

# Randomized Decode-and-Forward Strategies for Two-Way Relay Networks

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**Abstract**—Randomized space-time block coding (RSTBC) is a decentralized cooperative technique that ensures diversity gains through the recruitment of multiple uncoordinated relays, with virtually no signaling overhead. In this paper, RSTBC is applied to two-way relaying wireless networks which, when two terminals want to send a message to each other, can potentially improve the network throughput by allowing them to exchange data over two or three time slots via bidirectional relay communications. Specifically, two decode-and-forward relaying strategies are considered that take up only two time slots. In the first slot, the two sources transmit simultaneously. In the former scheme, which we refer to as *decode and forward both* (DFB) RSTBC, only relays that can reliably decode both source blocks via joint maximum likelihood decoding cooperate, and do so by modulating the bit-level XOR of the decoded data through a single RSTBC. In the latter scheme, called *decode and forward any* (DFA) RSTBC, the relays cooperate in the second slot also when they can decode only one of the two source data. In this case, each source data that is decoded is mapped into an independent RSTBC. If the relay decoded reliably both sources, after cancellation of the strong interference, then it sends the two RSTBCs encoding the symbol vectors from each of the sources. A randomized forwarding scheme is also proposed for three-time-slot relaying, which is also a DFA strategy, although without joint decoding or interference cancellation after the first slot. The diversity orders achievable through the three proposed schemes are calculated and the obtained theoretical results are validated by means of Monte Carlo numerical simulations.

**Index Terms**—Decode-and-forward relaying, diversity analysis, interference cancellation, maximum likelihood (ML) detection, space-time randomized coding, two-way cooperation.

## I. INTRODUCTION

**I**N ad hoc network applications or in distributed large scale wireless networks, several methods were proposed to exploit spatial diversity without using multiple antenna structures [1]–[5]. This can be achieved by exploiting the presence of multiple nodes distributed in space, which may cooperatively

serve as relay stations. One possible approach for involving more than one cooperative relay without a significant loss in spectral efficiency is to use *space-time block coding* (STBC) among the relays [3]–[5]. However, the use of conventional STBC rules [6] in a distributed fashion poses challenges, since it requires coordination among the source, the destination, and the relays, who need to be informed on how to encode the data and when to transmit. In an *ad hoc* scenario the set of relay candidates is generally unknown and randomly time-varying and the selection of the best relays adds considerable signaling burden for acquiring the channel state information that is needed to perform their selection, and inform the relays of the correct encoding action. A decode-and-forward relaying strategy that overcomes a significant portion of these coordination issues is *Randomized STBC* (RSTBC), proposed in [7], [8] to decentralize the recruitment of relays. RSTBC as an amplify-and-forward protocol has been recently proposed in [9]. However, the randomized designs in [7]–[9] are targeted at an *one-way* relay network.

*Two-way* communication has recently aroused great interest due to its potential of significantly enhancing the network throughput when two terminals want to exchange information to each other [10], [11]. Relaying schemes for two-way wireless networks have been recently proposed in [12], [13] based on linear dispersion (LD) STBC [5]. With appropriate code design, the achievable diversity order of these schemes is equal to the number of active relays  $N$  if the total power  $\mathcal{P}$  used in the whole network is very large, i.e.,  $\log(\mathcal{P}) \gg \log[\log(\mathcal{P})]$ , and the length  $K$  of the source symbol blocks is greater than  $N$ . In [14], the authors have provided achievable rate regions for different cooperation strategies, such as decode-and-forward based on block Markov superposition coding and compress-and-forward based on Wyner-Ziv source coding. They have shown that a combined strategy of block Markov superposition coding and Wyner-Ziv coding achieves the cut-set upper bound on the sum-rate of the two-way relay channel when the relay is in the proximity of one of the terminals.

In this paper, we propose three randomized two-way relaying schemes which are fully decentralized and we analyze their diversity performance by explicitly taken into account the randomness in the cooperating set. The idea is that to combine two-way relay codes with RSTBC. A preliminary version of our decentralized two-way decode-and-forward relaying scheme over two time slots was proposed in [15]; however, [15] includes the assumption that the two terminals employ a single antenna and each relay is equipped with two antennas.

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There are two important aspects that are missing in the theoretical analyses carried out in all the aforementioned papers: 1) due to the error-free detection constraint, the set of relays that are responsible for retransmission in the second time slot is random, which naturally prompts to use RSTBC to ease coordination; 2) unlike the one-way relay RSTBC setting, in the two-way relay case it is possible to have nodes capable of decoding only one of the two signals that are sent from the sources and this leads to interesting differences in what is the right strategy to encode at the relays. In particular, the second of these two points is what makes the extension of RSTBC to the two-way relay channel not straightforward. We refer to the case where cooperative relays are those who received reliably both messages as *Decode and Forward Both* (DFB), while we term *Decode and Forward Any* (DFA) the case where cooperative relays include those that have decoded one or both source signals. Herein, we focus on DFB and DFA relaying protocols using two and three time slots (T2 and T3 for the sake of brevity). In two-time-slot protocols, the two terminals who wish to communicate are allowed to transmit simultaneously, and the relays use multi-packet reception schemes to recover the source data. In the three-time-slot scheme, the two terminals transmit to the relays over the first and second time slot, respectively, without *colliding*. The key difference between DFA and DFB strategies is the method used to multiplex the source data. Specifically, in DFB, the relays retransmit the RSTBC of the XOR coded data. In DFA, a relay that has decoded both signals reliably, forwards the sum of RSTBCs of the decoded symbol vectors; otherwise, the relay forwards the RSTBC of the only symbol vector that it has decoded. Finally, each terminal can recover the desired symbol vector by cancelling from the received signal the contribution corresponding to its own message, i.e., using side information.

The decoding strategy for the DFB-T2 scheme is based on joint ML detection of the symbol blocks at the relays. Instead, in the DFA-T2 scheme, the relays perform Bayesian interference cancellation (BIC) and retransmit not only if both the two symbol blocks are correctly detected, but also if only one symbol block is detected reliably. The third scheme is a variant of DFA-T2 scheme. However, the nodes decode each source signal one at a time. For all the proposed schemes, we calculate theoretically the diversity order and show that, as in [7], the achievable diversity order in all the three protocols is  $\min\{L, N\}$ , where  $L$  is the number of virtual antennas. However, for DFA-T2 RSTBC, this is true only up to a certain very large signal-to-noise ratio (SNR) threshold.

The rest of the paper is organized as follows. Section II describes the network model. In Section III, DFB and DFA, which are based on joint ML detection and BIC at the relays, respectively, are introduced and analyzed. Section IV reports the design and analysis of the third proposed scheme using three time slots. Numerical Monte Carlo results in terms of symbol-error-probability (SEP) are presented in Section V, followed by conclusions in Section VI.

### A. Notations and preliminaries

The fields of complex, real, and integer numbers are denoted with  $\mathbb{C}$ ,  $\mathbb{R}$ , and  $\mathbb{Z}$ , respectively; matrices [vectors] are denoted

with upper [lower] case boldface letters; the field of  $m \times n$  complex [real] matrices is denoted as  $\mathbb{C}^{m \times n}$  [ $\mathbb{R}^{m \times n}$ ], with  $\mathbb{C}^m$  [ $\mathbb{R}^m$ ] used as a shorthand for  $\mathbb{C}^{m \times 1}$  [ $\mathbb{R}^{m \times 1}$ ]; the superscripts  $*$ ,  $T$ ,  $H$ , and  $-1$  denote the conjugate, the transpose, the Hermitian, and the inverse of a matrix, respectively;  $U(x)$  denotes the unit step function;  $\mathcal{A}^n$  and  $|\mathcal{A}|$  represent the  $n$ -dimensional Cartesian product and cardinality of the set  $\mathcal{A}$ ;  $\mathbf{0}_m \in \mathbb{R}^m$ ,  $\mathbf{O}_{m \times n} \in \mathbb{R}^{m \times n}$ , and  $\mathbf{I}_m \in \mathbb{R}^{m \times m}$  denote the null vector, the null matrix, and the identity matrix, respectively;  $\text{rank}(\mathbf{A})$  and  $\det(\mathbf{A})$  denote the rank and the determinant of  $\mathbf{A}$ , respectively;  $\|\mathbf{a}\|$  is the Euclidean norm of  $\mathbf{a} \in \mathbb{C}^n$ ;  $|\mathbf{A}|_{k+}$  denotes the product of the  $k \leq n$  smallest positive eigenvalues of the Hermitian matrix  $\mathbf{A} \in \mathbb{C}^{n \times n}$ ;  $P(A)$  denotes the probability that an event  $A$  occurs; the operator  $\mathbb{E}[\cdot]$  denotes ensemble averaging and a circular symmetric complex Gaussian random vector  $\mathbf{x} \in \mathbb{C}^n$  with mean  $\boldsymbol{\mu} \in \mathbb{C}^n$  and covariance matrix  $\mathbf{K} \in \mathbb{C}^{n \times n}$  is denoted as  $\mathbf{x} \sim \mathcal{CN}(\boldsymbol{\mu}, \mathbf{K})$ ;  $Q(x) \triangleq (1/\sqrt{2\pi}) \int_x^{+\infty} e^{-u^2/2} du$  denotes the Q function and  $Q^{-1}(\cdot)$  is its inverse; the function  $\Phi(n; p_1, p_2, \dots, p_N)$  denotes the Poisson binomial probability mass function (pmf) [16] and is defined as<sup>1</sup>

$$\Phi(n; p_1, p_2, \dots, p_N) \triangleq \sum_{\substack{\mathcal{C} \subseteq \{1, 2, \dots, N\} \\ \text{with } |\mathcal{C}| = n}} \left( \prod_{i \in \mathcal{C}} p_i \right) \cdot \left[ \prod_{i \in \{1, 2, \dots, N\} - \mathcal{C}} (1 - p_i) \right] \quad (1)$$

where the sum is done with respect to all the subsets  $\mathcal{C}$  of  $\{1, 2, \dots, N\}$  having cardinality  $n$ , whereas  $0 \leq p_i \leq 1$ , for  $i \in \{1, 2, \dots, N\}$ , and  $\sum_{i=1}^N p_i = 1$ ; the average probability of error for a communication system over a fading channel usually behaves as  $P_e(\gamma) \leq G_c \gamma^{-G_d}$  [19], where  $\gamma$  denotes the SNR without fading,  $G_c$  is the coding gain, and the *asymptotic* diversity order is defined as  $G_d \triangleq \lim_{\gamma \rightarrow +\infty} -\frac{\log[P_e(\gamma)]}{\log(\gamma)}$ .

## II. NETWORK MODEL

We consider a wireless network (see Fig. 1) composed of two terminal nodes  $T_m$ , with  $m \in \{1, 2\}$ , that wish to exchange information and  $N$  relay nodes  $R_i$  with  $i \in \{1, 2, \dots, N\}$ , which, for instance, could be placed at random, and can be used as two-way relays [12], [13]. All the nodes in the network have a single transmit/receive antenna.

The transmitted blocks of source symbols at the two terminals are denoted as  $\mathbf{s}_m \triangleq [s_m^{(1)}, s_m^{(2)}, \dots, s_m^{(K)}]^T \in \mathbb{C}^K$ , for  $m \in \{1, 2\}$ , which are composed of independent and identically distributed (i.i.d.) zero-mean unit-variance symbols assuming equiprobable values drawn from a finite alphabet  $\mathcal{A}$ . In addition, we assume that the average power of terminal  $T_m$  is  $\mathcal{P}_m$  and, for a fair comparison and simplicity, that the total average power on all the relays is  $\mathcal{P}_R$  and each relay node has

<sup>1</sup>The pmf  $\Phi(n; p_1, p_2, \dots, p_N)$  is the probability of having  $n$  successes in a sequence of  $N$  independent Bernoulli trials with success probabilities  $p_1, p_2, \dots, p_N$ . Its values can be calculated recursively [17] or, more efficiently, by using the discrete Fourier transform [18]. When all success probabilities are the same, i.e.,  $p_1 = p_2 = \dots = p_N$ , the Poisson binomial distribution ends up to the binomial distribution.

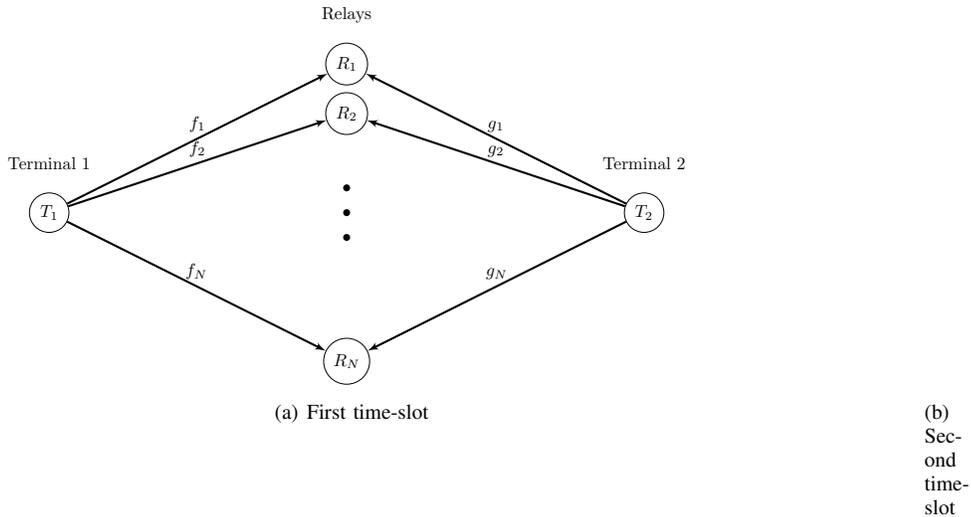


Fig. 1. Wireless two-way relay network model for two-time-slot protocols. (a) First time-slot: transmission from two terminals to the relays. (b) Second time-slot: transmission from the relays to two terminals.

equal average power  $\mathcal{P}_R/N$ . Hence, the total average power available in the network is  $\mathcal{P} = \mathcal{P}_1 + \mathcal{P}_2 + \mathcal{P}_R$ .

All the point-to-point channels are assumed frequency non-selective and quasi-stationary, i.e., they are characterized by a single fading coefficient that remains constant within one block of source symbols but may vary from block to block. In the first and second time slots, we denote the channel between  $T_1$  and  $R_i$  as  $f_i$  and the channel between  $T_2$  and  $R_i$  as  $g_i$ . All the underlying channels are assumed to be statistically independent, and are modeled as zero-mean circularly symmetric complex Gaussian random variables, i.e.,  $f_i \sim \mathcal{CN}(0, \sigma_{f_i}^2)$  and  $g_i \sim \mathcal{CN}(0, \sigma_{g_i}^2)$  for  $i \in \{1, 2, \dots, N\}$ . It is worth noting that, in contrast to the symmetric scenarios which are usually used in the literature [12], where all the relays are treated as being at the same location and midway between the two terminals, we consider herein an *asymmetric* network, where the relays are located in arbitrary positions. Following the related literature, e.g., [3], [7], [12], perfect synchronization is assumed at the symbol level among all the nodes.

### III. RSTBC PROTOCOLS FOR TWO-TIME-SLOT RELAYING (RSTBC-T2)

In two-time-slot relaying, the terminals  $T_1$  and  $T_2$  communicate with each other over two time slots. In the first time slot, the two terminal nodes simultaneously transmit to the relay nodes. In the second time slot, the relay nodes demodulate the received signals and forward the estimated symbol blocks to the two terminals  $T_1$  and  $T_2$ . Since each terminal transmits every two time slots, the transmit power is  $2\mathcal{P}_m$ , for  $m \in \{1, 2\}$ . For  $i \in \{1, 2, \dots, N\}$ , the received signal at the relay  $R_i$  can be written as

$$\mathbf{x}_i = \sqrt{2\mathcal{P}_1} f_i \mathbf{s}_1 + \sqrt{2\mathcal{P}_2} g_i \mathbf{s}_2 + \mathbf{n}_i \quad (2)$$

where noise at the  $i$ -th relay is modeled as  $\mathbf{n}_i \sim \mathcal{CN}(\mathbf{0}_K, \mathbf{I}_K)$ , and is statistically independent of  $\mathbf{s}_1$  and  $\mathbf{s}_2$ . The random vectors  $\mathbf{s}_1$  and  $\mathbf{s}_2$  are drawn from a set  $\mathcal{A}^K$  of  $|\mathcal{A}|^K$  different symbol vectors. Hereinafter, a particular element of the set

$\mathcal{A}^K$  will be referred to as  $\mathbf{s}^{(k)}$ , with  $k \in \{1, 2, \dots, |\mathcal{A}|^K\}$ . It is worth noting that  $\sqrt{2\mathcal{P}_1} f_i$  and  $\sqrt{2\mathcal{P}_2} g_i$  can be estimated at the relays by resorting to training-based channel estimation techniques [20], [21]. Specifically, the data transmission in the first time-slot is preceded by a training period wherein  $T_1$  and  $T_2$  simultaneously transmit orthogonal or almost orthogonal training vectors, which are known at the relays and can be used to jointly acquire  $\sqrt{2\mathcal{P}_1} f_i$  and  $\sqrt{2\mathcal{P}_2} g_i$  by means of linear or nonlinear estimators.

#### A. Decode and forward both (DFB)

In this section, we consider a relaying scheme where all relays perform joint ML detection of  $\mathbf{s}_1$  and  $\mathbf{s}_2$ . In this case, only those relays that correctly detect *both*  $\mathbf{s}_1$  and  $\mathbf{s}_2$  can then cooperate in the second time slot, by simultaneously forwarding the bit-level XOR of the detected symbols at the data link layer to the two terminals, using RSTBC [7]. The simple idea of DFB-T2 RSTBC is to improve the reliability of the two-way relay network coding strategy by leveraging on cooperative diversity at the physical layer.

1) *First time slot*: Under the assumption that  $\sqrt{2\mathcal{P}_1} f_i$  and  $\sqrt{2\mathcal{P}_2} g_i$  are perfectly known at the  $i$ th relay, joint ML detection of both  $\mathbf{s}_1$  and  $\mathbf{s}_2$  from (2) can be performed by exploiting the differences in the channels between each terminal and the  $i$ th relay, as done in a multiple access channel [22]. Such a detector is guaranteed to yield minimum probability of erroneous detection and, provided that the realizations of the coefficients  $f_i$  and  $g_i$  are different (which happens with probability one in fading channels with continuous distribution), it is able to assure good performance even if the signal model (2) represents an under-determined mathematical problem, i.e., each relay has to decode  $2K$  unknowns while it has only  $K$  data points. However, in this scenario, even with joint ML decoding, the error probability at the detector output is high in comparison with the case where the number of data points is at least the number of unknowns [22].

To find the SEP  $P_{\text{ml}}^{(i)}(e)$  at relay  $R_i$ , we rewrite (2) as  $\mathbf{x}_i = \mathbf{S} \mathbf{h}_i + \mathbf{n}_i$ , where  $\mathbf{S} \triangleq [\mathbf{s}_1, \mathbf{s}_2] \in \mathbb{C}^{K \times 2}$  is taken from a set  $\mathcal{A}^{2K}$  of  $|\mathcal{A}|^{2K}$  different symbol matrices and  $\mathbf{h}_i \triangleq [\sqrt{2\mathcal{P}_1} f_i, \sqrt{2\mathcal{P}_2} g_i]^T \in \mathbb{C}^2$ . An element of  $\mathcal{A}^{2K}$  is referred to as  $\mathbf{S}^{(k)}$ , with  $k \in \{1, 2, \dots, |\mathcal{A}|^{2K}\}$ . Joint ML detection of  $\mathbf{s}_1$  and  $\mathbf{s}_2$  amounts to

$$\widehat{\mathbf{S}}_i = \arg \min_{\mathbf{S} \in \mathcal{A}^{2K}} \|\mathbf{x}_i - \mathbf{S} \mathbf{h}_i\|^2 \quad (3)$$

and  $P_{\text{ml}}^{(i)}(e)$  is bounded by using the union bound [23] as

$$\begin{aligned} P_{\text{ml}}^{(i)}(e) &\leq \frac{1}{|\mathcal{A}|^{2K}} \sum_{k=1}^{|\mathcal{A}|^{2K}} \sum_{\substack{\ell=1 \\ \ell \neq k}}^{|\mathcal{A}|^{2K}} P[\mathbf{S}^{(k)} \rightarrow \mathbf{S}^{(\ell)} \text{ at } R_i] \\ &\leq (|\mathcal{A}|^{2K} - 1) \max_{\substack{k, \ell \in \{1, 2, \dots, |\mathcal{A}|^{2K}\} \\ \ell \neq k}} P[\mathbf{S}^{(k)} \rightarrow \mathbf{S}^{(\ell)} \text{ at } R_i] \end{aligned} \quad (4)$$

where  $P[\mathbf{S}^{(k)} \rightarrow \mathbf{S}^{(\ell)} \text{ at } R_i]$  is the pairwise error probability (PEP), which is the probability that  $\mathbf{S}^{(\ell)}$  is detected at the relay  $R_i$  when  $\mathbf{S}^{(k)}$  is transmitted. By using the Chernoff inequality [23], one has

$$P[\mathbf{S}^{(k)} \rightarrow \mathbf{S}^{(\ell)} \text{ at } R_i] \leq \exp \left[ -\frac{1}{4} \mathbf{h}_i^H \mathbf{M}^{(k, \ell)} \mathbf{h}_i \right] \quad (5)$$

where

$$\begin{aligned} \mathbf{M}^{(k, \ell)} &\triangleq [\mathbf{S}^{(k)} - \mathbf{S}^{(\ell)}]^H [\mathbf{S}^{(k)} - \mathbf{S}^{(\ell)}] \\ &= \begin{bmatrix} m_{11}^{(k, \ell)} & m_{12}^{(k, \ell)} \\ m_{21}^{(k, \ell)} & m_{22}^{(k, \ell)} \end{bmatrix} \in \mathbb{C}^{2 \times 2}. \end{aligned} \quad (6)$$

We observe that  $P[\mathbf{S}^{(k)} \rightarrow \mathbf{S}^{(\ell)} \text{ at } R_i]$  is maximized, with respect to  $k$  and  $\ell$ , when  $\mathbf{S}^{(k)}$  and  $\mathbf{S}^{(\ell)}$  differs only in one symbol. If this different symbol is in the first column of  $\mathbf{S}^{(k)}$  and  $\mathbf{S}^{(\ell)}$ , then  $m_{11}^{(k, \ell)} = d_{\min}^2$  and  $m_{12}^{(k, \ell)} = m_{21}^{(k, \ell)} = m_{22}^{(k, \ell)} = 0$ , where  $d_{\min}$  denotes the minimum distance between the symbols in  $\mathcal{A}$ . Similarly, if the different symbol is in the second column, then  $m_{22}^{(k, \ell)} = d_{\min}^2$  and  $m_{11}^{(k, \ell)} = m_{12}^{(k, \ell)} = m_{21}^{(k, \ell)} = 0$ . Consequently, we can write

$$\begin{aligned} &\max_{\substack{k, \ell \in \{1, 2, \dots, |\mathcal{A}|^{2K}\} \\ \ell \neq k}} P[\mathbf{S}^{(k)} \rightarrow \mathbf{S}^{(\ell)} \text{ at } R_i] \\ &\leq \max \left\{ e^{-\frac{1}{2} d_{\min}^2 \mathcal{P}_1 |f_i|^2}, e^{-\frac{1}{2} d_{\min}^2 \mathcal{P}_2 |g_i|^2} \right\}. \end{aligned} \quad (7)$$

It is seen from (4) and (7) that, besides  $|\mathcal{A}|$ ,  $K$ , and  $d_{\min}$ , the largest value of  $P_{\text{ml}}^{(i)}(e)$  is dictated by the variable  $\mathcal{Z}_i \triangleq \min\{\mathcal{P}_1 |f_i|^2, \mathcal{P}_2 |g_i|^2\}$ . Thus, we say that relay  $R_i$  can *reliably* detect both  $\mathbf{s}_1$  and  $\mathbf{s}_2$  if  $\mathcal{Z}_i \geq \tau_{\text{ml}}$ ; the threshold  $\tau_{\text{ml}} > 0$  is chosen to ensure a given SEP constraint  $P_{\text{ml}}^{(i)}(e) \leq \text{SEP}_{\text{target}}$  which, by virtue of (4) and (7), leads to the choice  $\tau_{\text{ml}} = \frac{2}{d_{\min}^2} \ln \left[ \frac{|\mathcal{A}|^{2K} - 1}{\text{SEP}_{\text{target}}} \right]$ . In the proposed scheme, all the relay nodes that reliably detected both  $\mathbf{s}_1$  and  $\mathbf{s}_2$  cooperate in order to transmit the source symbols to the destinations. This error-free constraint renders the number of active relays  $M$  in the second time slot a discrete random variable which can assume the values  $\{0, 1, \dots, N\}$ . Since all the underlying

channels are statistically independent and the relays make decisions independently of each other, it follows that, for  $n \in \{0, 1, \dots, N\}$ , the probability mass function (pmf) of  $M$  is given by  $P(M = n) = \Phi(n; p_1, p_2, \dots, p_N)$  where  $p_i \triangleq P(\mathcal{Z}_i \geq \tau_{\text{ml}})$  is the probability of cooperation for the  $i$ th relay. In order to completely specify the pmf of  $M$ , we have to calculate  $p_i$ . Let us define the random variables  $\mathcal{X}_i \triangleq \mathcal{P}_1 |f_i|^2$  and  $\mathcal{Y}_i \triangleq \mathcal{P}_2 |g_i|^2$ , which have exponential distributions with parameters  $L_1^{(i)} = 1/(\mathcal{P}_1 \sigma_{f_i}^2)$  and  $L_2^{(i)} = 1/(\mathcal{P}_2 \sigma_{g_i}^2)$ , respectively. The random variable  $\mathcal{Z}_i$  is the minimum of two independent exponential random variables and its cumulative distribution function (CDF) is  $F_{\mathcal{Z}_i}(\tau_{\text{ml}}) = 1 - e^{-[L_1^{(i)} + L_2^{(i)}] \tau_{\text{ml}}}$ . Thus, we have  $p_i = 1 - F_{\mathcal{Z}_i}(\tau_{\text{ml}}) = e^{-[L_1^{(i)} + L_2^{(i)}] \tau_{\text{ml}}}$ , which shows that  $p_i$  depends on  $i$  through the variances of  $f_i$  and  $g_i$ .<sup>2</sup> In the asymptotic SNR regime, i.e., when  $\mathcal{P}_1, \mathcal{P}_2 \rightarrow +\infty$ , it results that  $p_i \rightarrow 1$ , that is, all the relays cooperate with probability one.

*Corollary 1:* For a given  $\tau_{\text{ml}}$ , among the relay nodes with fixed received power  $\mathcal{P}_1 \sigma_{f_i}^2 + \mathcal{P}_2 \sigma_{g_i}^2 = \mathcal{C}$ , the probability  $p_i$  is maximized when  $\mathcal{P}_1 \sigma_{f_i}^2 = \mathcal{P}_2 \sigma_{g_i}^2$ .

In the case of  $\mathcal{P}_1 = \mathcal{P}_2$ , Corollary 1 leads to the conclusion that the probability of cooperation in relay node  $i$  for the DFB scheme is maximized when it is located at an equal distance from the two terminals.

2) *Second time slot:* Each cooperating node, which reliably detected both  $\mathbf{s}_1$  and  $\mathbf{s}_2$ , applies a deterministic space-time block code to  $\mathbf{s}_{\text{XOR}} \triangleq \mathbf{s}_1 \oplus \mathbf{s}_2 \in \mathcal{A}^K$ , where  $\oplus$  denotes the bit-level XOR operator performed prior to modulating the symbol on the alphabet [12], [13], thus obtaining the code matrix  $\mathcal{G}(\mathbf{s}_{\text{XOR}}) \in \mathbb{C}^{P \times L}$ , where  $L$  is the number of *virtual* antennas and  $P \geq K$  is the block length transmitted by each antenna. Without considering any specific code structure, we only assume that code matrix satisfies the rank criterion [6], which states that, for any pair  $\mathcal{G}^{(k)} \triangleq \mathcal{G}(\mathbf{s}^{(k)})$  and  $\mathcal{G}^{(\ell)} \triangleq \mathcal{G}(\mathbf{s}^{(\ell)})$ , where  $\mathbf{s}^{(k)}, \mathbf{s}^{(\ell)} \in \mathcal{A}^K$  with  $k \neq \ell$ , the matrix  $\mathcal{G}^{(k)} - \mathcal{G}^{(\ell)}$  is full rank, i.e., its rank is  $\min\{P, L\}$ . Let  $\mathbf{r}_i \in \mathbb{C}^L$  denote the randomization vector that contains the linear combination coefficients for the  $i$ -th node, in the proposed scheme the relay  $R_i$  transmits the RSTBC block  $\beta \mathcal{G}(\mathbf{s}_{\text{XOR}}) \mathbf{r}_i$  [7] to the two terminals during the second time slot, where  $\beta \triangleq \sqrt{(2\mathcal{P}_R)/N}$  denotes the scaling factor used by the relays in order to satisfy the average power constraint. For convenience, the deterministic code is normalized such that  $\mathbb{E}[\mathcal{G}^H(\mathbf{s}_{\text{XOR}}) \mathcal{G}(\mathbf{s}_{\text{XOR}})] = \mathbf{I}_L$ , and, moreover, we impose that  $\mathbb{E}[\|\mathbf{r}_i\|^2] = 1$ . In the following, we only consider the received signal at the second terminal, which is given by

$$\mathbf{y}_{2, \text{ml}} = \beta \sum_{i=1}^M g_i \mathcal{G}(\mathbf{s}_{\text{XOR}}) \mathbf{r}_i + \mathbf{w}_2 = \beta \mathcal{G}(\mathbf{s}_{\text{XOR}}) \mathbf{R} \mathbf{g} + \mathbf{w}_2 \quad (8)$$

where the noise vector at the second terminal is modeled as  $\mathbf{w}_2 \sim \mathcal{CN}(\mathbf{0}_P, \mathbf{I}_P)$ , which is statistically independent of  $\mathbf{s}_1, \mathbf{s}_2$ , and  $\mathbf{r}_i$ . Moreover, following the same notation as in [7],

<sup>2</sup>As a special case, when  $\sigma_{f_i}^2$  and  $\sigma_{g_i}^2$  do not depend on  $i$ , i.e., all the relays are at the same location, the pmf  $\Phi(n; p_1, p_2, \dots, p_N)$  boils down to a binomial distribution [16] of parameters  $N$  and  $p \triangleq p_1 = p_2 = \dots = p_N$ .

the matrix  $\mathbf{R} \triangleq [\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_M] \in \mathbb{C}^{L \times M}$  collects all the randomization coefficients of the active relays, whereas  $\mathbf{g} \in \mathbb{C}^M$  gathers the channel coefficients between the  $M$  cooperative nodes and terminal  $T_2$  which, under our assumptions, is a zero-mean complex circularly symmetric Gaussian vector, with diagonal autocorrelation matrix  $\mathbf{\Sigma}_g \triangleq \mathbb{E}[\mathbf{g}\mathbf{g}^H] \in \mathbb{R}^{M \times M}$ . We observe that the *effective* channel vector  $\tilde{\mathbf{g}} \triangleq \beta \mathbf{R}\mathbf{g} \in \mathbb{C}^L$  can be estimated at the terminal  $T_2$  by allowing each data transmission in the second time-slot be preceded by a training period, wherein all the relays, which reliably detected both  $\mathbf{s}_1$  and  $\mathbf{s}_2$ , transmit a symbol sequence known to the terminal; the randomization vectors used during the training phase will be maintained in the subsequent data transmission, as in [7].

Since  $\mathbf{s}_2$  is known at  $T_2$ , ML decoding can be used at the receiver to recover  $\mathbf{s}_{\text{XOR}}$ . Under the assumption that  $\tilde{\mathbf{g}}$  is perfectly known, the ML decoding of  $\mathbf{s}_{\text{XOR}}$  can be easily obtained from (8) as follows

$$\hat{\mathbf{s}}_{\text{XOR}} = \arg \min_{\mathbf{s} \in \mathcal{A}^K} \|\mathbf{y}_{2,\text{ml}} - \mathcal{G}(\mathbf{s})\tilde{\mathbf{g}}\|^2 \quad (9)$$

The intended symbol vector  $\mathbf{s}_1$  at  $T_2$ , then can be extracted using  $\hat{\mathbf{s}}_1 = \mathbf{s}_2 \oplus \hat{\mathbf{s}}_{\text{XOR}}$ . At this point, the evaluation of the average SEP at the output of the ML detector of the terminal  $T_2$  is similar to that of a one-way relay network employing RSTBC [7]. Conditioned on  $M > 1$ , we summarize the average performance of the scheme at hand in the lemma:

*Lemma 1:* Let  $M = n$ , for  $n \in \{1, 2, \dots, N\}$ , and  $\eta(n) \triangleq \min\{L, n\}$ . Assume that the matrix  $\mathbf{R}$  is full-rank with probability one, i.e.,  $\text{rank}(\mathbf{R}) = \eta(n)$ , and  $\mathbb{E}[|\mathbf{R}\mathbf{R}^H|_{\eta(n)+}^{-1}]$  is finite. The expected value  $\bar{P}_{2,\text{ml}}(e|M=n)$  of the SEP at the terminal  $T_2$  over the sample space of the pair  $\{\mathbf{R}, \mathbf{g}\}$  is bounded as

$$\bar{P}_{2,\text{ml}}(e|M=n) \leq G_c(n) \gamma^{-\eta(n)} \quad (10)$$

where  $\gamma \triangleq \beta^2$  is the SNR at  $T_2$  without fading and  $G_c(n) \triangleq \Gamma(n) \mathbb{E}[|\mathbf{R}\mathbf{R}^H|_{\eta(n)+}^{-1}]$ , with

$$\Gamma(n) \triangleq \frac{4^{\eta(n)} (|\mathcal{A}|^K - 1)}{\left[ \min_{\substack{k, \ell \in \{1, 2, \dots, |\mathcal{A}|^K\} \\ \ell \neq k}} \{|\mathbf{Q}^{(k, \ell)}|_{\eta(n)+}\} \right] |\mathbf{\Sigma}_g|_{\eta(n)+}} \quad (11)$$

and  $\mathbf{Q}^{(k, \ell)} \triangleq [\mathcal{G}^{(k)} - \mathcal{G}^{(\ell)}]^H [\mathcal{G}^{(k)} - \mathcal{G}^{(\ell)}] \in \mathbb{C}^{L \times L}$ .

*Proof:* See [7]. ■

We observe from (10) that, given  $M = n > 1$ , the randomized two-way relaying scheme with joint ML demodulation at the relays has asymptotic diversity order  $\eta(n) = \min\{L, n\}$  if the conditions in Lemma 1 are satisfied. However, due to the randomness in the number of active relays in the second time slot, such diversity order is not completely representative of the actual system performance.

By the total probability theorem [23] and accounting for (10), we obtain the following upper bound on the average (with respect to  $\mathbf{R}$ ,  $\mathbf{g}$ , and  $M$ ) SEP  $\bar{P}_{2,\text{ml}}(e)$  at the output of

the ML detector of  $T_2$ :

$$\begin{aligned} \bar{P}_{2,\text{ml}}(e) &= \sum_{n=0}^N \bar{P}_{2,\text{ml}}(e|M=n) P(M=n) \\ &\leq \left(1 - \frac{1}{|\mathcal{A}|^K}\right) P(M=0) + \sum_{n=1}^N G_c(n) \gamma^{-\eta(n)} P(M=n) \end{aligned} \quad (12)$$

where  $\bar{P}_{2,\text{ml}}(e|M=0)$  is the SEP at the receiver when none of the relays cooperate. If  $M=0$ , the received signal (8) contains noise only and, in this case, the terminal can only draw at random one symbol vector from the set  $\mathcal{A}^K$ , thus making a correct decision with probability  $1/|\mathcal{A}|^K$ . To evaluate the asymptotic diversity order, since  $\mathcal{P}_1$ ,  $\mathcal{P}_2$ , and  $\mathcal{P}_R$  are monotonically increasing functions of  $\mathcal{P}$ , we can directly take the limit as  $\mathcal{P} \rightarrow +\infty$ . Because  $p_i \rightarrow 1$  as  $\mathcal{P}_1, \mathcal{P}_2 \rightarrow +\infty$  by using its expression, it follows that  $\lim_{\mathcal{P} \rightarrow +\infty} P(M=n)$  is equal to one for  $n=N$  and zero otherwise. Consequently, we obtain

$$\lim_{\mathcal{P} \rightarrow +\infty} -\frac{\log[\bar{P}_{2,\text{ml}}(e)]}{\log(\mathcal{P})} = \eta(N) \quad (13)$$

which shows that the asymptotic diversity order is  $\min\{L, N\}$ .

The diversity scheme considered herein requires that the relays perform joint ML detection of  $\mathbf{s}_1$  and  $\mathbf{s}_2$ . For those systems with simple relay units such as wireless sensor networks and practical *ad hoc* or multihop wireless networks, this processing may be too computationally expensive for large values of  $K$  and/or  $|\mathcal{A}|$ . Even though (2) could be efficiently solved using generalized sphere decoder [24], the high decoding complexity could still be a practical issue. Even worse, in the under-determined scenario (2), existing low-complexity detectors such as zero-forcing or minimum-mean-square-error receivers would not work, apart from the performance degradation suffered from such suboptimal detectors due to receiver linearization. In addition, in some network topologies, some relay nodes are more likely to decode one symbol block corresponding to the closer terminal reliably and in such cases, correctly detecting both signals at those relays, happens with a low probability or is not even possible. In the next subsection, we investigate the diversity performance of a suboptimal nonlinear receiver, which involves an affordable processing burden at the relays.

### B. Decode and forward any (DFA-T2)

In the previous subsection, we have assumed that each relay node cooperates only if it has correctly detected both the symbol vectors  $\mathbf{s}_1$  and  $\mathbf{s}_2$ . However, due to fading effects and relative distance of the relays to two terminals, some relay nodes may be more likely to detect a single symbol block correctly rather than both of them. Therefore, in this section, we propose a relaying scheme where relay nodes cooperate using RSTBC not only if they have reliably detected both  $\mathbf{s}_1$  and  $\mathbf{s}_2$ , but also if they have correctly detected only one of  $\mathbf{s}_1$  and  $\mathbf{s}_2$ . The difference here is that the relay nodes have to multiplex the source signals in a way that makes them able to transmit concurrently, even though they cannot use network coding to combine the sources data.

1) *First time slot:* The computation of  $\hat{\mathbf{S}}_i$  in (3) can be reduced if one of the symbol vectors,  $\mathbf{s}_1$  or  $\mathbf{s}_2$ , can be canceled out. Therefore, we consider a scheme where  $\mathbf{s}_1$  and  $\mathbf{s}_2$  in (2) are sequentially detected one at a time, by treating the other symbol vector as Gaussian noise. Let  $\text{SINR}_m^{(i)}$  and  $\text{SNR}_m^{(i)}$  denote the signal-to-interference-plus-noise ratio (SINR) and the SNR, respectively, at relay  $R_i$  corresponding to the symbol vector  $\mathbf{s}_m$ , for  $m \in \{1, 2\}$ . Accounting for (2), one has

$$\begin{aligned} \text{SINR}_1^{(i)} &\triangleq \frac{E \left[ \left\| \sqrt{2\mathcal{P}_1} f_i \mathbf{s}_1 \right\|^2 \right]}{E \left[ \left\| \sqrt{2\mathcal{P}_2} g_i \mathbf{s}_2 + \mathbf{n}_i \right\|^2 \right]} = \frac{2 P_1 |f_i|^2}{2 P_2 |g_i|^2 + 1} \\ &= \frac{2 \mathcal{X}_i}{2 \mathcal{Y}_i + 1} \end{aligned} \quad (14)$$

$$\text{SNR}_1^{(i)} \triangleq \frac{E \left[ \left\| \sqrt{2\mathcal{P}_1} f_i \mathbf{s}_1 \right\|^2 \right]}{E \left[ \left\| \mathbf{n}_i \right\|^2 \right]} = 2 P_1 |f_i|^2 = 2 \mathcal{X}_i \quad (15)$$

where we recall that the coefficients  $\sqrt{2\mathcal{P}_1} f_i$  and  $\sqrt{2\mathcal{P}_2} g_i$  are known at  $R_i$ . Similarly, we define  $\text{SINR}_2^{(i)} \triangleq \frac{2 \mathcal{Y}_i}{2 \mathcal{X}_i + 1}$  and  $\text{SNR}_2^{(i)} \triangleq 2 \mathcal{Y}_i$ . The candidate relays initially attempt to firstly detect the symbol block with the highest SINR. For instance, if  $\text{SINR}_1^{(i)} \geq \text{SINR}_2^{(i)}$ , then  $\mathbf{s}_1$  is tentatively detected at the relay  $R_i$ , by treating  $\mathbf{s}_2$  as unknown interference. More precisely, from a Bayesian point of view, this means that, instead of using the fact that  $\mathbf{s}_2$  belongs to the set  $\mathcal{A}^K$ , we simply recall that  $E[\mathbf{s}_2] = \mathbf{0}_K$  and  $E[\mathbf{s}_2 \mathbf{s}_2^H] = \mathbf{I}_K$ ; then, the prior for  $\mathbf{s}_2$  is taken to the maximum entropy distribution satisfying such constraints, which turns out to be the Gaussian distribution  $\mathcal{CN}(\mathbf{0}_K, \mathbf{I}_K)$  [25]. In this case, we say that  $R_i$  can *reliably* detect  $\mathbf{s}_1$  if  $\text{SINR}_1^{(i)} \geq \tau_{\text{bic}}$ ; the threshold  $\tau_{\text{bic}} > 0$  is chosen such that to fulfill the SEP constraint  $P_{1,\text{bic}}^{(i)}(e) \leq \text{SEP}_{\text{target}}$  where, under the Gaussian approximation for  $\mathbf{s}_2$ , the SEP associated with the detection of  $\mathbf{s}_1$  is approximatively given by  $P_{1,\text{bic}}^{(i)}(e) = \alpha_1 Q \left( \sqrt{\alpha_2 \text{SINR}_1^{(i)}} \right)$ , with  $\alpha_1$  and  $\alpha_2$  being constants that depend on the symbol constellation. This leads to the choice  $\tau_{\text{bic}} = \frac{1}{\alpha_2} \left[ Q^{-1} \left( \frac{\text{SEP}_{\text{target}}}{\alpha_1} \right) \right]^2$ . If  $R_i$  can reliably detect  $\mathbf{s}_1$ , it adopts the decision rule

$$\hat{\mathbf{s}}_{1,\text{bic}}^{(i)} = \arg \min_{\mathbf{s}_1 \in \mathcal{A}^K} \left\| \mathbf{x}_i - \sqrt{2\mathcal{P}_1} f_i \mathbf{s}_1 \right\|^2 \quad (16)$$

and attempts to detect  $\mathbf{s}_2$  afterwards; otherwise, if  $\text{SINR}_1^{(i)} < \tau_{\text{bic}}$ , the  $i$ th relay is unable to correctly demodulate both  $\mathbf{s}_1$  and  $\mathbf{s}_2$  and, thus, it will be inactive in the second time slot. In particular, when  $\text{SINR}_1^{(i)} \geq \text{SINR}_2^{(i)}$  and  $\text{SINR}_1^{(i)} \geq \tau_{\text{bic}}$ , we say that  $R_i$  can *reliably* detect  $\mathbf{s}_2$  if  $\text{SNR}_2^{(i)} \geq \tau_{\text{bic}}$ ; in this case, an estimate of  $\mathbf{s}_2$  is determined according to the rule

$$\hat{\mathbf{s}}_{2,\text{bic}}^{(i)} = \arg \min_{\mathbf{s}_2 \in \mathcal{A}^K} \left\| \mathbf{x}_i - \sqrt{2\mathcal{P}_1} f_i \hat{\mathbf{s}}_{1,\text{bic}}^{(i)} - \sqrt{2\mathcal{P}_2} g_i \mathbf{s}_2 \right\|^2 \quad (17)$$

and the relay is capable of correctly detecting both  $\mathbf{s}_1$  and  $\mathbf{s}_2$ . On the other hand, if  $\text{SNR}_2^{(i)} < \tau_{\text{bic}}$ , only  $\mathbf{s}_1$  can be detected reliably. A similar algorithm is used at the relay  $R_i$  when  $\text{SINR}_2^{(i)} > \text{SINR}_1^{(i)}$ . Henceforth, in the proposed scheme, some relay nodes may only detect  $\mathbf{s}_1$  reliably, some

other nodes might only detect  $\mathbf{s}_2$  correctly, and some nodes could reliably detect both of them.

Let  $\mathcal{D}_1$ ,  $\mathcal{D}_2$  and  $\mathcal{D}_{1,2}$  denote the set of relays that have detected only  $\mathbf{s}_1$ , only  $\mathbf{s}_2$ , and both symbol vectors, respectively. Due to the error-free constraints, the cardinalities of these sets are random variables, which depend on the available power in the network and  $\tau_{\text{bic}}$ . In this respect, let us define the sets  $\mathcal{B}_1 \triangleq \{b_{1,1}, \dots, b_{1,M_1}\} = \mathcal{D}_1 \cup \mathcal{D}_{1,2}$  and  $\mathcal{B}_2 \triangleq \{b_{2,1}, \dots, b_{2,M_2}\} = \mathcal{D}_2 \cup \mathcal{D}_{1,2}$ , where the number of relay nodes that reliably detected  $\mathbf{s}_m$  is denoted by  $M_m$ , for  $m \in \{1, 2\}$ . In general,  $M_1$  and  $M_2$  are two discrete random variables which can assume the values  $\{0, 1, \dots, N\}$ . Since all the underlying channels are statistically independent by assumption and the relays make decisions independently of each other, it follows that, for  $n_m \in \{0, 1, \dots, N\}$ , the pmf of  $M_m$  can be expressed as  $P(M_m = n_m) = \Phi(n_m; p_{\mathcal{B}_m}^{(1)}, p_{\mathcal{B}_m}^{(2)}, \dots, p_{\mathcal{B}_m}^{(N)})$ , where  $p_{\mathcal{B}_m}^{(i)}$  is the probability that the  $i$ th relay belongs to  $\mathcal{B}_m$ , with  $m \in \{1, 2\}$ . In the case of  $\tau_{\text{bic}} \geq 1$ , the probability  $p_{\mathcal{B}_m}^{(i)}$  is calculated in Appendix A. As a special case, when  $\sigma_{f_i}^2$  and  $\sigma_{g_i}^2$  do not depend on  $i$ , i.e., all the relays are at the same location, the pmf  $P(M_m = n_m)$  ends up to a binomial distribution [16] of parameters  $N$  and  $p_{\mathcal{B}_m} \triangleq p_{\mathcal{B}_m}^{(1)} = p_{\mathcal{B}_m}^{(2)} = \dots = p_{\mathcal{B}_m}^{(N)}$ . Let us focus on the situation when  $\mathcal{P}_1 = \mathcal{P}_2 = \mathcal{P}/4$  and  $\mathcal{P}_R = \mathcal{P}/2$ , which is the optimal power allocation for some centralized relaying schemes [12]. In this case, it is interesting to observe that, in the high SNR region, i.e., as  $\mathcal{P} \rightarrow +\infty$ , one has (see Appendix A)

$$\bar{p}_{\mathcal{B}_1}^{(i)} \triangleq \lim_{\mathcal{P} \rightarrow +\infty} p_{\mathcal{B}_1}^{(i)} = \frac{\sigma_{f_i}^2}{\sigma_{f_i}^2 + \sigma_{g_i}^2 \tau_{\text{bic}}} + \frac{\sigma_{g_i}^2}{\sigma_{g_i}^2 + \sigma_{f_i}^2 \tau_{\text{bic}}}. \quad (18)$$

In words, the probability that the relay  $R_i$  reliably detects  $\mathbf{s}_1$  does not approach to one in the high SNR region: indeed, the exact value of  $\bar{p}_{\mathcal{B}_1}^{(i)}$  depends on the decoding threshold  $\tau_{\text{bic}}$ , as well as on the ratio  $\sigma_{f_i}^2/\sigma_{g_i}^2$ , which depends on the distance of relay node  $i$  from the two terminals.

*Corollary 2:* For a given  $\tau_{\text{bic}}$ , among the relay nodes with fixed amount of received power  $\mathcal{P}_1 \sigma_{f_i}^2 + \mathcal{P}_2 \sigma_{g_i}^2 = \mathcal{C}$ , the probability  $\bar{p}_{\mathcal{B}_1}^{(i)}$  is minimized when  $\mathcal{P}_1 \sigma_{f_i}^2 = \mathcal{P}_2 \sigma_{g_i}^2$ .

In the case of  $\mathcal{P}_1 = \mathcal{P}_2$ , Corollary 2 leads to the conclusion that the asymptotic probability of cooperation in relay node  $i$  for DFA-T2 scheme is minimized when it is located at an equal distance from two terminals. Therefore, we conclude from Corollary 2 that the performance of the DFA-T2 scheme will be improved if the relay nodes are located asymmetrically, because the probability of cooperation in this case will be increased in comparison with a symmetric network.

2) *Second time slot:* Each relay transmits a random linear combination of the columns of the deterministic space-time code matrix of the symbol block(s) detected correctly. More precisely, if the relay  $R_i$  reliably detected the only vector  $\mathbf{s}_m$ , with  $m \in \{1, 2\}$ , it transmits the block  $\beta \mathcal{G}(\mathbf{s}_m) \mathbf{r}_i$  to the two terminals  $T_1$  and  $T_2$  during the second time slot, where the code matrix  $\mathcal{G}(\cdot)$  and the randomization vector  $\mathbf{r}_i$  have been defined in Subsection III-A. On the other hand, if both  $\mathbf{s}_1$  and  $\mathbf{s}_2$  were reliably detected at the relay  $R_i$ , the block  $\frac{\beta}{\sqrt{2}} [\mathcal{G}(\mathbf{s}_1) + \mathcal{G}(\mathbf{s}_2)] \mathbf{r}_i$  is transmitted to  $T_1$  and  $T_2$  during the

second time slot. Thus, we can write the received signals at the terminal  $T_2$  as follows

$$\mathbf{y}_{2,\text{bic}} = \beta \left\{ \sum_{i \in \mathcal{D}_1} g_i \mathcal{G}(\mathbf{s}_1) \mathbf{r}_i + \sum_{i \in \mathcal{D}_2} g_i \mathcal{G}(\mathbf{s}_2) \mathbf{r}_i + \sum_{i \in \mathcal{D}_{1,2}} \frac{1}{\sqrt{2}} g_i [\mathcal{G}(\mathbf{s}_1) + \mathcal{G}(\mathbf{s}_2)] \mathbf{r}_i \right\} + \mathbf{w}_2 \quad (19)$$

where  $\beta$  and  $\mathbf{w}_2$  have been defined in Subsection III-A. It should be observed that, in this case, the bit-level XOR randomized coding rule cannot be used since there might be relays that reliably detected only one symbol block. Due to symmetry, the expression of received signal at the terminal  $T_1$  is omitted. Let  $\bar{g}_i = g_i$  if  $i \in \mathcal{D}_1$  or  $i \in \mathcal{D}_2$ , and  $\bar{g}_i = \frac{g_i}{\sqrt{2}}$  if  $i \in \mathcal{D}_{1,2}$ , eq. (19) can be more compactly written as

$$\begin{aligned} \mathbf{y}_{2,\text{bic}} &= \beta \left[ \sum_{i \in \mathcal{B}_1} \bar{g}_i \mathcal{G}(\mathbf{s}_1) \mathbf{r}_i + \sum_{i \in \mathcal{B}_2} \bar{g}_i \mathcal{G}(\mathbf{s}_2) \mathbf{r}_i \right] + \mathbf{w}_2 \\ &= \beta [\mathcal{G}(\mathbf{s}_1) \mathbf{R}_1 \mathbf{g}_1 + \mathcal{G}(\mathbf{s}_2) \mathbf{R}_2 \mathbf{g}_2] + \mathbf{w}_2 \end{aligned} \quad (20)$$

where  $\mathbf{R}_m \triangleq [\mathbf{r}_{b_{m,1}}, \mathbf{r}_{b_{m,2}}, \dots, \mathbf{r}_{b_{m,M_m}}] \in \mathbb{C}^{L \times M_m}$  and  $\mathbf{g}_m \triangleq [\bar{g}_{b_{m,1}}, \bar{g}_{b_{m,2}}, \dots, \bar{g}_{b_{m,M_m}}] \in \mathbb{C}^{M_m}$  is a zero-mean complex circularly symmetric Gaussian vector, with diagonal autocorrelation matrix given by  $\Sigma_{\mathbf{g}_m} \triangleq \mathbb{E}[\mathbf{g}_m \mathbf{g}_m^H] \in \mathbb{R}^{M_m \times M_m}$ , for  $m \in \{1, 2\}$ . Hereinafter, let  $M_m = n_m$  for  $n_m \in \{0, 1, \dots, N\}$ , we assume that  $\mathbf{R}_m$  is full-rank with probability one, i.e.,  $\text{rank}(\mathbf{R}_m) = \eta(n_m) = \min\{L, n_m\}$ , and  $\mathbb{E}[|\mathbf{R}_m \mathbf{R}_m^H|_{\eta(n_m)+}^{-1}]$  is finite. We underline that  $T_2$  can estimate the *partial* effective channel vectors  $\tilde{\mathbf{g}}_1 \triangleq \beta \mathbf{R}_1 \mathbf{g}_1 \in \mathbb{C}^L$  and  $\tilde{\mathbf{g}}_2 \triangleq \beta \mathbf{R}_2 \mathbf{g}_2 \in \mathbb{C}^L$  via training by using a procedure similar to that described in Subsection III-A.

Since  $\mathbf{s}_2$  is also known at  $T_2$ , under the assumption that  $\tilde{\mathbf{g}}_1$  and  $\tilde{\mathbf{g}}_2$  are perfectly acquired, the ML decoder at  $T_2$  can be obtained by solving the optimization problem

$$\begin{aligned} \hat{\mathbf{s}}_{1,\text{bic}} &= \arg \min_{\mathbf{s}_1 \in \mathcal{A}^K} \|\mathbf{y}_{2,\text{bic}} - \mathcal{G}(\mathbf{s}_1) \tilde{\mathbf{g}}_1 - \mathcal{G}(\mathbf{s}_2) \tilde{\mathbf{g}}_2\|^2 \\ &= \arg \min_{\mathbf{s}_1 \in \mathcal{A}^K} \|\tilde{\mathbf{y}}_{2,\text{bic}} - \mathcal{G}(\mathbf{s}_1) \tilde{\mathbf{g}}_1\|^2 \end{aligned} \quad (21)$$

where  $\tilde{\mathbf{y}}_{2,\text{bic}} \triangleq \mathbf{y}_{2,\text{bic}} - \mathcal{G}(\mathbf{s}_2) \tilde{\mathbf{g}}_2$ . It is apparent that the decision rule (21) is similar to that reported in (9). Consequently, reasoning as done in Subsection III-A, it is seen that the average (with respect to  $\mathbf{R}_1$ ,  $\mathbf{g}_1$ , and  $M_1$ ) SEP  $\bar{P}_{2,\text{bic}}(e)$  at the output of the ML detector of  $T_2$  can be upper bounded as

$$\begin{aligned} \bar{P}_{2,\text{bic}}(e) &\leq \bar{P}_{2,\text{bic}}^{\text{ub}}(e) \triangleq \left(1 - \frac{1}{|\mathcal{A}|^K}\right) P(M_1 = 0) \\ &\quad + \sum_{n_1=1}^N G_{c,1}(n_1) \gamma^{-\eta(n_1)} P(M_1 = n_1) \end{aligned} \quad (22)$$

with  $G_{c,1}(n_1) \triangleq \Gamma_1(n_1) \mathbb{E}[|\mathbf{R}_1 \mathbf{R}_1^H|_{\eta(n_1)+}^{-1}]$ , where  $\Gamma_1(n_1)$  is obtained from (11) by replacing  $\eta(n)$  and  $\Sigma_{\mathbf{g}}$  with  $\eta(n_1)$  and  $\Sigma_{\mathbf{g}_1}$ , respectively. As previously done, let us focus on the special case when  $\mathcal{P}_1 = \mathcal{P}_2 = \mathcal{P}/4$  and  $\mathcal{P}_R = \mathcal{P}/2$ , which implies that  $\gamma = \mathcal{P}/N$ . It is evident that, in this case, since  $p_{\mathcal{B}_1}^{(i)}$  does not converge to one as  $\mathcal{P} \rightarrow +\infty$  and, thus, one

has  $P(M_1 = 0) = \prod_{i=1}^N [1 - p_{\mathcal{B}_1}^{(i)}] \neq 0$ , the upper bound on  $\bar{P}_{2,\text{bic}}(e)$  is a constant in the high SNR regime, i.e.,

$$\lim_{\mathcal{P} \rightarrow +\infty} \bar{P}_{2,\text{bic}}^{\text{ub}}(e) = \left(1 - \frac{1}{|\mathcal{A}|^K}\right) \prod_{i=1}^N [1 - \bar{p}_{\mathcal{B}_1}^{(i)}] \quad (23)$$

where  $\bar{p}_{\mathcal{B}_1}^{(i)}$  is given by (18). This fact implies that the asymptotic diversity order of the considered scheme is zero and the system performance exhibits a SEP floor for high enough SNR values. However, Theorem 1 shows that there exists a threshold  $\gamma_{\text{th}}$ , which is very large in practical situations, such that, for values of SNR less than  $\gamma_{\text{th}}$ , the diversity order equal to  $\min\{L, N\}$  is achieved.

*Theorem 1:* Suppose that  $\mathcal{P}_1 = \mathcal{P}_2 = \mathcal{P}/4$  and  $\mathcal{P}_R = \mathcal{P}/2$ . DFA-T2 achieves the diversity order  $\min\{L, N\}$  up to a threshold  $\gamma_{\text{th}}$  for the SNR. This threshold is approximately lower bounded as  $\gamma_{\text{th}} \geq \gamma_{\text{th}}^{\text{lb}}$ , with

$$\gamma_{\text{th}}^{\text{lb}} = \left\{ \frac{\Gamma_1(N) \mathbb{E}[|\mathbf{R}_1 \mathbf{R}_1^H|_{\min\{L, N\}+}^{-1}] \Upsilon}{1 - \left(1 - \frac{1}{|\mathcal{A}|^K}\right) \prod_{i=1}^N [1 - \bar{p}_{\mathcal{B}_1}^{(i)}]} \right\}^{1/L} \quad (24)$$

where  $\bar{p}_{\mathcal{B}_1}^{(i)}$  is defined in (18) and

$$\Upsilon \triangleq \begin{cases} \sum_{n_1=L}^N \Phi(n_1; \bar{p}_{\mathcal{B}_1}^{(1)}, \bar{p}_{\mathcal{B}_1}^{(2)}, \dots, \bar{p}_{\mathcal{B}_1}^{(N)}), & \text{for } N \geq L; \\ \prod_{i=1}^N \bar{p}_{\mathcal{B}_1}^{(i)}, & \text{for } L > N. \end{cases} \quad (25)$$

*Proof:* See Appendix B.  $\blacksquare$

The result of Theorem 1 can be further explicated by assuming a specific randomization rule [7]. It is shown in Section V that  $\gamma_{\text{th}}^{\text{lb}}$  takes on large values which are well beyond the SNR range of practical interest. Hence, the loss of diversity of the considered scheme is basically a problem of theoretical interest, and has limited impact on the actual system performance.

It is worth mentioning that existing relaying techniques (e.g., [12]) can achieve zero-approaching error probability as  $\mathcal{P} \rightarrow +\infty$ , while the proposed BIC scheme shows a SEP floor. The reason is that, for such protocols, a specific *nonempty* set of nodes is assumed to cooperate, by ignoring the network cost of such an assumption. Under the same assumption, the probability  $P(M_1 = 0)$  in (22) is zero and, thus, DFA-T2 does not exhibit a SEP floor. It is possible to use control signals at the medium access control (MAC) sublayer that may prevent the situation that gives rise to the SEP floor to occur. To this aim, following [26], a control signal sent at unison from the relays by using RSTBC might be provided after the first time-slot, which acknowledges their capability to cooperate. The lack of such a response signal can let the terminals know that there are no available relays in the second-time slot and, hence, cooperation is not possible. However, the analysis of such a PHY/MAC protocol is beyond the scope of this paper.

#### IV. RSTBC PROTOCOL FOR THREE-TIME-SLOT RELAYING (RSTBC-T3)

In three-time-slot relaying, the communication between terminals  $T_1$  and  $T_2$  consumes three time slots. In the first time slot, terminal  $T_1$  transmits  $\mathbf{s}_1$  to the relay nodes. In the

second time slot, the other terminal  $T_2$  transmits  $s_2$  to the relay nodes. In the third time slot, each relay utilizes the ML rule to detect the received symbol block at the first and second time slots and, then, they jointly forward the estimated symbol vectors to the two terminal nodes. We refer to this scheme as DFA-T3. For  $i \in \{1, 2, \dots, N\}$ , the received signals in the first and second time slots at the relay  $R_i$  can be written, respectively, as

$$\mathbf{x}_{i,1} = \sqrt{3\mathcal{P}_1} f_i \mathbf{s}_1 + \mathbf{n}_{i,1} \quad \text{and} \quad \mathbf{x}_{i,2} = \sqrt{3\mathcal{P}_2} g_i \mathbf{s}_2 + \mathbf{n}_{i,2} \quad (26)$$

where  $\mathbf{n}_{i,1}$  and  $\mathbf{n}_{i,2}$  are i.i.d. with distribution  $\mathcal{CN}(\mathbf{0}_K, \mathbf{I}_K)$  and are statistically independent of  $\mathbf{s}_1$  and  $\mathbf{s}_2$ . As in Section III, the coefficients  $\sqrt{3\mathcal{P}_1} f_i$  and  $\sqrt{3\mathcal{P}_2} g_i$  are assumed to be exactly known at  $R_i$ , since they can be acquired by using conventional point-to-point training-based estimation methods.

Let  $P_{1,\text{ml}}^{(i)}(e)$  and  $P_{2,\text{ml}}^{(i)}(e)$  denote the SEP at the output of the ML detector of the relay  $R_i$  in the first and second time-slots, respectively, one has  $P_{1,\text{ml}}^{(i)}(e) = \alpha_1 Q(\sqrt{3\alpha_2} \mathcal{X}_i)$  and  $P_{2,\text{ml}}^{(i)}(e) = \alpha_1 Q(\sqrt{3\alpha_2} \mathcal{Y}_i)$  [23], where  $\alpha_1$  and  $\alpha_2$  depend on the symbol constellation, whereas  $\mathcal{X}_i = \mathcal{P}_1 |f_i|^2$  and  $\mathcal{Y}_i = \mathcal{P}_2 |g_i|^2$  have been already defined in Subsection III-B. We say that relay  $R_i$  can *reliably* detect  $s_1$  if  $\mathcal{X}_i \geq \tilde{\tau}_{\text{ml}}$  and it can *reliably* detect  $s_2$  if  $\mathcal{Y}_i \geq \tilde{\tau}_{\text{ml}}$ ; the threshold  $\tilde{\tau}_{\text{ml}}$  is chosen so as to assure the SEP constraints  $P_{1,\text{ml}}^{(i)}(e) \leq \text{SEP}_{\text{target}}$  and  $P_{2,\text{ml}}^{(i)}(e) \leq \text{SEP}_{\text{target}}$ , respectively, and it is given by  $\tilde{\tau}_{\text{ml}} = \frac{1}{3\alpha_2} \left[ Q^{-1} \left( \frac{\text{SEP}_{\text{target}}}{\alpha_1} \right) \right]^2$ . The probability  $\tilde{p}_{\mathcal{B}_m}^{(i)}$  that the  $i$ th relay belongs to  $\mathcal{B}_m$  can be expressed as  $\tilde{p}_{\mathcal{B}_m}^{(i)} = e^{-L_m^{(i)} \tilde{\tau}_{\text{ml}}}$ . It is worth mentioning that, in this scheme, detectors at the relay nodes do not suffer from interference. As a result, by comparing  $\tilde{p}_{\mathcal{B}_m}^{(i)}$  with the expression of  $p_{\mathcal{B}_m}^{(i)}$  calculated in Appendix A, the probability of cooperation is higher than the probability of cooperation in two-time-slot relaying schemes, for  $\tilde{\tau}_{\text{ml}} = \tau_{\text{bic}}$ .

In the third time slot, as in Subsection III-B, each relay operates as a DFA-T2 RSTBC relay and transmits a random linear combination of the columns of an orthogonal space-time code  $\mathcal{G}(\cdot)$  that encodes whatever the relay was able to decode, by using the randomization vector  $\mathbf{r}_i$ . From this point on, the signal model at the terminal  $T_2$  and the corresponding SEP analysis strictly follow the treatment developed in Subsection III-B for the second time slot. Thus, under the same assumptions, the average SEP at the output of the ML detector of the second terminal can be upper bounded as in (22), by only replacing  $\gamma$  with  $\tilde{\gamma} \triangleq \tilde{\beta}^2$ , which is the SNR at  $T_2$  without fading, where  $\tilde{\beta} \triangleq \sqrt{(3\mathcal{P}_R)/N}$  denotes the scaling factor used by the relays in order to satisfy the average power constraint. Since  $\tilde{p}_{\mathcal{B}_m}^{(i)} \rightarrow 1$  as  $\mathcal{P}_1, \mathcal{P}_2 \rightarrow +\infty$ , it follows that the asymptotic diversity order of the proposed three-time-slot randomized scheme is  $\min\{L, N\}$ .

## V. NUMERICAL PERFORMANCE ANALYSIS

In this section, we present the performance of the proposed randomized distributed space-time codes. We obtain the average SEP (ASEP) through the Monte Carlo method and compare the performance of the proposed randomized schemes

TABLE I  
AVERAGE VARIANCES OF CHANNEL LINKS BETWEEN TERMINALS AND RELAY NODES.

	Topology I	Topology II
$\sum_{i=1}^N \sigma_{f_i}^2 / N$	0.310	0.296
$\sum_{i=1}^N \sigma_{g_i}^2 / N$	0.349	0.289

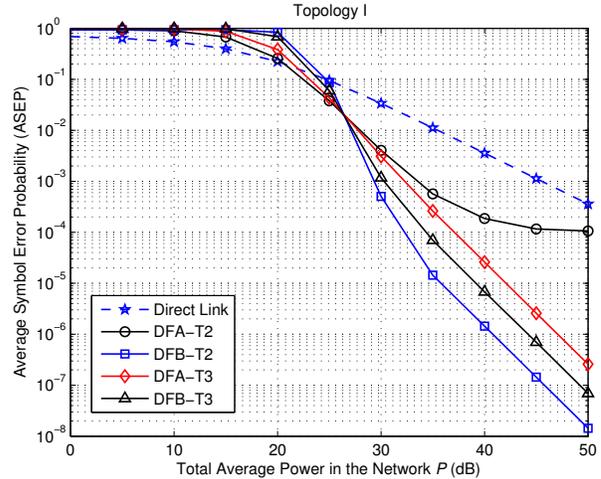


Fig. 2. ASEP versus total available power  $\mathcal{P}$  for all proposed schemes in Topology I ( $N = 10$ ).

for  $N = 10$  with uniform complex spherical randomization rule which performs slightly better than the other common randomization rules (see [7] for details). We only report the ASEP for terminal  $T_2$ . The Alamouti space-time code [27] is used with  $L = 2$  and, to equalize the data rate, QPSK modulation is employed at  $T_1$  and  $T_2$  for the two-time-slot case, whereas 8-PSK is used in the case of the three-time-slot relaying. We investigate the performance of the proposed schemes in two network topologies. In the first topology (Topology I), the relays are distributed randomly and independently in two circles of radius 0.5 centered at two terminals located at the positions  $(-1, 0)$  and  $(1, 0)$  in the  $xOy$  Cartesian coordinate system. In the second topology (Topology II), the relays are distributed randomly and independently in a circle with center at origin  $O$  and radius 1. The variance of the channel coefficient between relay node  $i$  and terminal  $m$  characterizing the link between them is modeled as  $(d_0 + \|\mathbf{x}_i - \mathbf{t}_m\|)^{-2}$ , where  $\|\mathbf{x}_i - \mathbf{t}_m\|$  denotes the Euclidean distance between them and  $d_0$  is a modelling parameter. In these simulations, we have set  $d_0 = 1$ . In Table I, we have summarized the average path-loss in both topologies to demonstrate the value of the channel links variances. The thresholds  $\tau_{\text{ml}}$ ,  $\tau_{\text{bic}}$ , and  $\tilde{\tau}_{\text{ml}}$  are chosen by fixing  $\text{SEP}_{\text{target}} = 10^{-3}$ . In this section, we also consider the three-time-slot variant of DFB scheme for comparison and call it DFB-T3. In DFB-T3, the relay nodes that decoded both signals reliably cooperate in the third time slot by forwarding the RSTBC of the bit-level XOR of the detected symbol vectors.

Figs. 2 and 3 illustrate the performance of all the proposed strategies versus total average available power in the network  $\mathcal{P}$ , for Topology I and Topology II, respectively. We set  $\mathcal{P}_1 = \mathcal{P}_2 = \mathcal{P}/4$  and  $\mathcal{P}_R = \mathcal{P}/2$  in the case of two time

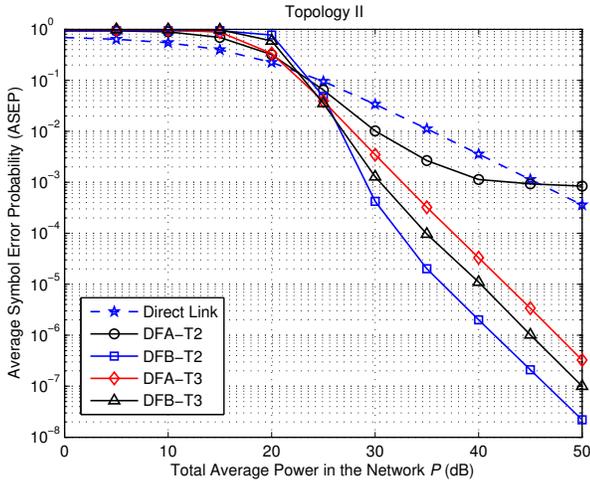


Fig. 3. ASEP versus total available power  $\mathcal{P}$  for all proposed schemes in Topology II ( $N = 10$ ).

slots, whereas  $\mathcal{P}_1 = \mathcal{P}_2 = \mathcal{P}/3$  and  $\mathcal{P}_R = \mathcal{P}/3$  for three-time-slot relaying. We have also included the performance of the case where the direct link between two terminals exists for the purpose of comparison. In Fig. 2, DFA-T2 outperforms the other strategies up to 27 dB and then it loses its diversity. In both topologies, the performance of DFB-T2 is better than other strategies for SNR values larger than a threshold. In Figs. 2 and 3, DFB-T3 shows 3 dB improvement over DFA-T3 in large SNR, because in high SNR, all the nodes are able to decode and it is more power efficient to transmit the bit-level XOR of the decoded signals. In Topology II, the performance of DFA-T2 is poor in comparison with other strategies. As mentioned in Subsection III-B, we can verify that in both topologies, DFA-T2 achieves the diversity order  $\min\{L, N\} = 2$  up to the threshold  $\gamma_{th}^{lb}$  given by (24) and as predicated in Subsection III-A, the diversity order equal to  $\min\{L, N\} = 2$  is achieved for DFB-T2. In Table II, the results obtained from simulations are compared with analytical ones for DFA-T2 in both topologies. The simulation results suggest that based on the network topology, deployment of the relay nodes in the network and available power in the network, one scheme may be preferable to the others.

In Figs. 4 and 5, we compare the performance of the proposed schemes by fixing the average terminal powers ( $\mathcal{P}_1 = \mathcal{P}_2 = 20$  dB or  $\mathcal{P}_1 = \mathcal{P}_2 = 30$  dB) and change the available power in each relay, for Topology I and Topology II, respectively. Since the source power is fixed, the probability of cooperation in relays is constant and varying the relay power shows how the performance changes with respect to relay power. In Topology I, DFB-T2 has the best performance for source power equal to 30 dB and the worst performance when source power is 20 dB. In Topology II, DFB-T2 has the best performance for source power equal to 30 dB. DFA-T2 has the worst performance when source power is 20 dB. In both topologies, all the strategies exhibit ASEP floor for source power equal to 20 dB, because the probability of cooperation in relay nodes is constant and less than 1. In Fig. 4, DFA-T3 has the smallest ASEP floor. In summary, we can observe that

TABLE II  
COMPARISON OF SIMULATION WITH ANALYTICAL RESULTS OF DFA-T2 IN THE CASE OF UNIFORM COMPLEX SPHERICAL RANDOMIZATION.

	Simulation result	Analytical value
Asymptotic SEP, Topology I	$1.06 \times 10^{-4}$	$9.9733 \times 10^{-5}$
Threshold $\gamma_{th}^{lb}$ , Topology I	38 dB	35.85 dB
Asymptotic SEP, Topology II	$8.4 \times 10^{-4}$	$8.8759 \times 10^{-4}$
Threshold $\gamma_{th}^{lb}$ , Topology II	36 dB	34.43 dB

choosing the strategy with the best performance depends on the source power and the available power in relay nodes. The former determines the probability and pattern of cooperation in the relay nodes and the latter affects the amount of received power at the terminals.

Figs. 6 and 7 demonstrate the ratio of average transmitted power to the available power in relays versus average terminal power in all strategies, for Topology I and Topology II, respectively. All schemes except DFA-T2 reach the ratio 1 for some large values of terminal SNR. This is due to the fact that for large SNR values, all relay nodes become able to cooperate (the probability of cooperation goes to 1 as terminal power goes to infinity). Since the asymptotic probability of cooperation in DFA-T2 is not 1, the asymptotic transmitted power ratio in this case is a constant smaller than 1. Clearly, for each strategy, the terminal power should be greater than some threshold to make relay nodes capable of decoding reliably and result in transmitted power ratio greater than 0. We observe that, the smallest terminal power threshold occurs for DFA-T2 and the largest terminal power threshold appears for DFB-T2.

## VI. CONCLUSIONS

We studied two-way wireless relaying protocols using RSTBC at relays and analyze the performance of the proposed two- and three-time-slots relaying schemes. More specifically, the first proposed scheme DFB takes up only two time slots and achieves the asymptotic diversity order  $\min\{L, N\}$ , but it requires more complex detectors at the relay nodes. We proposed another two-time-slot protocol (DFA-T2), which achieves the same diversity order up to a threshold for the SNR that, however, assumes very large values in practice. Finally, a randomized scheme is proposed for three-time-slot relaying (DFA-T3), whose asymptotic diversity order is  $\min\{L, N\}$ , but a smaller throughput is achieved for large values of  $N$ .

## APPENDIX A CALCULATION OF $p_{\mathcal{B}_m}^{(i)}$

Since  $\mathcal{D}_1$ ,  $\mathcal{D}_2$ , and  $\mathcal{D}_{1,2}$  are disjoint sets, the probability that the  $i$ th relay node belongs to the set  $\mathcal{B}_m$  is  $p_{\mathcal{B}_m}^{(i)} = p_{\mathcal{D}_m}^{(i)} + p_{\mathcal{D}_{1,2}}^{(i)}$ , for  $m \in \{1, 2\}$ , where  $p_{\mathcal{D}_1}^{(i)} \triangleq P[R_i \in \mathcal{D}_1]$ ,  $p_{\mathcal{D}_2}^{(i)} \triangleq P[R_i \in \mathcal{D}_2]$ , and  $p_{\mathcal{D}_{1,2}}^{(i)} \triangleq P[R_i \in \mathcal{D}_{1,2}]$ . Since all the relays resort to interference cancellation for demodulating the source blocks  $s_1$  and  $s_2$ , these probabilities can be explicitly written as

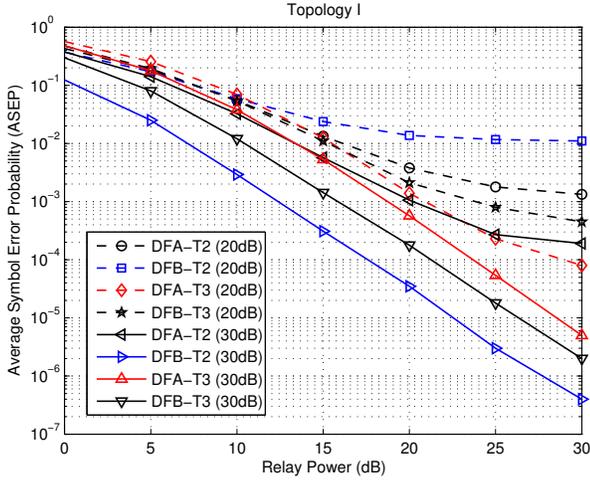


Fig. 4. ASEP versus relay power for all proposed schemes in Topology I ( $N = 10$ ).

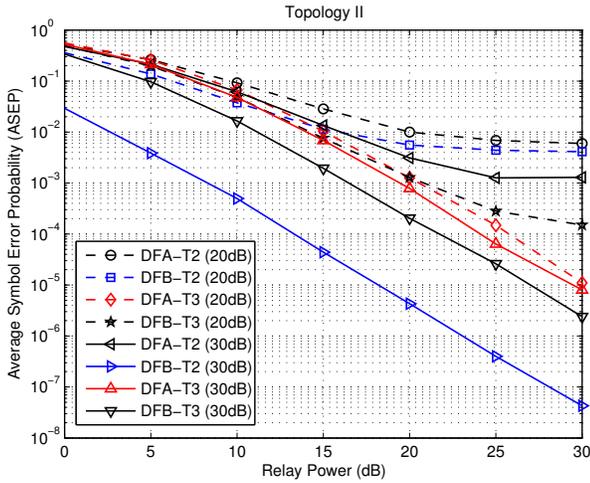


Fig. 5. ASEP versus relay power for all proposed schemes in Topology II ( $N = 10$ ).

$$p_{D_1}^{(i)} = P[(\text{SINR}_1^{(i)} \geq \text{SINR}_2^{(i)}) \cap (\text{SINR}_1^{(i)} \geq \tau_{\text{bic}}) \cap (\text{SNR}_2^{(i)} < \tau_{\text{bic}})] \quad (27)$$

$$p_{D_2}^{(i)} = P[(\text{SINR}_2^{(i)} > \text{SINR}_1^{(i)}) \cap (\text{SINR}_2^{(i)} \geq \tau_{\text{bic}}) \cap (\text{SNR}_1^{(i)} < \tau_{\text{bic}})] \quad (28)$$

$$p_{D_{1,2}}^{(i)} = P[(\text{SINR}_1^{(i)} \geq \text{SINR}_2^{(i)}) \cap (\text{SINR}_1^{(i)} \geq \tau_{\text{bic}}) \cap (\text{SNR}_2^{(i)} \geq \tau_{\text{bic}})] \\ + P[(\text{SINR}_2^{(i)} > \text{SINR}_1^{(i)}) \cap (\text{SINR}_2^{(i)} \geq \tau_{\text{bic}}) \cap (\text{SNR}_1^{(i)} \geq \tau_{\text{bic}})] \cdot (29)$$

Since we are interested in the high SNR regime, we assume that  $\tau_{\text{bic}} \geq 1$ . This assumption simplifies the expressions in

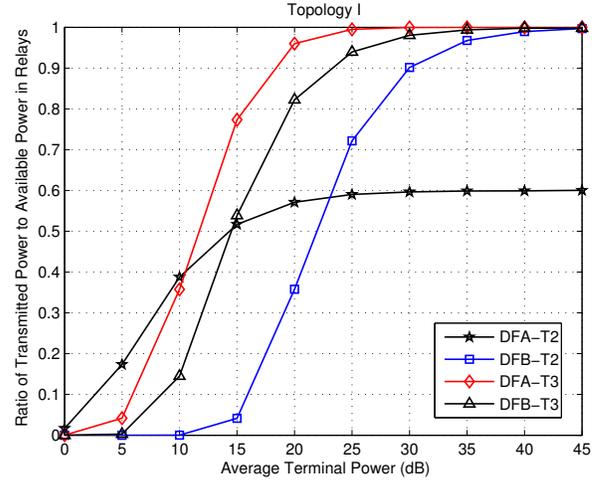


Fig. 6. Ratio of average transmitted power to available power in relays versus terminal power for all proposed schemes in Topology I ( $N = 10$ ).

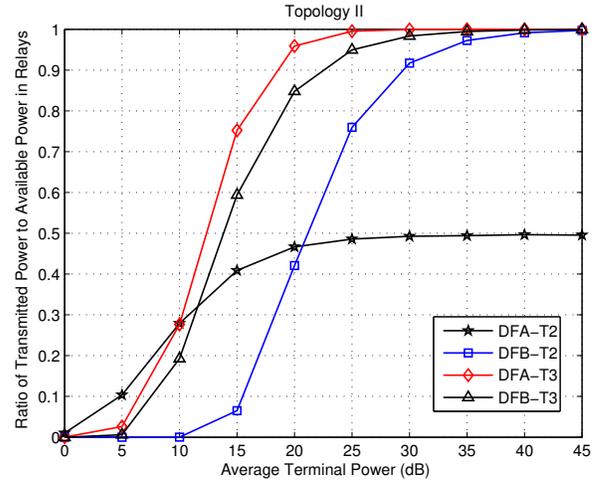


Fig. 7. Ratio of average transmitted power to available power in relays versus terminal power for all proposed schemes in Topology II ( $N = 10$ ).

(27)–(29), which can be rewritten as follows

$$p_{D_1}^{(i)} = P[(\text{SINR}_1^{(i)} \geq \tau_{\text{bic}}) \cap (\text{SNR}_2^{(i)} < \tau_{\text{bic}})] \\ = P\left[\left(\mathcal{X}_i \geq \tau_{\text{bic}} \mathcal{Y}_i + \frac{\tau_{\text{bic}}}{2}\right) \cap \left(\mathcal{Y}_i < \frac{\tau_{\text{bic}}}{2}\right)\right] \quad (30)$$

$$p_{D_2}^{(i)} = P[(\text{SINR}_2^{(i)} \geq \tau_{\text{bic}}) \cap (\text{SNR}_1^{(i)} < \tau_{\text{bic}})] \\ = P\left[\left(\mathcal{Y}_i \geq \tau_{\text{bic}} \mathcal{X}_i + \frac{\tau_{\text{bic}}}{2}\right) \cap \left(\mathcal{X}_i < \frac{\tau_{\text{bic}}}{2}\right)\right] \quad (31)$$

$$p_{D_{1,2}}^{(i)} = P[(\text{SINR}_1^{(i)} \geq \tau_{\text{bic}}) \cap (\text{SNR}_2^{(i)} \geq \tau_{\text{bic}})] \\ + P[(\text{SINR}_2^{(i)} \geq \tau_{\text{bic}}) \cap (\text{SNR}_1^{(i)} \geq \tau_{\text{bic}})] \\ = P\left[\left(\mathcal{X}_i \geq \tau_{\text{bic}} \mathcal{Y}_i + \frac{\tau_{\text{bic}}}{2}\right) \cap \left(\mathcal{Y}_i \geq \frac{\tau_{\text{bic}}}{2}\right)\right] \\ + P\left[\left(\mathcal{Y}_i \geq \tau_{\text{bic}} \mathcal{X}_i + \frac{\tau_{\text{bic}}}{2}\right) \cap \left(\mathcal{X}_i \geq \frac{\tau_{\text{bic}}}{2}\right)\right] \quad (32)$$

where we have also used (14)–(15) and their counterparts  $\text{SINR}_2^{(i)}$  and  $\text{SNR}_2^{(i)}$ . At this point, we recall that, since  $f_i \sim \mathcal{CN}(0, \sigma_{f_i}^2)$  and  $g_i \sim \mathcal{CN}(0, \sigma_{g_i}^2)$ , the random variables

$\mathcal{X}_i \triangleq \mathcal{P}_1 |f_i|^2$  and  $\mathcal{Y}_i \triangleq \mathcal{P}_2 |g_i|^2$  are independent and have exponential distributions with parameters  $L_1^{(i)} = 1/(\mathcal{P}_1 \sigma_{f_i}^2)$  and  $L_2^{(i)} = 1/(\mathcal{P}_2 \sigma_{g_i}^2)$ , respectively. Due to the symmetry of these probabilities, it can be easily shown that  $p_{\mathcal{D}_1}^{(i)} = f(\mathcal{P}_1, \mathcal{P}_2; \sigma_{f_i}^2, \sigma_{g_i}^2)$ ,  $p_{\mathcal{D}_2}^{(i)} = f(\mathcal{P}_2, \mathcal{P}_1; \sigma_{g_i}^2, \sigma_{f_i}^2)$ , and  $p_{\mathcal{D}_{1,2}}^{(i)} = g(\mathcal{P}_1, \mathcal{P}_2; \sigma_{f_i}^2, \sigma_{g_i}^2) + g(\mathcal{P}_2, \mathcal{P}_1; \sigma_{g_i}^2, \sigma_{f_i}^2)$ , where

$$f(\mathcal{P}_1, \mathcal{P}_2; \sigma_{f_i}^2, \sigma_{g_i}^2) \triangleq \int \int_{\mathcal{S}_1} \frac{1}{\mathcal{P}_1 \mathcal{P}_2 \sigma_{f_i}^2 \sigma_{g_i}^2} e^{-\frac{x}{\mathcal{P}_1 \sigma_{f_i}^2}} e^{-\frac{y}{\mathcal{P}_2 \sigma_{g_i}^2}} \cdot U(x) U(y) dx dy \quad (33)$$

$$g(\mathcal{P}_1, \mathcal{P}_2; \sigma_{f_i}^2, \sigma_{g_i}^2) \triangleq \int \int_{\mathcal{S}_2} \frac{1}{\mathcal{P}_1 \mathcal{P}_2 \sigma_{f_i}^2 \sigma_{g_i}^2} e^{-\frac{x}{\mathcal{P}_1 \sigma_{f_i}^2}} e^{-\frac{y}{\mathcal{P}_2 \sigma_{g_i}^2}} \cdot U(x) U(y) dx dy \quad (34)$$

with  $\mathcal{S}_1 \triangleq \{(x, y) \in \mathbb{R}^2 : x \geq \tau_{\text{bic}} y + \tau_{\text{bic}}/2, y < \tau_{\text{bic}}/2\}$  and  $\mathcal{S}_2 \triangleq \{(x, y) \in \mathbb{R}^2 : x \geq \tau_{\text{bic}} y + \tau_{\text{bic}}/2, y \geq \tau_{\text{bic}}/2\}$ . Calculation of the integrals in (33) and (34) leads to the following results:

$$f(\mathcal{P}_1, \mathcal{P}_2; \sigma_{f_i}^2, \sigma_{g_i}^2) = \frac{\sigma_{f_i}^2}{\sigma_{f_i}^2 + \sigma_{g_i}^2 \frac{\mathcal{P}_2 \tau_{\text{bic}}}{\mathcal{P}_1}} \left\{ \exp\left(-\frac{\tau_{\text{bic}}}{2 \mathcal{P}_1 \sigma_{f_i}^2}\right) - \exp\left[-\frac{\tau_{\text{bic}}}{2 \mathcal{P}_2 \sigma_{g_i}^2} - \frac{\tau_{\text{bic}}(\tau_{\text{bic}} + 1)}{2 \mathcal{P}_1 \sigma_{f_i}^2}\right] \right\} \quad (35)$$

$$g(\mathcal{P}_1, \mathcal{P}_2; \sigma_{f_i}^2, \sigma_{g_i}^2) = \frac{\sigma_{f_i}^2}{\sigma_{f_i}^2 + \sigma_{g_i}^2 \frac{\mathcal{P}_2 \tau_{\text{bic}}}{\mathcal{P}_1}} \cdot \exp\left[-\frac{\tau_{\text{bic}}}{2 \mathcal{P}_2 \sigma_{g_i}^2} - \frac{\tau_{\text{bic}}(\tau_{\text{bic}} + 1)}{2 \mathcal{P}_1 \sigma_{f_i}^2}\right] \quad (36)$$

from which readily follows that  $p_{\mathcal{B}_1}^{(i)} = k(\mathcal{P}_1, \mathcal{P}_2; \sigma_{f_i}^2, \sigma_{g_i}^2)$  and  $p_{\mathcal{B}_2}^{(i)} = k(\mathcal{P}_2, \mathcal{P}_1; \sigma_{g_i}^2, \sigma_{f_i}^2)$ , where

$$k(\mathcal{P}_1, \mathcal{P}_2; \sigma_{f_i}^2, \sigma_{g_i}^2) = \frac{\sigma_{f_i}^2}{\sigma_{f_i}^2 + \sigma_{g_i}^2 \frac{\mathcal{P}_2 \tau_{\text{bic}}}{\mathcal{P}_1}} \exp\left(-\frac{\tau_{\text{bic}}}{2 \mathcal{P}_1 \sigma_{f_i}^2}\right) + \frac{\sigma_{g_i}^2}{\sigma_{g_i}^2 + \sigma_{f_i}^2 \frac{\mathcal{P}_1 \tau_{\text{bic}}}{\mathcal{P}_2}} \exp\left[-\frac{\tau_{\text{bic}}}{2 \mathcal{P}_1 \sigma_{f_i}^2} - \frac{\tau_{\text{bic}}(\tau_{\text{bic}} + 1)}{2 \mathcal{P}_2 \sigma_{g_i}^2}\right]. \quad (37)$$

#### APPENDIX B PROOF OF THEOREM 1

To derive a lower bound on the SNR threshold  $\gamma_{\text{th}}$  over which the considered scheme loses diversity, let us consider the following ‘‘diversity’’ function  $G_d(\gamma) \triangleq -\frac{\log[\bar{P}_2^{\text{ub}}(e)]}{\log(\gamma)}$ , where  $\gamma = \mathcal{P}/N$  and  $\bar{P}_2^{\text{ub}}(e)$  is defined in (22). It should be observed that if one takes the limit of  $G_d(\gamma)$  as  $\gamma \rightarrow +\infty$ , one obtains the asymptotic diversity order  $G_d$  of the system, which turns out to be zero. Our aim is to study the behavior of  $G_d(\gamma)$  for (finite) large values of  $\gamma$ , i.e., we assume that  $1 \ll \gamma < +\infty$ . For such values of  $\gamma$ , we approximate  $p_{\mathcal{B}_1}^{(i)}$  with  $\bar{p}_{\mathcal{B}_1}^{(i)}$  given by (18), i.e.,  $p_{\mathcal{B}_1}^{(i)} \approx \bar{p}_{\mathcal{B}_1}^{(i)}$ . Moreover, for simplicity

of notation, we define the (positive) coefficients

$$a_0 \triangleq \left(1 - \frac{1}{|\mathcal{A}|^K}\right) \prod_{i=1}^N \left[1 - \bar{p}_{\mathcal{B}_1}^{(i)}\right] \quad (38)$$

$$a_{n_1} \triangleq G_{c,1}(n_1) \Phi(n_1; \bar{p}_{\mathcal{B}_1}^{(1)}, \bar{p}_{\mathcal{B}_1}^{(2)}, \dots, \bar{p}_{\mathcal{B}_1}^{(N)}) \quad (39)$$

$$a_L \triangleq \sum_{n_1=L}^N a_{n_1}. \quad (40)$$

It follows that  $G_d(\gamma) = \min\{L, N\} - \eta(\gamma)$ , with  $\eta(\gamma)$  given by (41) shown at the top of the next page, where the approximation holds for  $\gamma \gg 1$ . At this point, to calculate a lower bound  $\gamma_{\text{th}}^{\text{lb}}$  on  $\gamma_{\text{th}}$ , we have to study for which value of  $\gamma$  it results that  $\eta(\gamma) = \min\{L, N\}$ , whose solution is given by  $\gamma_{\text{th}}^{\text{lb}} = \left(\frac{a_{\min\{L, N\}}}{1 - a_0}\right)^{1/\min\{L, N\}}$ . Eq. (24) easily follows by substituting  $a_0$ ,  $a_L$ , and  $a_N$  in  $\gamma_{\text{th}}^{\text{lb}}$ , and observing that: (i) for  $N \geq L$ , the factor  $G_{c,1}(n_1)$  is independent of  $n_1$ , since  $\eta(n_1) = \min\{L, n_1\} = L$ , for  $n_1 \in \{L, L+1, \dots, N\}$ ; (ii) for  $L > N$ , the factor  $G_{c,1}(N)$  clearly depends only on  $N$ .

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$$\eta(\gamma) \triangleq \frac{\log \left[ a_0 \gamma^{\min\{L,N\}} + \sum_{n_1=1}^{\min\{L,N\}-1} a_{n_1} \gamma^{\min\{L,N\}-n_1} + a_{\min\{L,N\}} \right]}{\log(\gamma)} \approx \frac{\log \left[ a_0 \gamma^{\min\{L,N\}} + a_{\min\{L,N\}} \right]}{\log(\gamma)} \quad (41)$$

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