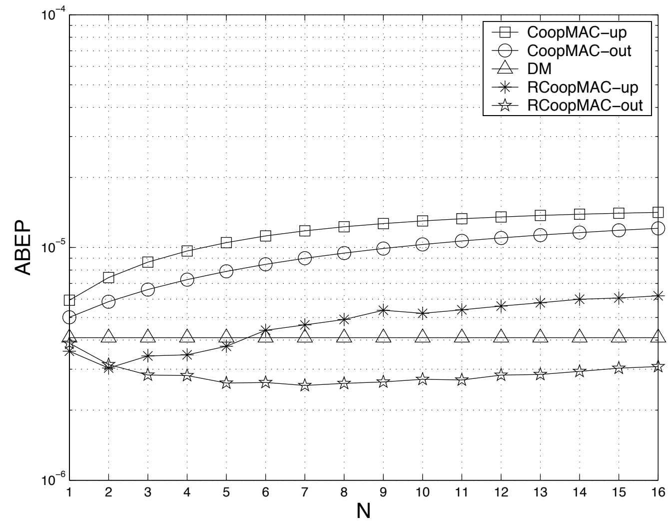


Fig. 8. ABEP versus $N$ ($\eta_{\text{relay}} = 0.2\,\eta_{s,d}$, $L = 18$ and $R_{\text{code}} = 5/9$). CoopMAC-up: the CoopMAC method based on the perfectly updated matrix $\mathbf{A}_s$. CoopMAC-out: the CoopMAC method based on the partially outdated matrix $\widetilde{\mathbf{A}}_s$. DM: direct transmission from $S$ to $D$. RCoopMAC-up: the proposed method detailed in Table I. RCoopMAC-out: the proposed method detailed in Table II.

while satisfying at the same time the target ABEP constraint (see Fig. 8).

## VI. Conclusions and directions for future work

We considered the problem of joint designing PHY and MAC layers by means of randomized cooperative coding, which allows the simultaneous recruitment of multiple relays in a totally automatic fashion, i.e., without requiring a dedicated handshaking between the source, the destination and the relays. It is shown that, with respect to both the direct transmission and the CoopMAC approach, the proposed RCoopMAC framework enables a significantly higher data rate, even when the destination has partial knowledge of the mean CSI of the neighboring nodes. The proposed cross-layer framework essentially focuses on the distributed optimization of the data rates in Phases I and II such that to minimize the overall transmission time with a QoS constraint in terms of ABEP. Such an optimization is carried by considering a single $(S, D)$ pair. A first interesting research subject consists of studying the extension of the RCoopMAC approach to large-scale wireless networks with heavy traffic conditions, where the sought performance metric is not only the transmission time over the individual links, but also the aggregate network throughput. In this scenario, the right balance between data-rate gains and the amount of interference introduced into the network by relaying requires a selection of a different power level for the relays and/or a modification of the cooperation decision. The first is an easier change, but it requires a more complex assessment at the nodes. The second requires more significant book-keeping and adequate training strategies to determine when and how to cooperate. Finally, we considered demodulate-and-forward relays and we assumed that source and relays are perfectly frequency-synchronized. Henceforth, two additional research issues are to consider different relay strategies (e.g., amplify-and-forward) and account for possible synchronization errors.

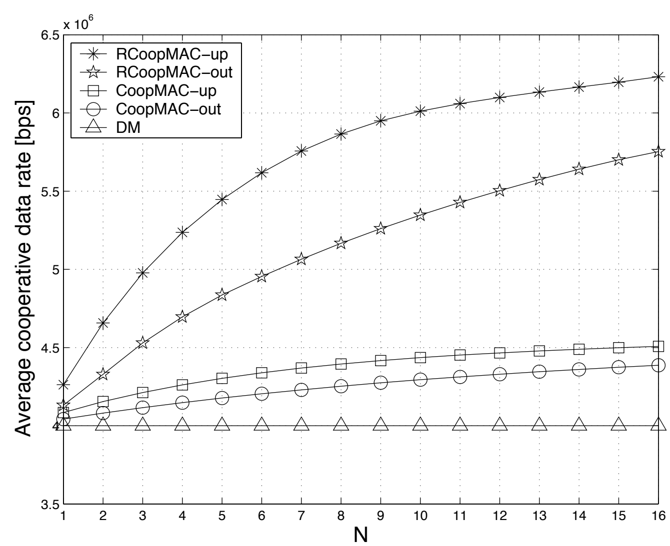


Fig. 9. Average cooperative data rate versus $N$ ($\eta_{\text{relay}} = 0.2\,\eta_{s,d}$, $L = 18$ and $R_{\text{code}} = 5/9$). RCoopMAC-up: the proposed method detailed in Table I. RCoopMAC-out: the proposed method detailed in Table II. CoopMAC-up: the CoopMAC method based on the perfectly updated matrix $\mathbf{A}_s$. CoopMAC-out: the CoopMAC method based on the partially outdated matrix $\widetilde{\mathbf{A}}_s$. DM: direct transmission from $S$ to $D$.

## Appendix A
## Proof of Theorem 2.1

Let $b \triangleq \frac{4}{\log_2(M'')}\left(1 - \frac{1}{\sqrt{M''}}\right)$, $c \triangleq \frac{3\,\gamma}{L\,(M''-1)}$ and $\mathrm{E}_{\boldsymbol{\mathcal{R}},\mathbf{g}_d}[\cdot]$ denote the expectation over the sample space of the pair $\{\boldsymbol{\mathcal{R}}, \mathbf{g}_d\}$, the conditional expectation rule ensures that $P_d(e\,|\,\mathcal{E}^c) = \mathrm{E}_{\boldsymbol{\mathcal{R}},\mathbf{g}_d}\left[P_d(e\,|\,\mathcal{E}^c, \boldsymbol{\mathcal{R}}, \mathbf{g}_d)\right] = \mathrm{E}_{\boldsymbol{\mathcal{R}}}\left\{\mathrm{E}_{\mathbf{g}_d|\boldsymbol{\mathcal{R}}}\left[P_d(e\,|\,\mathcal{E}^c, \boldsymbol{\mathcal{R}}, \mathbf{g}_d)\right]\right\}$, where $\mathrm{E}_{\boldsymbol{\mathcal{R}}}[\cdot]$ denotes the expectation over $\boldsymbol{\mathcal{R}}$ and $\mathrm{E}_{\mathbf{g}_d|\boldsymbol{\mathcal{R}}}[\cdot]$ is the expectation over $\mathbf{g}_d$ with $\boldsymbol{\mathcal{R}}$ being fixed. By resorting to the Chernoff bound, the probability $P_d(e\,|\,\mathcal{E}^c, \boldsymbol{\mathcal{R}}, \mathbf{g}_d)$ can be upper bounded [13] as

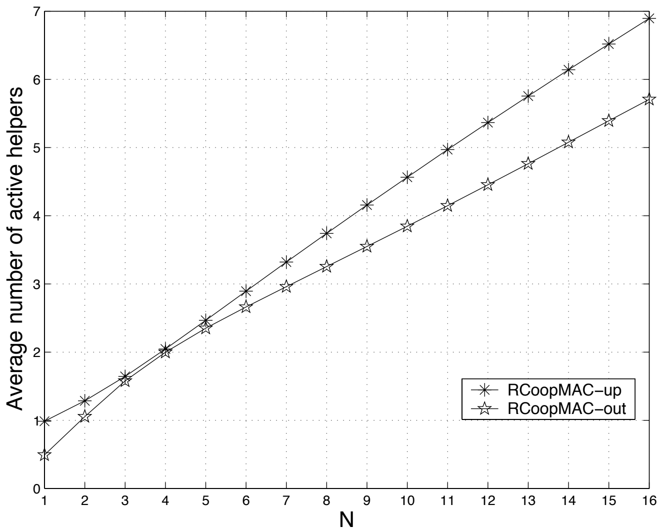


Fig. 10. Average number of active relays versus $N$ ($\eta_{\text{relay}} = 0.2\,\eta_{s,d}$, $L = 18$ and $R_{\text{code}} = 5/9$). RCoopMAC-up: the proposed method detailed in Table I. RCoopMAC-out: the proposed method detailed in Table II.

follows

$$P_d(e\,|\,\mathcal{E}^c, \boldsymbol{\mathcal{R}}, \mathbf{g}_d) \le b \cdot \exp\left[-c\,\mathbf{g}_d^H \boldsymbol{\Sigma}_{\mathbf{g}_d}^{-1/2}\left(\boldsymbol{\Sigma}_{\mathbf{g}_d}^{1/2}\overline{\boldsymbol{\mathcal{R}}}^H\overline{\boldsymbol{\mathcal{R}}}\,\boldsymbol{\Sigma}_{\mathbf{g}_d}^{1/2}\right)\boldsymbol{\Sigma}_{\mathbf{g}_d}^{-1/2}\mathbf{g}_d\right]. \quad \text{(A.1)}$$

The nonsingularity of $\boldsymbol{\Sigma}_{\mathbf{g}_d}$ implies that $\text{rank}(\boldsymbol{\Sigma}_{\mathbf{g}_d}^{1/2}\overline{\boldsymbol{\mathcal{R}}}^H\overline{\boldsymbol{\mathcal{R}}}\,\boldsymbol{\Sigma}_{\mathbf{g}_d}^{1/2}) = \text{rank}(\overline{\boldsymbol{\mathcal{R}}}) = r = \min(N_h+1, L)$, thus the eigenvalue decomposition holds $\boldsymbol{\Sigma}_{\mathbf{g}_d}^{1/2}\overline{\boldsymbol{\mathcal{R}}}^H\overline{\boldsymbol{\mathcal{R}}}\,\boldsymbol{\Sigma}_{\mathbf{g}_d}^{1/2} = \mathbf{U}\,\boldsymbol{\Lambda}\,\mathbf{U}^H$, where $\boldsymbol{\Lambda} \triangleq \text{diag}[\lambda_1, \lambda_2, \ldots, \lambda_r] \in \mathbb{R}^{r\times r}$ contains the nonzero eigenvalues of $\boldsymbol{\Sigma}_{\mathbf{g}_d}^{1/2}\overline{\boldsymbol{\mathcal{R}}}^H\overline{\boldsymbol{\mathcal{R}}}\,\boldsymbol{\Sigma}_{\mathbf{g}_d}^{1/2}$ and $\mathbf{U} \in \mathbb{C}^{(N_h+1)\times r}$ collects the corresponding eigenvectors. Consequently, we get

$$P_d(e\,|\,\mathcal{E}^c, \boldsymbol{\mathcal{R}}, \mathbf{g}_d) \le b\,\exp\Big[-c\,\underbrace{\mathbf{g}_d^H\boldsymbol{\Sigma}_{\mathbf{g}_d}^{-1/2}\mathbf{U}}_{\boldsymbol{\gamma}_d^H}\,\boldsymbol{\Lambda}\,\underbrace{\mathbf{U}^H\boldsymbol{\Sigma}_{\mathbf{g}_d}^{-1/2}\mathbf{g}_d}_{\boldsymbol{\gamma}_d}\Big]$$
$$= b\prod_{i=1}^{r}\exp\left(-c\,\lambda_i\,|\gamma_{i,d}|^2\right), \quad \text{(A.2)}$$

where, since $\mathbf{U}^H\mathbf{U} = \mathbf{I}_r$, one has $\boldsymbol{\gamma}_d \triangleq [\gamma_{1,d}, \gamma_{2,d}, \ldots, \gamma_{r,d}]^T \sim \mathcal{CN}(\mathbf{0}_r, \mathbf{I}_r)$. Therefore, we get

$$\text{E}_{\mathbf{g}_d|\boldsymbol{\mathcal{R}}}\left[P_d(e\,|\,\mathcal{E}^c, \boldsymbol{\mathcal{R}}, \mathbf{g}_d)\right] \le b\prod_{i=1}^{r}\text{E}_{\mathbf{g}_d|\boldsymbol{\mathcal{R}}}\left[\exp\left(-c\,\lambda_i\,|\gamma_{i,d}|^2\right)\right]$$
$$= b\prod_{i=1}^{r}\frac{1}{1+c\,\lambda_i}, \quad \text{(A.3)}$$

where in the last equality we have used the fact that $|\gamma_{i,d}|^2$ is a unit-mean exponential random variable. If we denote with $\mu_1, \mu_2, \ldots, \mu_r$ the nonzero eigenvalues of $\overline{\boldsymbol{\mathcal{R}}}^H\overline{\boldsymbol{\mathcal{R}}}$ arranged in increasing order, and with $\zeta_1, \zeta_2, \ldots, \zeta_{N_h+1}$ the diagonal entries of $\boldsymbol{\Sigma}_{\mathbf{g}_d}$ arranged in increasing order, by applying the Ostrowski theorem [19], it follows that, for each $i \in \{1, 2, \ldots, r\}$, there exists a positive real number $\theta_i$ such that $\zeta_1 \le \theta_i \le \zeta_{N_h+1}$ and $\lambda_i = \theta_i\,\mu_i$. It can be proven that a by-product of this result is the inequality $\prod_{i=1}^r \lambda_i = \prod_{i=1}^r \theta_i\,\mu_i \ge (\prod_{i=1}^r \mu_i)(\prod_{i=1}^r \zeta_i)$, where the equality holds if $r = N_h+1$, i.e., the matrix $\overline{\boldsymbol{\mathcal{R}}}^H\overline{\boldsymbol{\mathcal{R}}}$ is nonsingular with probability one. By virtue of this inequality and (A.3), we can write

$$P_d(e\,|\,\mathcal{E}^c) \le b\,\text{E}_{\boldsymbol{\mathcal{R}}}\left[\prod_{i=1}^{r}\frac{1}{1+c\,\lambda_i}\right]$$
$$\le \frac{b}{c^r}\left(\prod_{i=1}^{r}\frac{1}{\zeta_i}\right)\text{E}_{\overline{\boldsymbol{\mathcal{R}}}}\left[\prod_{i=1}^{r}\frac{1}{\mu_i}\right], \quad \text{(A.4)}$$

where, because all quantities at hand are positive (with probability one), we have used the fact that $1 + c\,\lambda_i \ge c\,\lambda_i$. Since the entries of $\overline{\boldsymbol{\mathcal{R}}}$ are i.i.d. circular symmetric complex Gaussian random variables with zero mean and unit variance, and $r = \text{rank}(\overline{\boldsymbol{\mathcal{R}}}) = \min(N_h+1, L)$ with probability one, by invoking well-known results regarding the joint probability density function of the eigenvalues of a matrix having central Wishart distribution, one has (see, e.g., [10])

$$\text{E}_{\overline{\boldsymbol{\mathcal{R}}}}\left[\prod_{i=1}^{r}\frac{1}{\mu_i}\right] = \frac{(|N_h - L + 1| - 1)!}{(\max(N_h+1, L) - 1)!}, \quad \text{with } N_h + 1 \ne L, \quad \text{(A.5)}$$

which can be substituted in (A.4), thus obtaining (II.6) after straightforward manipulations.

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